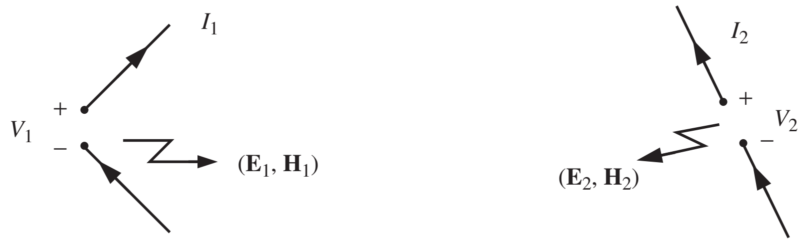

FIGURE 1-9 ■ Reaction between two antennas.

which is identical to that for a two-port network in circuit theory. Reciprocity and the reaction theorem will now prove useful in determining the elements Z_{ij} of the impedance matrix. These elements can be easily determined by shorting or open-circuiting the antennas one at a time. Setting $I_2 = 0$, gives

$$Z_{21} = \frac{V_2^{(1)}}{I_1}$$

and by referring to (1.126) we may express Z_{21} as

$$Z_{21} = -\frac{\langle 1, 2\rangle}{I_1 I_2}. \tag{1.129}$$

By invoking the reciprocity theorem (1.123), we also have $Z_{12} = Z_{21}$ and in general

$$Z_{ij} = -\frac{\langle j, i\rangle}{I_i I_j}. \tag{1.130}$$

This expression is valid for computing the self-impedance elements Z_{ii} as well and is useful in numerical simulations of antenna and scattering problems.

1.12 APPROXIMATE BOUNDARY CONDITIONS

In Section 1.4, we discussed the boundary conditions that must be imposed on material interfaces. These are the usual natural or exact boundary conditions. However, in many cases, it is possible to employ approximate boundary conditions that effectively account for the presence of some inhomogeneous interface, a material coating on a conductor, or a dielectric layer without actually having to include their geometry explicitly in the analysis.

1.12.1 Impedance Boundary Conditions

The most common approximate boundary condition (ABC) is the impedance boundary condition attributed to Leontovich (1948), which often carries his name in the literature. It can be derived by considering the simple problem of a plane wave incidence on a material half space. Choosing the interface to be the plane $y = 0$ with the y axis directed out of the half space, the Leontovich impedance boundary condition takes the form

$$E_z = -\eta Z_o H_x, \qquad E_x = \eta Z_o H_z \tag{1.131}$$

where $Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$, and η is a function of the material properties of the half space. These conditions are applied at $y = 0^+$ (just above the interface) and can be combined to yield the vector form

$$\hat{n} \times (\hat{n} \times \mathbf{E}) = -\eta Z_o \hat{n} \times \mathbf{H} \tag{1.132}$$

where $\hat{n}$ is the unit vector normal in the outward direction (see Figure 1-10). As can be seen, the form of the impedance boundary condition is independent of the geometry of the interface or the boundary where it is enforced and is thus applicable to planar as well as curved surfaces. Further, it can be generalized to the case of anisotropic material surfaces by writing it as

$$\hat{n} \times (\hat{n} \times \mathbf{E}) = -Z_o \overline{\overline{\eta}} \cdot \hat{n} \times \mathbf{H} \tag{1.133}$$

where $\overline{\overline{\eta}}$ is a tensor (a 2×2 matrix).

One way to derive the appropriate normalized impedance parameter η is to demand that the equivalent impedance surface satisfying the condition (1.132) reproduces the same reflected field. In doing so, for the planar dielectric interface we readily find that

$$\eta = \sqrt{\frac{\mu_r}{\epsilon_r}} \tag{1.134}$$

and for this choice of η the condition (1.132) becomes an approximation for simulating curved dielectric boundaries (see Figure 1-10b) provided (Senior, 1960)

$$\left| Im(\sqrt{\epsilon_r \mu_r}) \right| k_o \rho_i \gg 1 \tag{1.135}$$

where ρ_i are the principal radii of curvature associated with the surface. This ensures that the material is sufficiently lossy so that the fields penetrating the surface do not reemerge at some other point.

For the coated conductor in Figure 1-10c, the value of η is generally chosen to be the actual impedance of the corresponding planar structure illuminated by a plane wave, typically at normal incidence. Accordingly, for a homogeneous coating of thickness τ (Harrington and Mautz, 1975)

$$\eta = j\sqrt{\frac{\mu_r}{\epsilon_r}} \tan(k_o \sqrt{\epsilon_r \mu_r} \tau) \tag{1.136}$$

and we can readily compute the corresponding impedance for multilayer coatings. However, as can be expected, the accuracy of the proposed impedance boundary condition deteriorates for oblique angles of incidences, requiring that τ be kept small with respect to the wavelength to achieve resonable accuracies.

Provided the material parameters change slowly from one point of the simulated surface to another, the impedance boundary condition (1.132) is still applicable. In this case, the normalized surface impedance for the coating is computed from (1.136) with the material parameters now being functions of the location on the surface. For

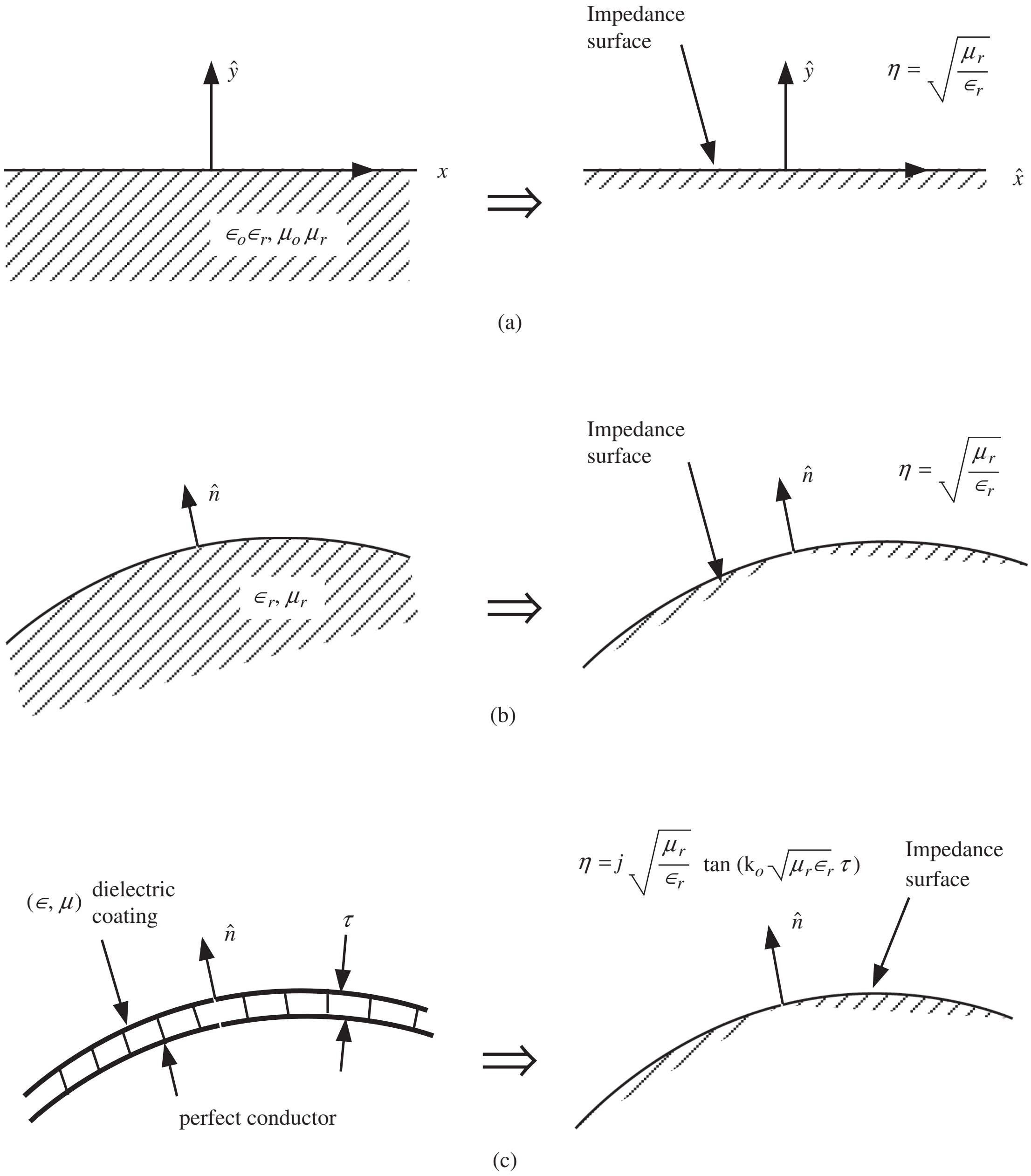

FIGURE 1-10 ■ Simulation of dielectric boundaries and coatings with equivalent impenetrable impedance surfaces. (Reference FEM book by Volakis et al.)

a planar interface, if ϵ_r and μ_r vary with respect to y, Rytov (1940) has shown that

$$\eta = \sqrt{\frac{\mu_r}{\epsilon_r}}\left\{1 + \frac{1}{2jk_oN}\frac{\partial}{\partial y}\ln(Z_oN) + O(N^{-2})\right\} \tag{1.137}$$

where $N = \sqrt{\mu_r\epsilon_r}$ is the refractive index, and the derivative is evaluated at the surface.

1.12.2 Resistive and Conductive Sheet Transition Conditions

For certain applications, it is desirable to replace a thin dielectric layer with an equivalent model in an effort to simplify the analysis. To illustrate this idea, let us consider a thin dielectric slab of thickness τ as shown in Figure 1-11. The slab has a conductivity σ, and it will thus support a current density given by (see (1.36))

$$\mathbf{J} = \sigma \mathbf{E} \tag{1.138}$$

where $\mathbf{E}$ denotes the field within the slab. However, since $\tau \ll \lambda$, we may replace $\mathbf{J}$ by an equivalent sheet current (having units in A/m)

$$\mathbf{J}_s = \tau \mathbf{J} \tag{1.139}$$

and thus from (1.138)

$$\mathbf{E} = \mathbf{J}_s/\sigma\tau = Z_o R_e \mathbf{J}_s. \tag{1.140}$$

This condition is a mathematical definition for a *resistive* sheet supporting a sheet current $\mathbf{J}_s$. The parameter $Z_o R_e$ is referred to as the *resistivity* of the sheet and is measured in Ω/square (e.g., Ω/cm^2 or Ω/m^2) (Senior and Volakis, 1995).

In deriving (1.140) it has been assumed that $\mathbf{E}$ is tangential to the layer or sheet, and therefore a more precise definition of the condition is

$$\hat{n} \times (\hat{n} \times \mathbf{E}) = -Z_o R_e \mathbf{J}_s \tag{1.141}$$

where $\hat{n}$ denotes the upper unit normal to the sheet. Further, it is desirable to work with field quantities that are measured outside the layer or sheet, and since $\hat{n} \times \mathbf{E}$ is continuous across the layer we may rewrite (1.141) as

$$\hat{n} \times [\hat{n} \times (\mathbf{E}^+ + \mathbf{E}^-)] = -2Z_o R_e \mathbf{J}_s \tag{1.142a}$$

$$\hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0 \tag{1.142b}$$

The superscripts $\pm$ denote the fields above and below the sheet or layer, and it was necessary to introduce (1.142b) to maintain the equivalence of (1.142) with (1.141). Alternatively, by employing the natural boundary condition (1.60), (1.142) can be rewritten as

$$\hat{n} \times [\hat{n} \times (\mathbf{E}^+ + \mathbf{E}^-)] = -2Z_o R_e\, \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) \tag{1.143a}$$

$$\hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0 \tag{1.143b}$$

By allowing $\hat{n}$ to be other than constant, these can be employed for the simulation of curved layers, provided again there is sufficient loss in the layer to suppress field repenetration from one layer location to another.

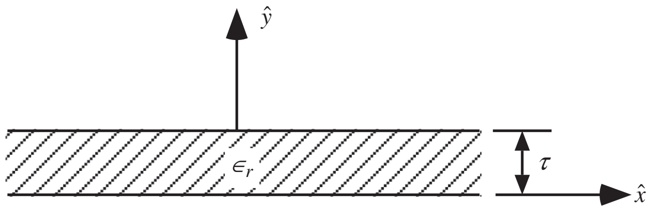

FIGURE 1-11 A thin sheet of dielectric material.

The dual to (1.143) are

$$\begin{aligned} \hat{n} \times [\hat{n} \times (\mathbf{H}^+ + \mathbf{H}^-)] &= 2Y_o R_m \, \hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) \\ \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) &= 0 \end{aligned} \tag{1.144}$$

and these define a *conductive* sheet capable of supporting a magnetic current $\mathbf{M}_s = -\hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-)$. The parameter, $Y_o R_m$, is now referred to as the *conductivity* (Senior and Volakis, 1995) of the magnetic sheet measured in Siemens/square. The utility of this sheet is not yet apparent but it will be shown to be essential for a sheet simulation of dielectric layers with nontrivial permeability. Also, it has been shown (Senior, 1985) that a special combination of coincident electric and magnetic current sheets is equivalent to an impenetrable impedance sheet. This equivalence holds when we set

$$R_e = \frac{\eta}{2}, \qquad R_m = \frac{1}{2\eta} \tag{1.145}$$

implying $4R_e R_m = 1$, where η is the normalized impedance of the sheet. Because co-planar electric and magnetic currents are independent of each other, (1.145) is important in simplifying the analysis with flat impedance surfaces.

Let us now consider a dielectric layer having a relative permittivity ϵ_r and thickness τ such that $k_o \tau \ll 1$ as illustrated in Figure 1-11. Based on the volume equivalence theorem, this layer can be replaced by the equivalent polarization currents

$$\begin{aligned} J_x &= jk_o Y_o(\epsilon_r - 1)E_x \\ J_y &= jk_o Y_o(\epsilon_r - 1)E_y \\ J_z &= jk_o Y_o(\epsilon_r - 1)E_z. \end{aligned} \tag{1.146}$$

On the assumption of $k_o \tau \ll 1$, the J_y component may be neglected, and the current densities $J_{x,z}$ can then be replaced by the equivalent sheet currents

$$J_{sx} = \tau J_x, \qquad J_{sz} = \tau J_z. \tag{1.147}$$

From (1.140), it now follows that

$$E_x = Z_o R_e J_{sx}, \qquad E_z = Z_o R_e J_{sz} \tag{1.148}$$

with

$$R_e = \frac{-j}{k_o \tau (\epsilon_r - 1)}. \tag{1.149}$$

Equation (1.148) are clearly identical to (1.140) except that R_e is now complex. Coordinate independent transition conditions for the dielectric layer are thus given by (1.141), (1.142), or (1.143) with the new definition for R_e.

When the dielectric slab is associated with nonunity μ_r, (1.143) must be complemented with a conductive sheet defined by (1.144) and in accordance with the volume equivalence theorem the normalized conductivity R_m is given by (Senior and Volakis, 1995)

$$R_m = \frac{-j}{k_o \tau (\mu_r - 1)}. \tag{1.150}$$

Thus, possible sheet transition conditions for a thin ferrite layer are

$$\begin{aligned} \hat{n} \times [\hat{n} \times (\mathbf{E}^+ + \mathbf{E}^-)] &= -2Z_o R_e \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) \\ \hat{n} \times [\hat{n} \times (\mathbf{H}^+ + \mathbf{H}^-)] &= +2Y_o R_m \hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) \end{aligned} \tag{1.151}$$

where R_e and R_m are defined in (1.149) and (1.150), and $\hat{n}$ denotes the upward unit normal to the layer. More accurate, higher-order transition and impedance conditions can also be derived. These involve higher-order derivatives of the field above and below the sheet or impedance surface and can permit modeling of thicker or higher contrast material layers and coatings (see, e.g., Senior and Volakis, 1987, 1991).

PROBLEMS

1. A chiral medium has the constitutive relations

$$\mathbf{D} = \epsilon \mathbf{E} - j\chi \mathbf{B}, \quad \mathbf{H} = -j\chi \mathbf{E} + \frac{\mathbf{B}}{\mu}$$

where χ is the chirality parameter.

(**a**) Show that the vector wave equation for this medium takes the form (see (1.109) and (1.110))

$$\nabla \times \nabla \times \mathbf{F} - k^2 \mathbf{F} + \mathbf{V} = 0$$

where $\mathbf{F} = \mathbf{E}, \mathbf{H}, \mathbf{D}$, or $\mathbf{B}$, and $\mathbf{V}$ is a vector to be found from your solution.

(**b**) Assume now a circularly polarized plane wave (RCP or LCP) propagating along the z-direction. Find the propagation constants k_{RCP} and k_{LCP} so that the wave equation found in (a) is satisfied.

(**c**) If a linearly polarized plane wave is incident upon a chiral interface as shown in Figure 1.P1, find the reflected and transmitted fields (by enforcing tangential field continuity at the interface).

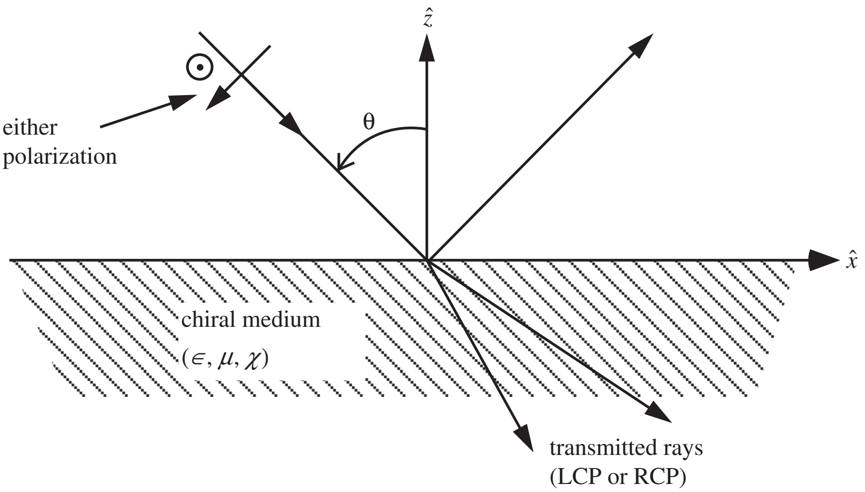

FIGURE 1.P1 Plane wave incident on a chiral interface.