

Mechanics of Materials Lab manual

EMA 3702L

Spring 2015

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Attachments:

1. General Course Objective and Syllabus
2. Course elementary and fundamental knowledge
3. Report Writing and Error Analysis
4. Lab Procedures:
 - a. Experiment One, Tensile Stress Testing
 - b. Experiment Two, Beam Deflection and Bending
 - c. Experiment Three, Torsional Tests of Solid circular Bars
 - d. Experiment Four, Principal of Stress and Strains
 - e. Experiment Five, Elastic and Inelastic buckling of Columns
 - f. Experiment Number Six, Hardness Tests of Material surfaces.

I. COURSE OBJECTIVES:

To verify mechanics of materials theories on real mechanical specimens.

Student Learning Outcome: The student will demonstrate the ability to:

- a. Identify when theory applies and when theory is limited by simplifying assumptions
- b. Identify the reasons why actual measurements will differ from theoretical calculations.

To learn to run an experiment

Students learning outcomes: The students will demonstrate the ability to :

- a. Perform pre- laboratory calculations to estimate the experimental parameters, outcomes and limits
- b. Develop an organized and meaningful data sheets
- c. Use software tools to reduce and analysis the data
- d. Organize a team to share responsibilities for operating equipment ad collecting data.

To learn to use testing equipment and measurement instrumentations

Student learning outcomes: The student will demonstrate the ability to:

- a. Use laboratory equipment correctly and safely to perform all experiments.

To learn to write a laboratory Report:

Student learning outcome: The student will demonstrate the ability to:

- a. Write experimental objectives and procedure for practical experiment set up and design
- b. Present results in organized and clear manner
- c. Draw graphs and figures and summarize key findings
- d. Put together a complete professional report including tables of content, references and appendices.

II. Pre-Lab CALCULATIONS

For each experiment discussed in this Manual, a set of calculations that must be completed and questions that must be answered in order to prepare the collection of data in the lab. Examples of types of calculations you will make include:

- Determination of the maximum allowable load on a specimen.
- Estimation or prediction of experimental results.

- Determination of simple relationships between independent and dependent variables.
- A data sheet, as described in the next section of this Manual.

Your laboratory preparation work will be completed and turned in during the session that meets prior to the session when actual lab is performed. (Usually first 15 minutes of each practical lab session). This insures that you are prepared so that accurate and complete data is collected and that a high quality report can be written. Incomplete or inadequate laboratory work will results in a similarly inadequate report. **[Please be advised, if the preparation has not done in advance and sample problems were not solved, you will not be allowed to conduct the experiment at this time and will affect your lab grades].**

Late laboratory preparation work will NOT receive credits.

Pre-lab calculations may varies from semester to semester, so do your own work.

In –Lab quiz questions on the theory and set up of the lab will be conducted and graded as part of your lab report and preparations.

If possible, calculations should be set up as a template in a spreadsheet or other computer applications so that the changes could be easily made. This can save time and effort if the experimental setup is found to differ from the description in this manual.

III. DATA SHEETS:

A prepared paper or spreadsheet template data sheet is required as part of each pre-lab. A copy of your paper data sheet or a printout of the spreadsheet template must be handed in with your pre-lab.

It is essential that you put some thought into how to plan to collect data before you come to the laboratory session. You must design it carefully in order to help you to collect complete and accurate data. Remember that you are very unlikely to write a good report with bad or incomplete data. Items to consider:

- Your data sheet should include a sketch of your test setup including the gauge location and appropriate dimensions so that you can check the experiment is set up correctly before you start. (Usually you can get this sketch from Laboratory manual).
- How many columns of data you need? You will generally need to have two columns for each data variable and sometimes three so that scale factor and/or zero offset calculations can be made.
- Each column should be labeled so that you can remember where the data came from and what it relates to.
- The units of the data you are taking should be in your column labels.
- How many rows you need? The rows will needed to be labeled as well.

- Is there a location to put comments that relate the test setup, procedure or the collection of data?

If you plan to use a spreadsheet and enter data directly into a computer, you need to design the spreadsheet with the same consideration as above. You want to design different sheets for different part of experiment.

A sample data sheet is included in the section of this manual for laboratory #1.

IV. ACCURACY AND ERROR ANALYSIS:

The number of significant digits displayed is an indication of the accuracy of the number. With digital calculators or computer spreadsheets, it is easy to overstate the accuracy of a number. When taking measurements, the least reliable digit relative to decimal point determines the precision of your measurement. Data should not be recorded more precisely than warranted. For example, assume you are reading load of the analog dial where are tick marks every 10 pounds. If the needle is between 50 and 60 pound tick mark you could guess that the load is 54 pounds, but the 4 is not very reliable. Certainly you would not want to write down 54.3 pounds. It is somehow harder to determine a reliable with digital machines, and one would have to read the testing machine manual to determine the reliability of each range, but normally two or three significant digits is all that can be expected for the equipment used in this lab.

Finally when doing calculations, the answer cannot be more precise than the data and constants you begin with. For example if you know that the dimensions of a rectangular bar are 25.5 mm (3 significant figure) by 30.01 mm (4 significant figure) the area of specimen would be:

$$25.5 \times 30.01 = 765.255 \text{ mm}^2 \cong 765 \text{ mm}^2 \text{ (3 significant figure).}$$

From time to time you may want your spreadsheet computer or calculator to have more significant figure to avoid round-off and truncation error, but your final answer should be more precise than the original data.

Error analysis and error propagation as results of experimental data and mathematical relations shall be established and calculated. **[There will be a lecture class, normally the first session, a discussions of error analysis and propagations and method of calculations will be discussed]**

V. LABORATORY REPORTS:

The format and requirements for your laboratory reports will be set forth either in each laboratory handout or by guidelines listed below:

1. Objective of Lab Reports:

The primary objective for a lab report is to convey to the client (general public, other engineers, the professors, etc...) the information that the experimenter has collected. The report should be organized so that someone who is not familiar with the particular experiment or test set-up can understand:

- What assumptions were made,
- What procedures were followed,
- What data collected,
- What analysis was completed, and
- Which conclusions were made.

The fundamental purpose of your report is to clearly demonstrate to the professor and the instructor that you fully understand the theories, methods, variable, accuracy, data analysis involved in the particular lab.

Furthermore, engineering reports, shall be organized such that one clearly could follow your methods and procedures step by step to come to same data, calculations, and conclusions under the same circumstances.

2. Report Organization:

In general each individual lab may have its own requirements, but in general a complete report shall have at least the following:

TITLE PAGE, listing the experiment number and title, laboratory professor and instructor, laboratory sessions identification (date and time of the week), date of submittal, and report author(s).

TABLE OF CONTENT, indicating sections, sub-sections and page numbers
MAIN BODY

Objective of the experiment (or introduction to problem statements)

Procedure

Theoretical Estimation of Experimental Results (pre-lab description)

Experimental set-up and apparatus arrangements

Presentation of data, in a clean format (not raw data)

Method of analysis and sample calculations

Results of experiments and final analyzed data presentations

Conclusions (how the objective were satisfied? Was the accuracy adequate? Does the theory work? In detail explanations, etc...)

REFERENCES (if needed, but will usually at least contain this lab manual as reference)

TABULATED DATA, and detail calculations, on a clearly labeled data sheet

GRAPHS AND FIGURES (properly labeled and numbered)

APPENDICES

Appendix A- Raw Data (including the actual data sheet used in the lab)

Appendix B. – Supporting Calculations

Appendix C. –Data reduction

All pages should be numbered. Often numbering starts over in the appendices and numbering such as A-1, A-2,..., B-1, B-2, etc. is used.

3. Objective of Experiment and procedures

The objective of the Experiment and the procedure may be adapted from the objective of the experiment manual, be sure the manual is cited in the text, and listed in the references section of your report. Procedures should explain in the text configuration as well as the steps to follow to complete the experiment. Use of bullets or numbering system in this section is acceptable and often makes for clearer, more concise organization. Be sure and clearly describe if and how your procedure or the setup varied from that provided in the experiment manual.

4. Theoretical Estimation of Experiment Results:

Each lab has a theoretical component, usually summarized and performed as part of Pre-Lab assignment. The theory should be clearly explained, including the key components of the theory, equations used, and any assumptions there were made and used. You should include sample of all calculations and then a table to summarize estimates of key experimental results completed during the Pre-Lab assignment. For example a table such as below may be useful:

Table1: Estimated Strains on Beam #2

Gauge Number	Load (lbs.)	Estimated Strains (μ inch/inch)
1	P	0.02P
2	P	0.04P
3	P	-0.03P

Equations should be written using appropriate software (such as Microsoft word, or Microsoft Equation maker, or similar software). Simple equation may be created using a traditional keyboard. All equations should be indented and numbered. Equation numbers should occur at the same tab stop for all equations. As an example:

$$\beta = \frac{1/\theta}{\pi} \int_{-2}^{+5} x dx \quad (\text{Eq. 1})$$

$$\sigma = \frac{P}{A} \quad (\text{Eq. 2})$$

5. Results of Experiments

The experimental measurements should be summarized and explained in this section. This section usually will not include raw data, since that will be reserved for Appendix A. Often you will want to include tables and plots to show the summarized results. Be very careful to present numerical results with appropriately accurate significant figures. In charts and graphs, you should not curve fit a line through data points since errors (which is always present) cause the individual data points to be offset from actual phenomena being measured. A chart that presents a theoretical relationship may be presented by a continuous line if theory results in a continuous equation. An example of a complete table is shown here:

Table 2: Results From Experiment 2			
Actual Load (lbs) (measured)	Strain (μinch/inch) (Measured)	Stress (psi) (Calculated)	Predicated Strains (μinch/inch)
0	0	0	0
1020	35	6700	32
2200	72	13700	69
2350	106	20200	105

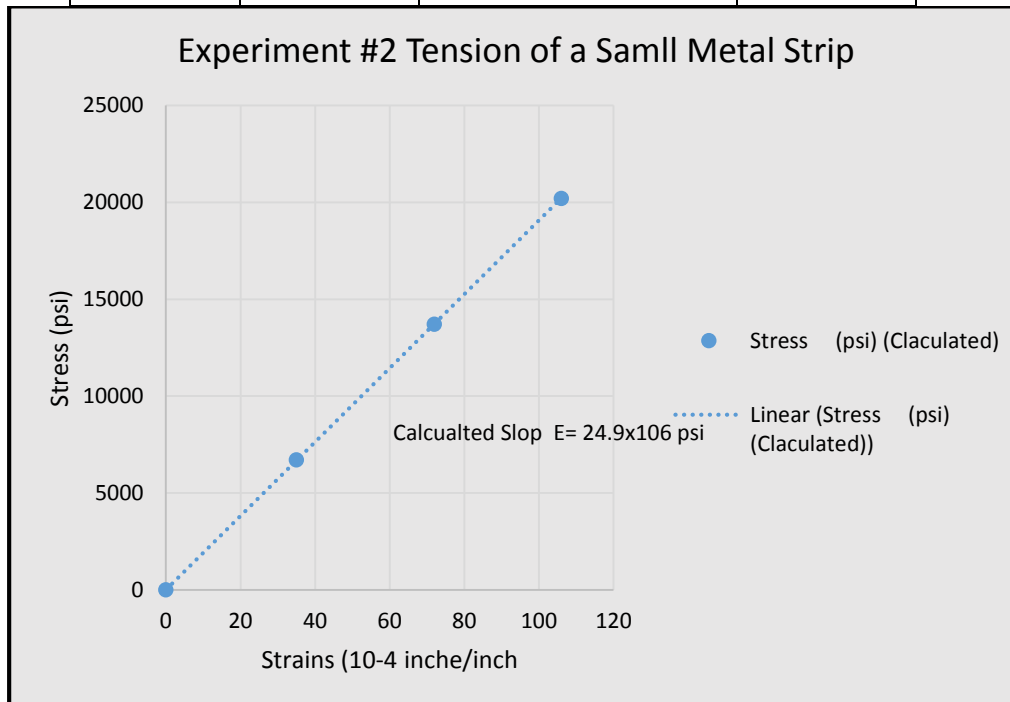


Figure 1. Stress –Strain relationship calculated from data taken in Experiment #2 on January 15, 2015 the curve was calculated using the least square function in Excel.

Note: If possible, you should present your data and prediction as plot for comparison. All plots for comparison should be the same scale. Present numerical comparisons in tables.

6. Conclusions

This section is for student to discuss the objective (found in the Manual) of the experiment were achieved and make observations and draw conclusions based on the comparison of the experimental data and the theoretical predictions Here are few examples of points that could be included:

- A qualification of difference between theoretical prediction and measured results.
- If the data and theoretical predictions are not the same, a discussion of reasons why this may have happened;
- Whether the theoretical model is appropriate for the real world specimen;
- Whether the test procedure is an appropriate method of validation of the theory;
- Difficulties that occurred in completing the experiment or data analysis.

In addition the conclusion should include at least one technical aspect of the experiment that the author understands clearly. This technical item could be related to the theory, or the assumptions that were made that would prevent the experiment from matching theory, or some functional aspects of the instrumentation, etc. If the report is a group report, the conclusion should contain one item clearly understood by each student in the group.

7. References

Any material, that is not written or derived by the lab report author(s), need to be referenced. The purpose of the citation is to provide the reader with information necessary to find item in library or internet.

8. Tables

Tables must include a table number, a title above the table, and labels at the top of each column that completely describes the content of the column, In some cases you may also need labels for the rows or subsections of the table. All labels should include units where appropriate. Since tables must be readable, you should not use a font smaller than 10 point. Tables may be presented in portrait or landscape mode.

9. Graphs

It is likely that you may use either excel or Matlab for drawing the graphs. It is very important that you understand the assumptions being made and limitations of the software

when you draw a graph. For example you should know that the difference between scatter graph and line graph in Excel.

In general data should be shown on the graph as markers without connecting lines. Theoretical predictions, or estimates of relationships developed from analysis of the data, should be shown as lines without markers. Generally a graph should be at least one half of a page so that you could fit all the relevant information on it and still use the a font that is 10 points or larger. A full page for each graph is normally recommended.

Do not fit curves to your data unless the curve being fitted is part of the theory being tested. If there is a theory being tested, the theory will provide an appropriate equation.

Graphs shall include a figure number with caption that give information about the data such as experiment number and date collected, a title at the top of the graph, the axis names and units, and an indication of which is the experiment data. You may use legend or may label points and lines. Example of a graph was given in preceding section.

10. Figures

Figures are any types of graphics that serve to support the laboratory report. Drawings of the test apparatus or specimens, as well as the graphs developed to display data are all figures. Drawings of the test apparatus and specimens are simple way to show dimensions, orientations, and loading configuration of each experiment. Figures of test set may be copied from laboratory manual if manual is properly cited, both in the figure and Appendices. Digital cameras are excellent way to acquire images to be used as figures in your report. Figures must include a figure number and a caption below the figure. The caption should clearly explain what is in the figure. If a figure is not drawn by the authors, but instead taken from another source, it must contain a citation to the reference.

Theories

A. The Stress - Strain Relationship for Solids

This report first gives the definition of stress and strain, and then gives their relationship in elastic region. The relationship between stress and strain includes uni-axial stress state, pure shear stress state, bi-axial stress state (plane stress), biaxial strain (plane strain) state and tri-axial stress state cases. For uni-axial case, the general stress – strain curve for ductile materials and brittle materials are provided. About ductile materials, some important terms are also introduced.

Some definitions

Normal stress (axial stress)

Normal stress (axial stress) results when a member is subject to an axial load applied through the centric of the cross section. It can be defined in terms of forces applied to a uniform rod [1], numerically it is the ratio of the perpendicular force applied to a specimen divided by its original cross sectional area [2]:

$$\sigma = \frac{F}{A}$$

Figure 1& 2 show the diagram.

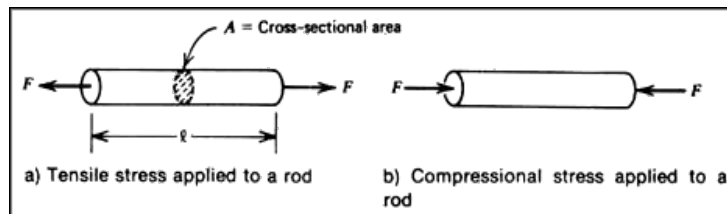


Figure 1

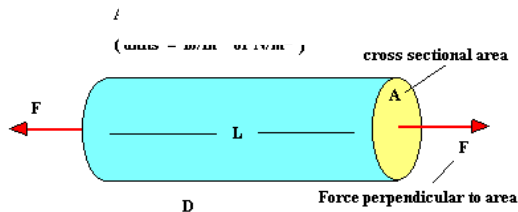


Figure 2

Normal stress includes tensile and compressive stress, the conventional sign for normal stresses are: tensile stresses are positive (+), compressive stresses are negative (-). The unit of stress is pascals (Pa) ($1\text{Pa}=1\text{N/m}^2$). Because in practice 1Pa is too small, Mpa , N/mm^2 are usually used. In America, English unit psi (or ksi or Msi) is often used: $1\text{psi} \approx 7000\text{Pa}$.

Normal strain

Normal strain is defined as the ratio of change in length due to deformation to the original length of the specimen [2]:

$$\epsilon = \frac{\Delta L}{L_0}$$

Figure 3 shows the diagram.

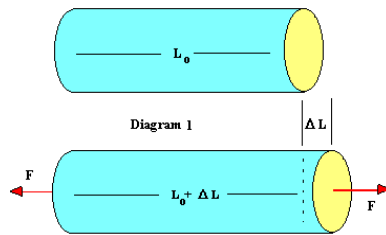


Figure 3

Strain is unitless, but often units of m/m (or mm/mm) are used. In America, inch/inch is often used.

Shear stress

Shear stress is defined in terms of a couple that tends to deform a joining member (Figure 4) [1]. It is used in cases where purely shear force is applied to a specimen, the formula for calculation and units remain the same as tensile stress (Figure 5):

$$\tau = \frac{F}{A_0}$$

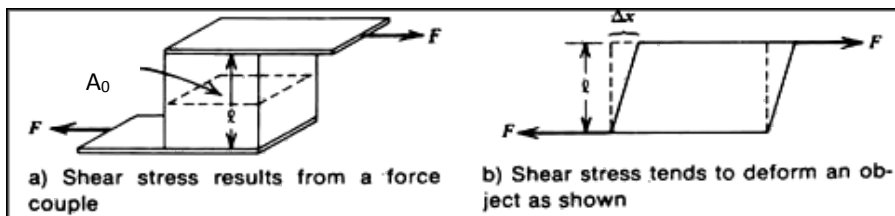


Figure 4

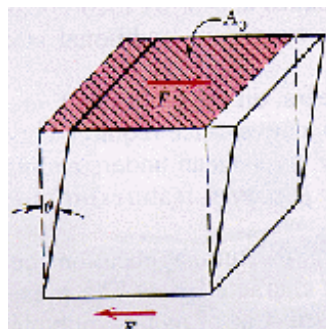


Figure 5

The unit of shear stress is same as normal stress.

Shear strain

Shear strain is defined as the tangent of the angle theta, and, in essence, determines to what extent the plane was displaced (Figure 5) [2]:

$$\gamma = \tan \theta = \frac{\Delta x}{l}$$

Same as normal strain, shear strain is unitless.

True stress and true strain

In practical, true stress and true strain are sometime used. Similar to the definitions of stress and strain mentioned above, but instead of original cross sectional area, true stress and true strain are based upon instantaneous values of cross sectional area and gage length.

Poisson's ratio

Poisson's ratio is the ratio of the lateral to axial strains [2] (Figure 6):

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = \frac{\epsilon_y}{\epsilon_z}$$

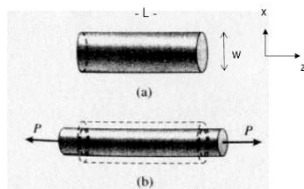


Figure 6

Theoretically, isotropic materials will have a value for Poisson's ratio of 0.25. The maximum value of n is 0.5. Most metals exhibit value between 0.25 and 0.35.

Stress-strain Curves

In practice, through the tension or compress testing of a material, we can get the stress-strain curves of the material. Figure 7 and Figure 8 show the general stress-strain curves of general ductile material and brittle material.

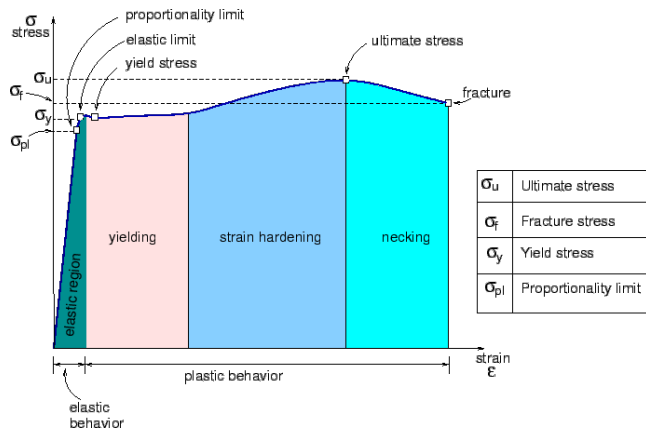


Figure 7

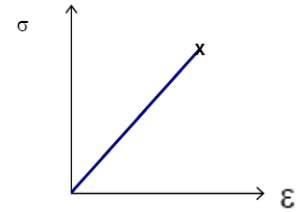
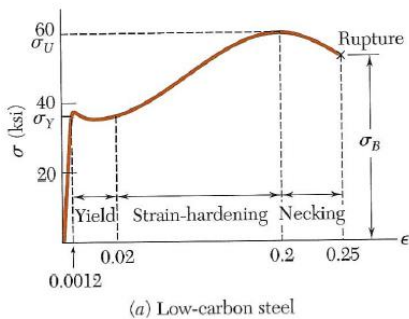


Figure 8

As examples, Figure 9, 10, and 11 show stress-Strain curves of low-carbon steel, Aluminum and glass, respectively [3]. Low-carbon steel and Aluminum belong to ductile material, whereas glass is a kind of typical brittle material.



ε

Figure 9

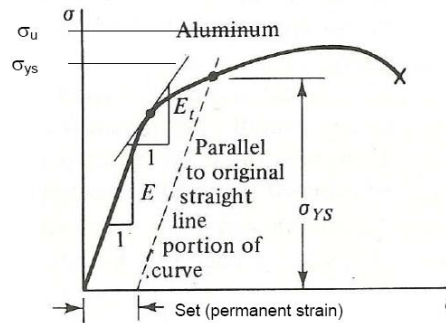


Figure 10

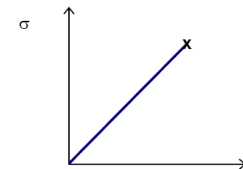


Figure 11

Yield strength is a very important parameter for a material. For any material, yield strength is defined the maximum stress that can be applied without exceeding a specified value of permanent strain (typically 0.2% = .002 in/in). Precision elastic limit or micro-yield strength is another parameter to define the property of a material; it is defined as the maximum stress that can be applied without exceeding a permanent strain of 1 ppm or 0.0001% [3].

For general ductile material, there are two regions in the stress-Strain curve: elastic region and plastic region (Figure 12). Within the elastic region, there exists a linear relationship between stress and strain, but in plastic region, their relationship is non-linear.

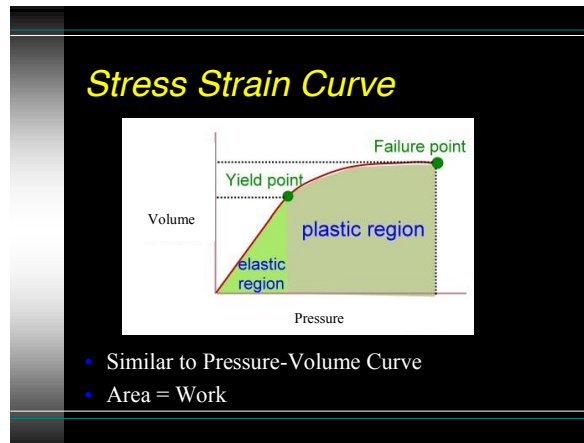


Figure 12

Uni-axial Stress State Elastic analysis

Figure 13 shows uni-axial stress state:



Figure 13

Stresses on inclined planes

Stresses on inclined planes (Figure 14) are expressed in following [4]:

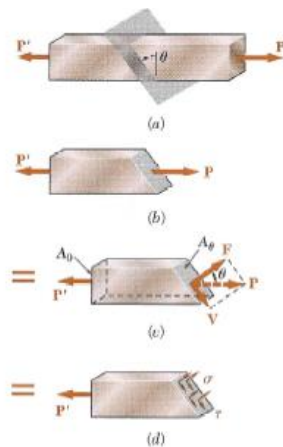


Figure 14

$$\sigma = \frac{F}{A_\theta} = \frac{P \cos \theta}{\frac{A_o}{\cos \theta}} = \frac{P}{2A_o} (1 + \cos 2\theta)$$

$$\tau = \frac{V}{A_\theta} = \frac{P \sin \theta}{\frac{A_o}{\cos \theta}} = \frac{P}{2A_o} \sin 2\theta$$

Stress-strain relationship

For materials at relatively low levels, normal stress and strain are proportional through:

$$\sigma = E\varepsilon$$

The constant E is called Young's modulus.

Thermal strain

Strain caused by temperature changes is called thermal strain (Figure 15).

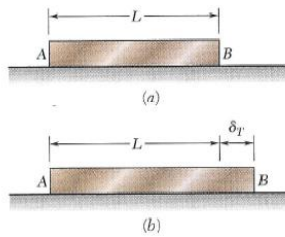


Figure 15

It can be calculated by the following formula:

$$\varepsilon_T = \alpha\Delta T$$

α is a material characteristic called the coefficient of thermal expansion. Strain caused by temperature changes and strain caused by applied loads are essentially independent. Therefore, the total amount of strain may be expressed as follows:

$$\begin{aligned} \varepsilon_{\text{total}} &= \varepsilon_{\sigma} + \varepsilon_T \\ \varepsilon_{\text{total}} &= \frac{\sigma}{E} + \alpha\Delta T \end{aligned}$$

Pure shear stress state elastic analysis

As a special case of biaxial stress state, pure shear stress state is shown in Figure 16.

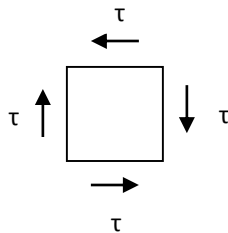


Figure 16

Stress-strain relationship

Shear stress and strain are related in a similar manner as normal stress and strain, but with a different constant of proportionality:

$$\tau = G\gamma$$

The constant G is called the shear modulus and relates the shear stress and strain in the elastic region. For linear, isotropic materials, E and G are related as [3]:

$$G = \frac{E}{2(1+\nu)}$$

Bi-axial stress state elastic analysis

(1) Plane stress state

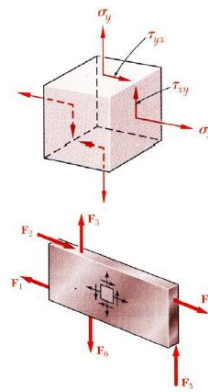


Figure 17

For linear, isotropic material, plane stress state assumes that [5]:

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \quad \gamma_{xz} = \gamma_{yz} = 0,$$

State of plane stress occurs in a thin plate subjected to forces acting in the mid-plane of the plate. State of plane stress also occurs on the free surface of a structural element or machine component, i.e., at any point of the surface not subjected to an external force (Figure 17).

Transformation of stresses

Figure 18 schematically shows two elements with angle θ between them.

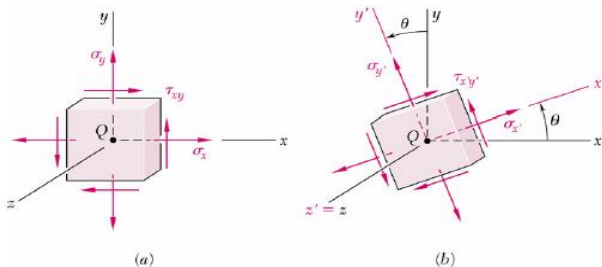


Figure 18

The transformation of stresses between the two planes is [6]:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Mohr's circle for plane stress state

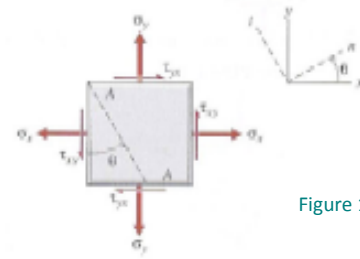
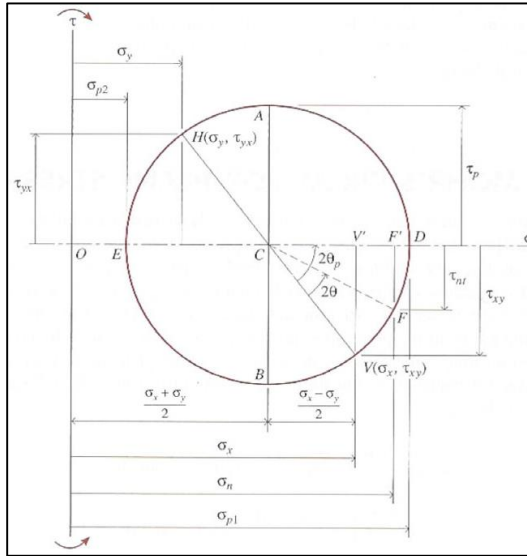


Figure 19

Figure 19 is the Mohr's circle for plane stress state. From the Mohr's circle, any inclined plane's stresses can be numerically obtained. For example, the coordinates of the two points H, V are the stresses of an inclined plane with inclined angle θ . The instruction to make the Mohr's circle is [4]:

1. Determine the point on the body in which the principal stresses are to be determined.
2. Treating the load cases independently and calculated the stresses for the point chosen.
3. Choose a set of x-y reference axes and draw a square element centered on the axes.
4. Identify the stresses σ_x , σ_y , and $\tau_{xy} = \tau_{yx}$ and list them with the proper sign.
5. Draw a set of $\sigma - \tau$ coordinate axes with σ being positive to the right and τ being positive in the upward direction. Choose an appropriate scale for the each axis.
6. Using the rules on the previous page, plot the stresses on the x face of the element in this coordinate system (point V). Repeat the process for the y face (point H).
7. Draw a line between the two point V and H. The point where this line crosses the σ axis establishes the center of the circle.
8. Draw the complete circle.
9. The line from the center of the circle to point V identifies the x axis or reference axis for angle measurements (i.e. $\theta = 0$).

Note: The angle between the reference axis and the σ axis is equal to $2\theta_p$.

Principal Stresses and principal planes

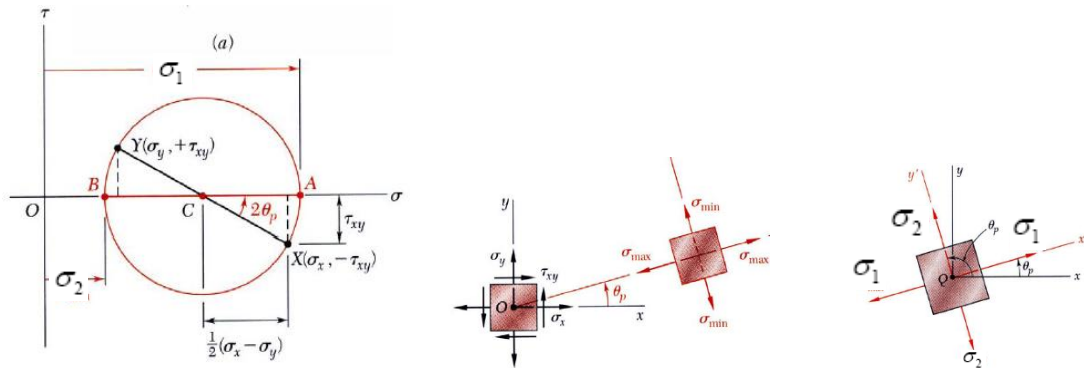


Figure 20

Principal Stresses σ_1, σ_2 (shown in Figure 20) are the stresses that act on an inclined planes where shear stresses = 0. From Mohr's circle, principal Stresses can be obtained [4]:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The inclined planes on which principal stresses act are called principal planes. The angle between principal planes and reference plane is:

$$2\theta_p = \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum Shearing Stress:

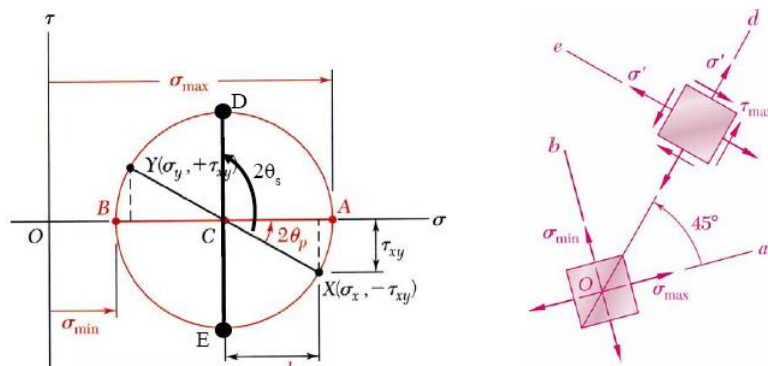


Figure 21

From the Mohr's circle, we can get the maximum shear stress (Figure 21), which will be:

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

The angle of the plane where maximum shear stress occurs is:

$$2\theta_s = \tan^{-1} \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note: On Mohr's circle, the planes of maximum shearing stress (points D and E) are at 90° to the principal planes (points A and B), but on the element, the planes of maximum shearing stress are at 45° to the principal planes.

Stress-strain relationship

With plane stress assumption, for isotropic material, in elastic region, the stress-strain relationship should be [5]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Note: $\varepsilon_z = \frac{1}{E}(-\nu)(\sigma_x + \sigma_y)$

Where, E and ν are the Young's modulus and Poisson's ratio of the material.

(2) Plane strain state

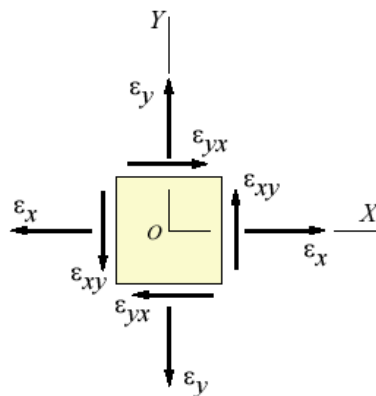


Figure 22

For linear, isotropic material, plane stress state assumes that [7] (Figure 22):

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0 \quad \tau_{xz} = \tau_{yz} = 0$$

Coordinate Transformation

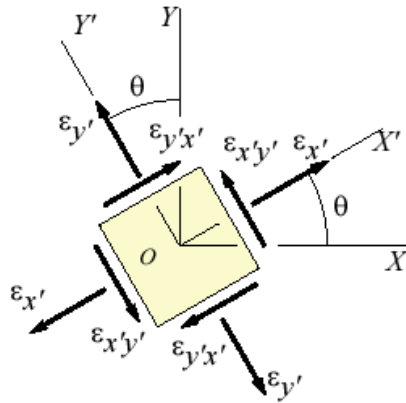


Figure 23

The transformation of strains with respect to the reference $\{x,y,z\}$ coordinates to the strains with respect to $\{x',y',z'\}$ (Figure 23) is performed via the equations [7]:

$$\begin{cases} \epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \epsilon_{xy} \sin 2\theta \\ \epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \epsilon_{xy} \sin 2\theta \\ \epsilon_{x'y'} = -\frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta + \epsilon_{xy} \cos 2\theta \end{cases}$$

Mohr's circle for plane strain state

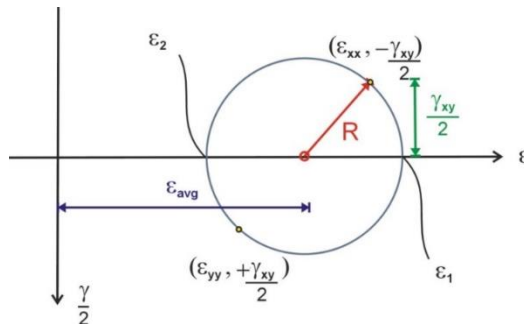


Figure 24

Figure 24 is the Mohr's circle for plane strain state. The method to construct the circle is similar to that construct Mohr's circle for plane stress state.

Principal Strains and the directions

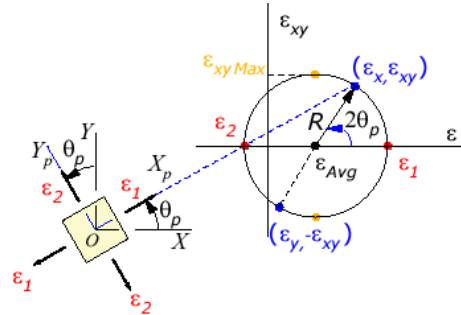
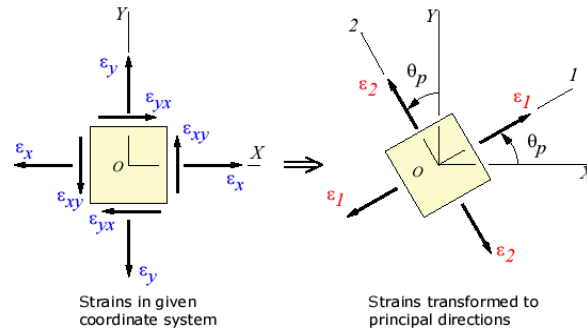


Figure 25

Same as principal stresses, principal strains ϵ_1, ϵ_2 (shown in Figure 25) are the strains that occur on an inclined planes where shear strains = 0. From the Mohr's circle for plain strain, we can get [7]:

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \epsilon_{xy}^2}$$

The direction of ϵ_1, ϵ_2 are shown in Figure 24, too. The angle can be calculated by:

$$\tan 2\theta_p = \frac{2\epsilon_{xy}}{\epsilon_x - \epsilon_y}$$

Maximum shear strain and the directions

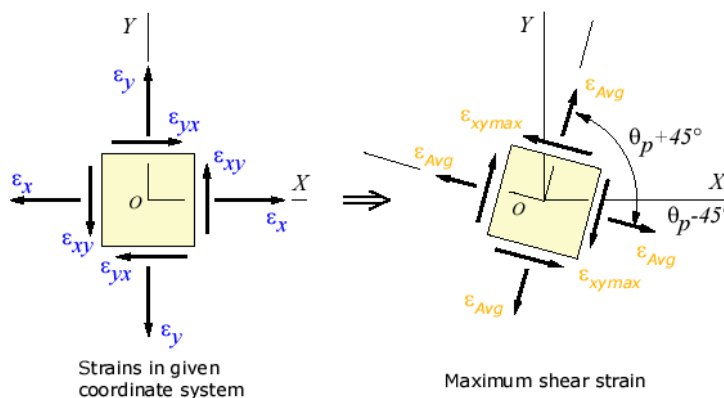


Figure 26

The maximum shear strain is [7]:

$$\varepsilon_{\max} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^2} = \frac{\varepsilon_1 - \varepsilon_2}{2}$$

The angle between the plane that maximum shear strain occurs and reference plane is:

$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{2\varepsilon_{xy}}$$

$$\Rightarrow \theta_s = \theta_p \pm 45^\circ$$

Stress-strain relationship

With plane stress assumption, for isotropic material, in elastic region, the stress-strain relationship should be [5]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{Bmatrix}$$

Note: $\sigma_z = \frac{E}{1+\nu} \left[\frac{\nu}{1-2\nu} (\varepsilon_x + \varepsilon_y) \right]$

Tri-axial stress state elastic analysis

In general, at a point, 3 normal stresses may act on faces of the cube, as well as, 6 components of shear stress (Figure 27). For the 6 shear stresses, only 3 of them are independent, because:

$$\begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{xz} &= \tau_{zx} \\ \tau_{zy} &= \tau_{yz} \end{aligned}$$

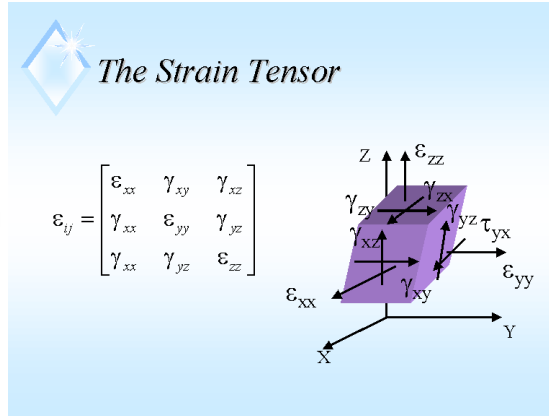
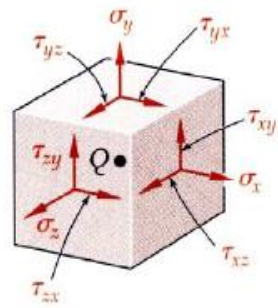


Figure 27

The strain components are shown in Figure 27, too.

The stress on an inclined plane

To simplify the case, choose a set of reference coordinates (Figure 28) coincident to the principle stresses, and then the stresses σ_n and τ_n in any inclined plane with normal direction (l, m, n) can be calculated by following equations [9]:

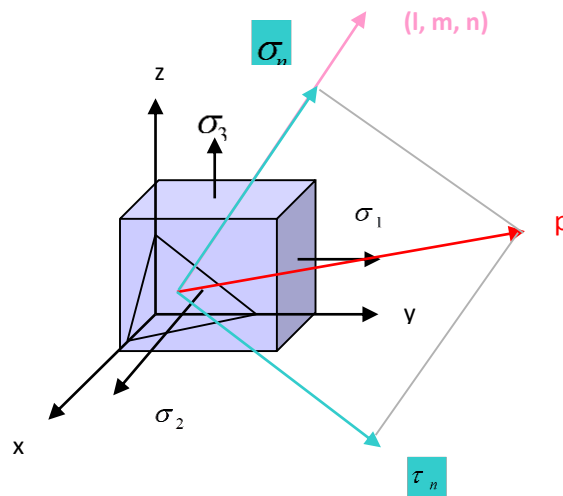


Figure 28

$$\left(\sigma_n + \frac{\sigma_2 + \sigma_3}{2}\right)^2 + \tau_n^2 = \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2 + l^2(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)$$

$$\left(\sigma_n + \frac{\sigma_3 + \sigma_1}{2}\right)^2 + \tau_n^2 = \left(\frac{\sigma_3 - \sigma_1}{2}\right)^2 + m^2(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)$$

$$\left(\sigma_n + \frac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau_n^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + n^2(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)$$

Mohr's circle

Figure 29 is the Mohr's circle for general tri-axial stress state. The 3 equations shown above that calculate the stresses on an inclined plane express 3 circles. The 3 circles intersect in point D, which should be located within the 3 reference circles. The value of coordinates of D is the stresses of the inclined plane.

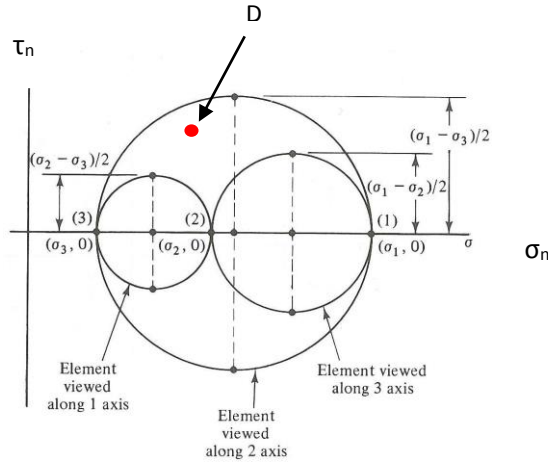


Figure 29

Stress-strain relationship

In general tri-axial stress state, for isotropic material and in elastic region, if taking into account thermal effect, the stress-strain relationship should be ^[10]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} + \frac{E\alpha\Delta T}{1-2\nu} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

1. Bending of a Beam