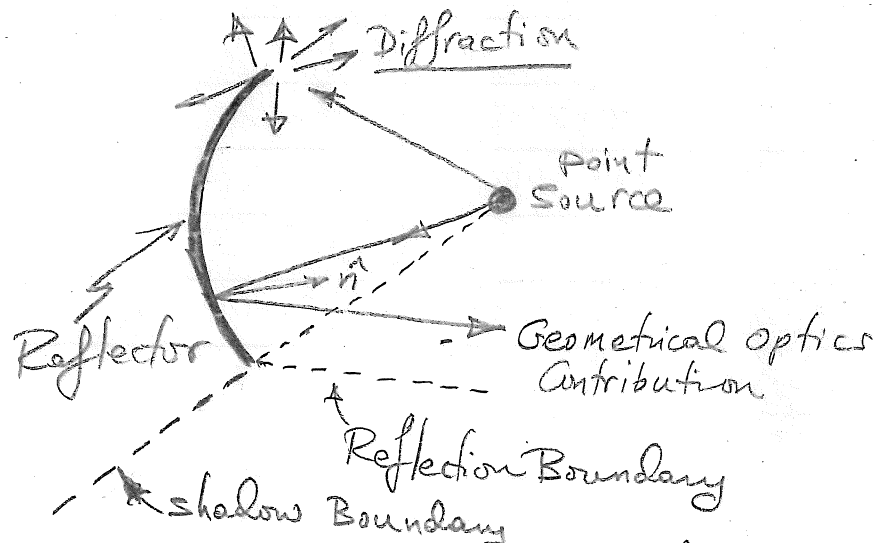
The Geometrical Uniform Theory of Diffraction & its Uniform Extensions

- It Extends Geometrical Optics Theory to Account for Diffraction effects

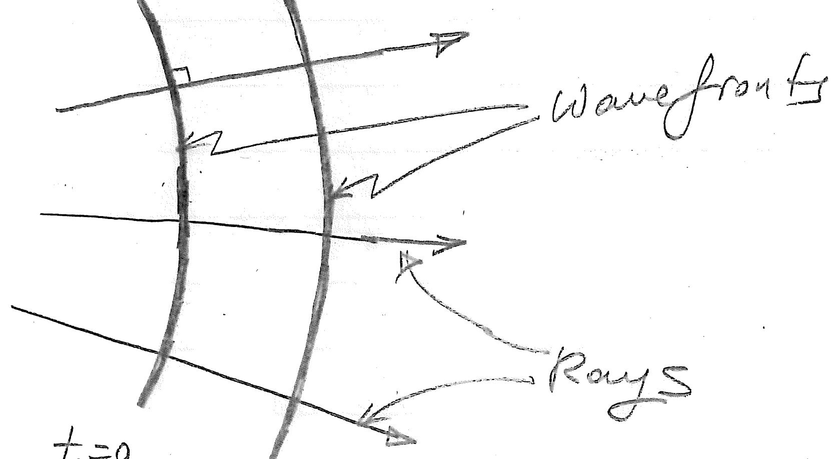


- Geometrical Optics Contributions are those attributed to surface reflections or Direct Contributions from the far zone source. A Stationary phase evaluation of the Physical Optics Integral generally yields the Geometrical Optics fields.

- Uniform extensions of the GTD provide corrections at the shadow boundaries of the reflected and source field

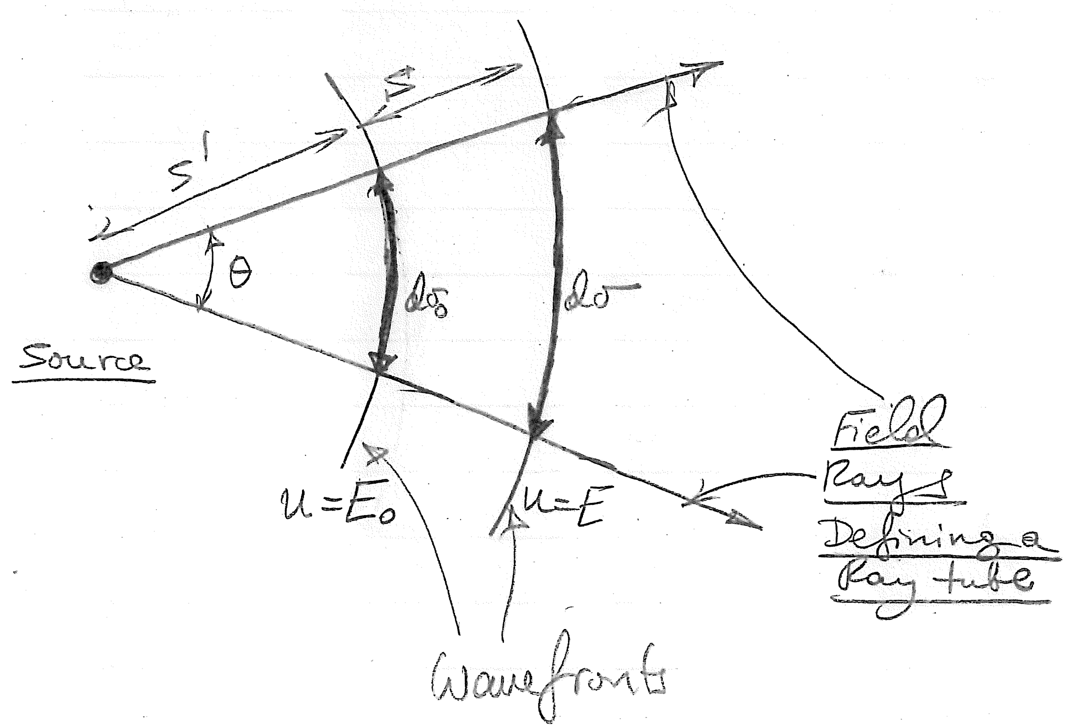
Principles of the Geometrical Optics Field

- Electromagnetic Energy Travel can be represented by Light Rays
  -- High Frequency Phenomenon --
- Each Light Ray Emanating from the Source Travels independent of the others
  --- Local Phenomenon --
- Light Rays Obey Fermat's and Huygen's Principles. That is, they travel through the shortest (time) paths. These are straight lines in homogeneous media. Consequently an optical disturbance/wavefront described by the surface $\varphi(x, y, z) = c_0$ at $t=0$, propagates normal to that surface



- Light Rays Obey the laws of Reflection and Refraction at Material Discontinuities

Example



u = Field intensity

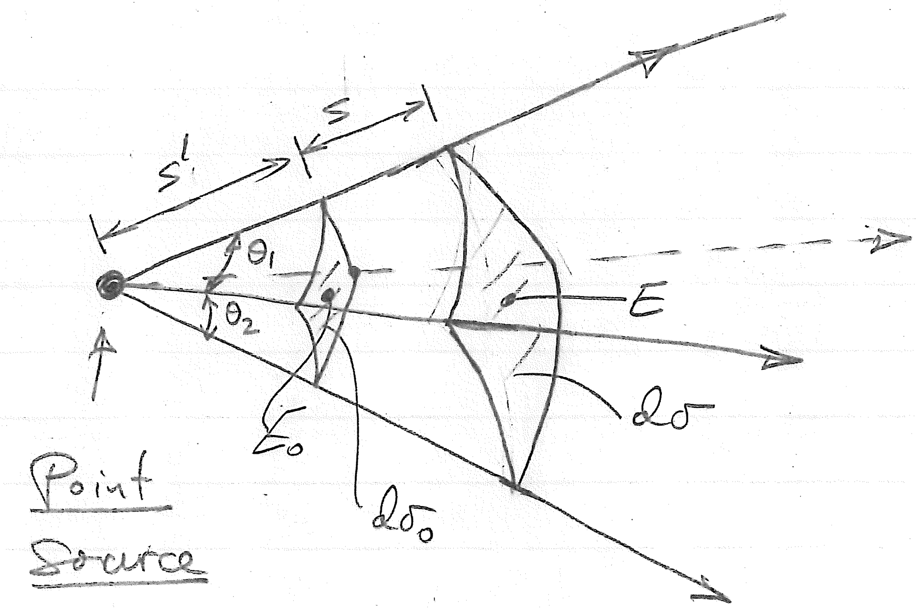
By energy conservation (flux in a ray tube is constant)

$$E_0^2 \, d\sigma_0 = E^2 \, d\sigma \Rightarrow \quad E = E_0 \sqrt{\frac{d\sigma_0}{d\sigma}}$$

$$\frac{d\sigma_0}{d\sigma} = \frac{\theta s'}{\theta(s+s')} = \frac{s'}{s+s'}$$

Thus,

$$E = E_0 \sqrt{\frac{s'}{s+s'}} \times \underbrace{e^{-jk_0 s}}_{\text{accounts for phase delay}}$$



Point Source

From Conservation of Energy

$$E_0^2 d\sigma_0 = E^2 d\sigma \Rightarrow \boxed{E = E_0 \sqrt{\frac{d\sigma_0}{d\sigma}}}$$

$$d\sigma_0 = (\theta_1 s')(\theta_2 s')$$

$$d\sigma = [\theta_1 (s+s')][\theta_2 (s+s')]$$

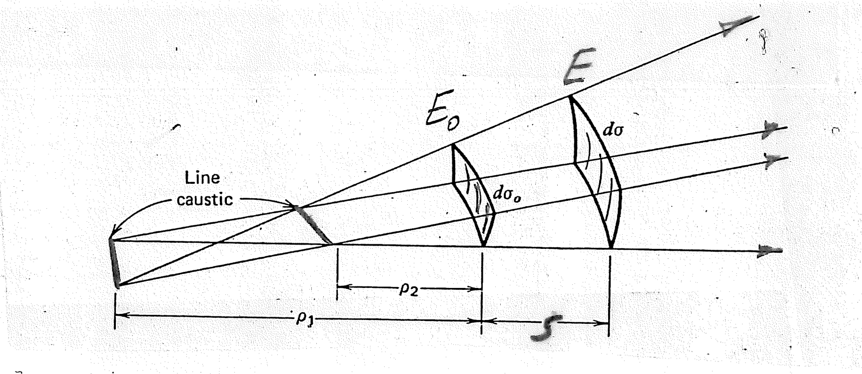
Thus,

$$E = E_0 \sqrt{\left(\frac{s'}{s+s'}\right)^2} = E_0 \frac{s'}{s+s'}$$

To account for phase delay

$$E = E_0 \left(\frac{s'}{s+s'}\right) e^{-jk_0 s}$$

General Ray tube Configuration



Astigmatic Ray tube

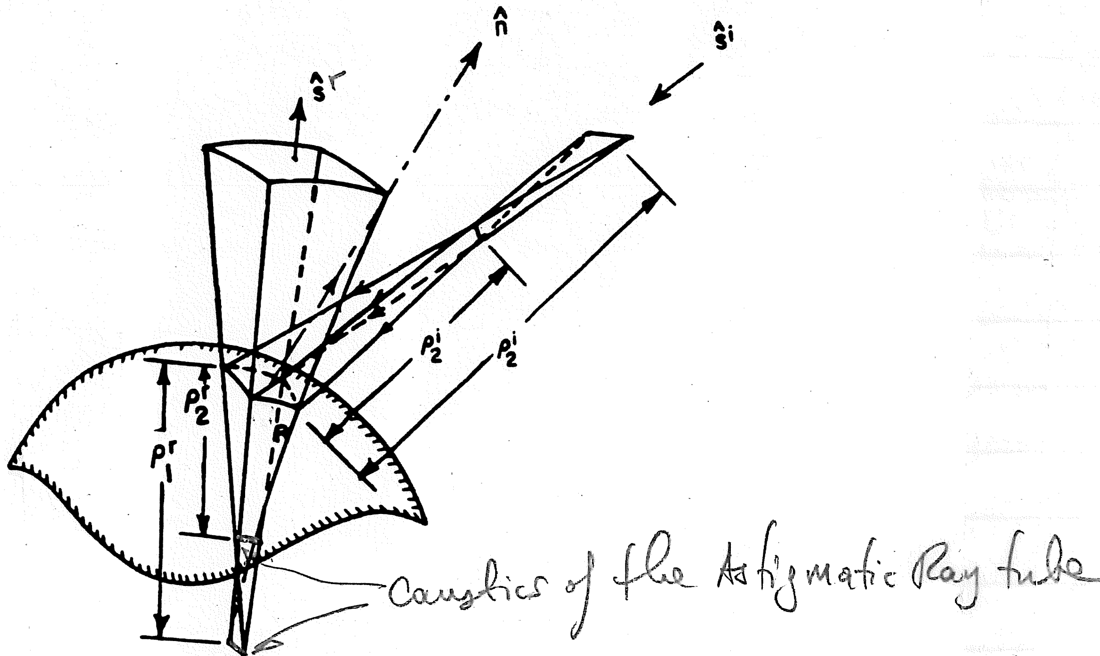
$$E = E_0 \sqrt{\frac{d\sigma_0}{d\sigma}}\, e^{-jk_0 s}$$

$$E = E_0 \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}}\, e^{-jk_0 s}$$

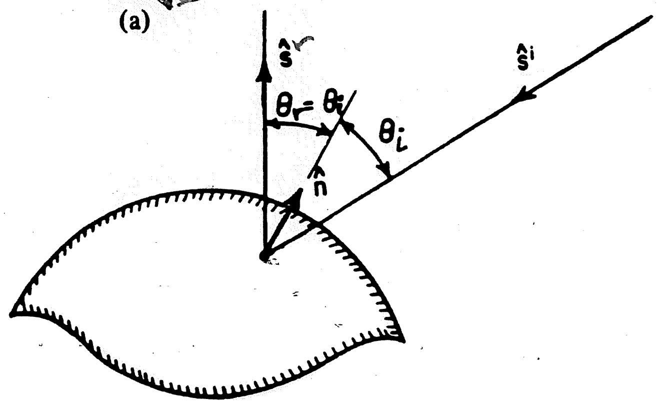
where,

ρ_1 and ρ_2 are the principal radii of curvature associated with the wave front at the reference point.

G. O. Reflected Field



(a)



(b)

reflection coefficient

$$E^r_{\parallel,\perp}(s) = \underbrace{E^i_{\parallel,\perp}(s_R)}_{\text{Reference field}} R_{\parallel,\perp} \underbrace{\sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r+s^r)(\rho_2^r+s^r)}}}_{\text{Decay or Spread factor}} \underbrace{e^{-jk_0 s^r}}_{\text{phase delay factor}}$$

Spread factor $\rightarrow \sqrt{\dfrac{d\sigma_0}{d\sigma}}$

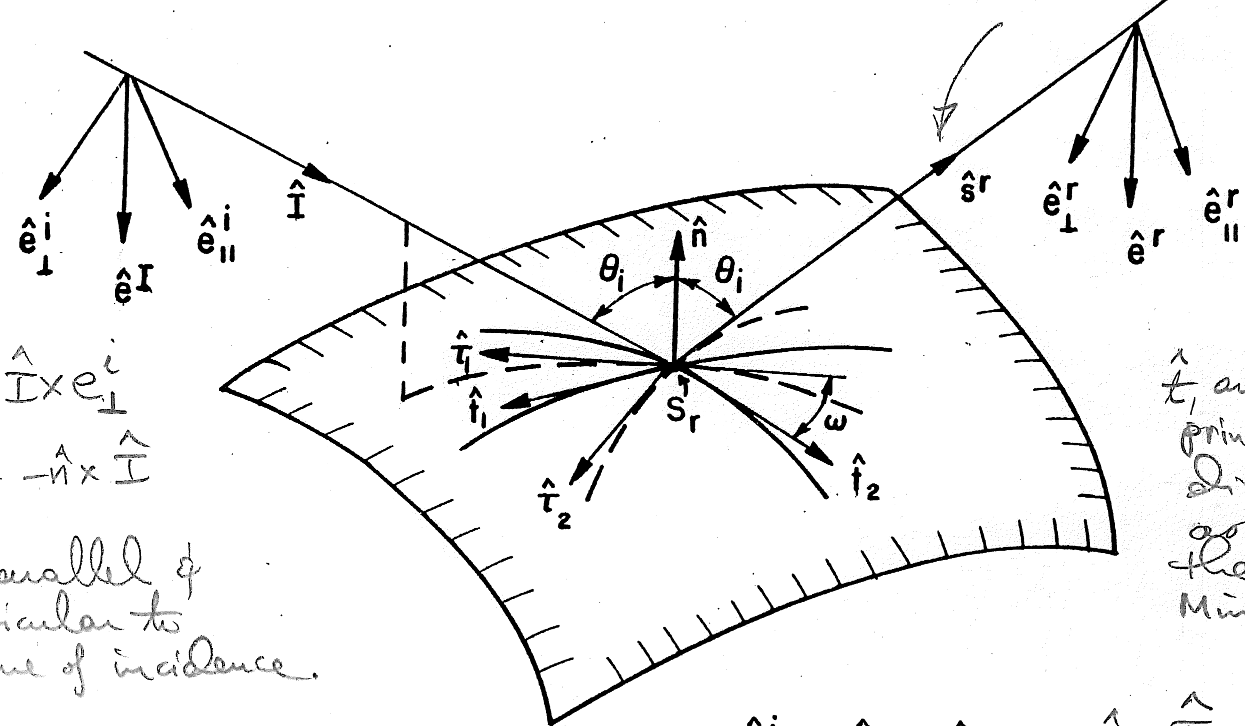
$\rho_{1,2}^r$: principal radii of the reflected wavefront.

7

Arbitrary Polarization of incidence

$$\bar{E}^i = \left(\hat{x} E_x^i + \hat{y} E_y^i + \hat{z} E_z^i\right) \frac{e^{-jk_0 s'}}{s'}$$

$$\hat{s}^r = \hat{I} - 2(\hat{I}\cdot\hat{n})\hat{n}$$



$\hat{e}^i_\parallel = \hat{I} \times e^i_\perp$

$\hat{e}^i_\perp = -\hat{n} \times \hat{I}$

Vectors parallel & perpendicular to the plane of incidence.

$\hat{t}_1$ and $\hat{t}_2$ are the principal surface directions associated with the Max and Min Radii of curvature.

$$\hat{e}^i_\perp = \hat{\tau}_2 = \hat{e}^r_\perp = -\hat{n} \times \hat{I}$$

$$\hat{e}^r_\parallel = \hat{s}^r \times \hat{e}^i_\perp$$

Figure 2.4. Geometry for reflection from a smooth arbitrary surface.

$$\bar{E}^r(s^r) = \bar{E}^i(S_r) \cdot \bar{\bar{R}} \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + s^r)(\rho_2^r + s^r)}}\, e^{-jk_0 s^r}$$

$$\bar{\bar{R}} = R_s\, \hat{e}^i_\perp e^i_\perp + R_h\, \hat{e}^i_\parallel \hat{e}^r_\parallel$$

Principal Radii of the Reflected Wavefront

$$\frac{1}{\rho^r_{1,2}} = \frac{1}{\rho^i_m} + \frac{1}{\cos\theta_i}\left[\frac{\sin^2\omega+\cos^2\theta_i\cos^2\omega}{R_1} + \frac{\cos^2\omega+\cos^2\theta_i\sin^2\omega}{R_2}\right]$$

$$\pm\frac{1}{2}\sqrt{\left(\frac{1}{\rho^i_1}-\frac{1}{\rho^i_2}\right)^2 + \frac{4}{\cos\theta_i}\left(\frac{1}{\rho^i_1}-\frac{1}{\rho^i_2}\right)\left[\frac{\sin^2\omega-\cos^2\theta_i\cos^2\omega}{R_1}+\frac{\cos^2\omega-\cos^2\theta_i\sin^2\omega}{R_2}\right] + \frac{4}{\cos^2\theta_i}\left\{\left[\frac{\sin^2\omega+\cos^2\theta_i\cos^2\omega}{R_1}+\frac{\cos^2\omega+\cos^2\theta_i\sin^2\omega}{R_2}\right]^2 - \frac{4\cos^2\theta_i}{R_1R_2}\right.}$$

$$\frac{1}{\rho^i_m} = \frac{1}{2}\left(\frac{1}{\rho^i_1}+\frac{1}{\rho^i_2}\right)$$

ω: angle of rotation of the plane of incidence from the principal direction

$R_{1,2}$: principal surface radii of curvature (max and min radii of curvature)

θ_i: angle of incidence with respect to normal

$\rho^i_{1,2}$: principal radii of curvature associated with incident wavefront. For a point source $\rho^i_{1,2}$ = distance from source

Simplifications when plane of Incidence coincides with one of the principal planes of the surface (principal planes of the surface are defined by the normal $\hat{n}$ and $\hat{t}_1$ or $\hat{t}_2$.

$$\hat{\tau}_1 = \pm \hat{t}_1$$

$$\frac{1}{\rho_1^r} = \frac{1}{\rho_1^i} + \frac{2}{R_1 \cos\theta_i}$$

$$\frac{1}{\rho_2^r} = \frac{1}{\rho_2^i} + \frac{2\cos\theta_i}{R_2}$$

$$\hat{\tau}_1 = \pm \hat{t}_2$$

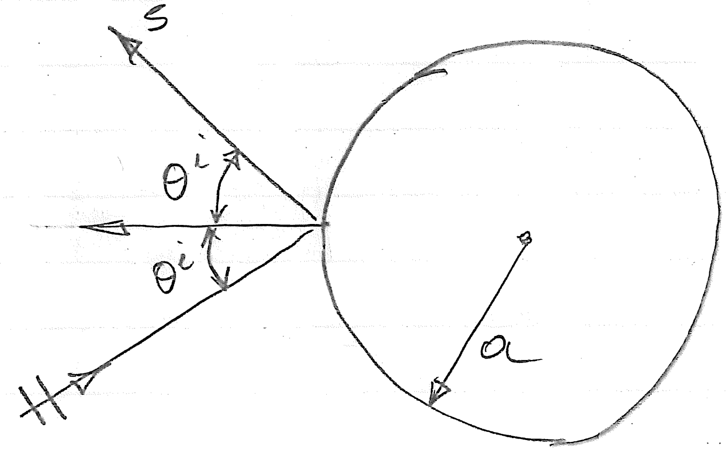
$$\frac{1}{\rho_1^r} = \frac{1}{\rho_1^i} + \frac{2\cos\theta_i}{R_1}$$

$$\frac{1}{\rho_2^r} = \frac{1}{\rho_2^i} + \frac{2}{R_2 \cos\theta_i}$$

For a spherical surface, incidence is always in the principal surface planes and

$$R_1 = R_2 = \text{sphere radius}$$

Reflection by a sphere with plane wave incidence



$\rho^i_{1,2} = s' = \infty$ for plane wave incidence

then

$$\rho_1^r = \frac{R_1 \cos\theta_i}{2} = \frac{a \cos\theta_i}{2} \qquad \rho_2^r = \frac{a}{2\cos\theta_i}$$

Reflected field

reflection coeff.

$$E^r = E^i (\pm 1) \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + s)(\rho_2^r + s)}}\, e^{-jk_0 s} \Bigg|_{s \to \infty} = E^i (\pm 1) \frac{\sqrt{\rho_1^r \rho_2^r}}{s} e^{-jk_0 s}$$

$$|E^r| = |E^i| \sqrt{\frac{a^2 \cancel{\cos\theta_i}}{4 \cancel{\cos\theta_i}}}\, \frac{1}{s} = |E^i| \frac{a}{2} \frac{1}{s}$$

$$\sigma = \lim_{s\to\infty} 4\pi s^2 \left|\frac{E^r}{E^i}\right|^2 = 4\pi s^2 \frac{a^2}{4}\frac{1}{s^2} = \pi a^2$$

sphere radar
Cross section
Valid for large a
(no creeping wave contributions)

In the case of Cylinder, reflection,

$$R_2 \to \infty \Rightarrow$$

$$\rho_1^r = \frac{a\cos\theta_i}{2} \quad \& \quad \rho_2^r = \frac{R_2}{2\cos\theta_i} \to \infty$$

Thus

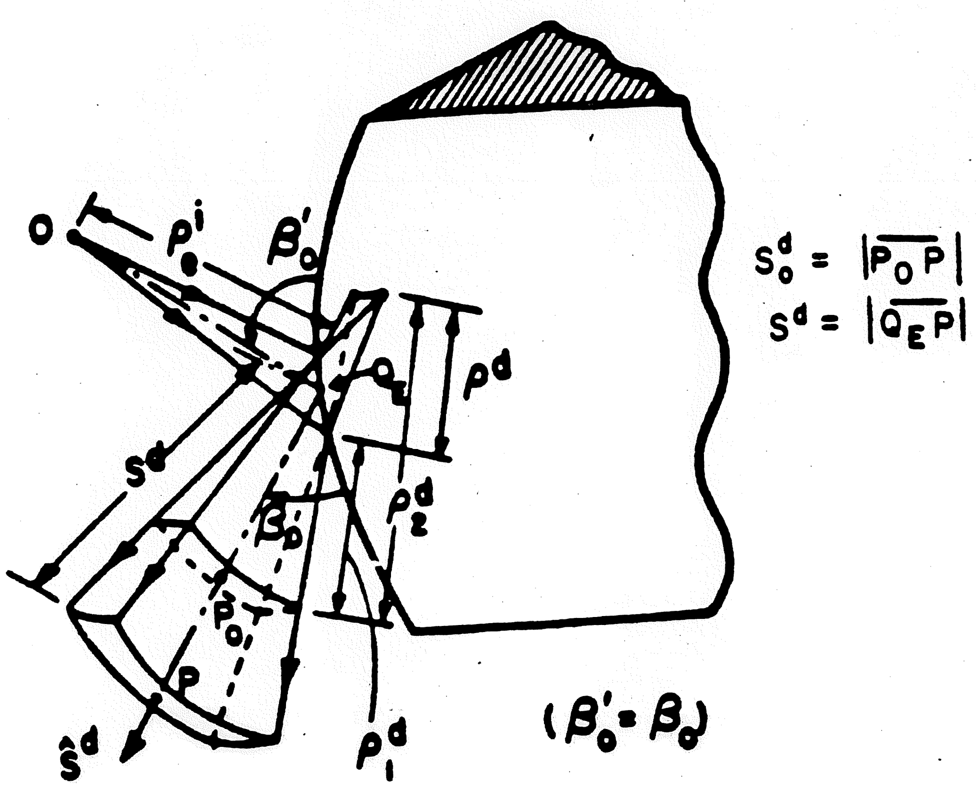
$$E^r = \pm E^i \sqrt{\frac{\rho_1^r \cancel{\rho_2^r}}{(\rho_1^r+s)\cancel{(\rho_2^r+s)}}}\, e^{-jk_0 s}$$

$$E^r = \pm E^i \sqrt{\frac{\rho_1^r}{\rho_1^r+s}}\Bigg|_{s\to\infty} e^{-jk_0 s} = \pm E^i \frac{\sqrt{\rho_1^r}}{\sqrt{s}}\, e^{-jk_0 s}$$

$$\sigma = \lim_{s\to\infty} 2\pi s \left|\frac{E^r}{E^i}\right|^2 = 2\pi \cancel{s} \frac{\rho_1^r}{\cancel{s}} = \cancel{2}\pi \frac{a\cos\theta_i}{\cancel{2}}$$

$$\sigma = \pi a \cos\theta_i \Big|_{\text{normal incidence}} = \pi a$$

Diffracted field

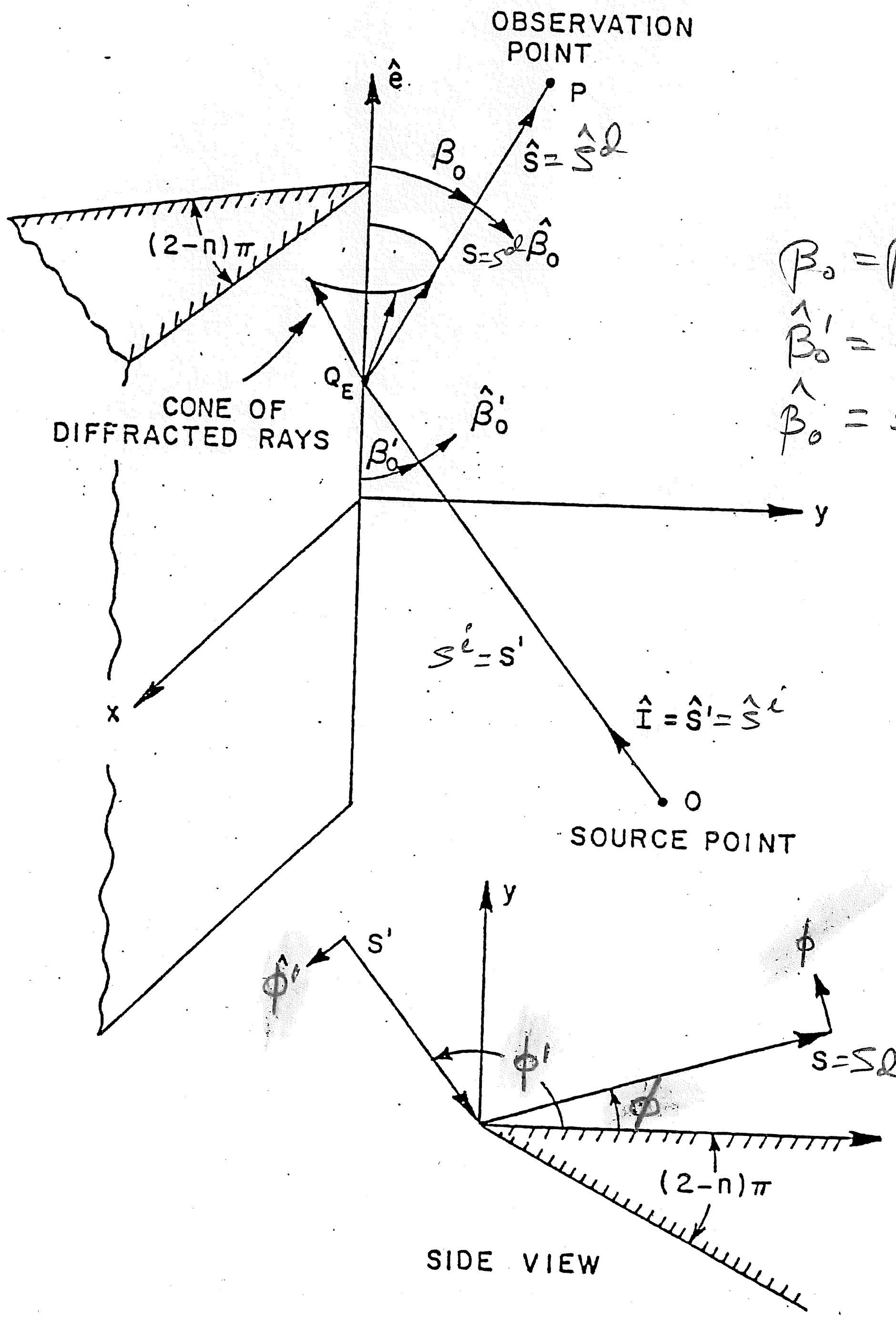


$$\bar{E}^d(P) \approx \bar{E}^d(P_0)\sqrt{\frac{\rho_1^d \rho_2^d}{(\rho_1^d + s_0^d)(\rho_2^d + s_0^d)}}\, e^{-jk_0 s_0^d}$$

letting $\rho_1^d \to 0$

$$\bar{E}^d(P) \approx \underbrace{\lim_{\rho_1^d \to 0}\left[\sqrt{\rho_1^d}\,\bar{E}^d(P_0)\right]}_{\bar{E}^i(Q_E)\cdot\bar{\bar{D}}}\sqrt{\frac{\rho_2^d}{(\cancel{\rho_1^d} + s_0^d)(\rho_2^d + s_0^d)}}\, e^{-jk_0 s_0^d}$$

$$\underbrace{\bar{E}^i(Q_E)}_{\begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix}\begin{bmatrix} E^i_{\beta_0'} \\ E^i_{\phi'} \end{bmatrix}} \cdot \bar{\bar{D}} \leftarrow \text{diffraction coefficient}$$



$$\beta_o = \beta_o'$$
$$\hat{\beta}_o' = \hat{s}^i \times \hat{\phi}'$$
$$\hat{\beta}_o = \hat{s}^d \times \hat{\phi}$$

Figure 2.7. Geometry for three-dimensional wedge diffraction problem.

Given that:

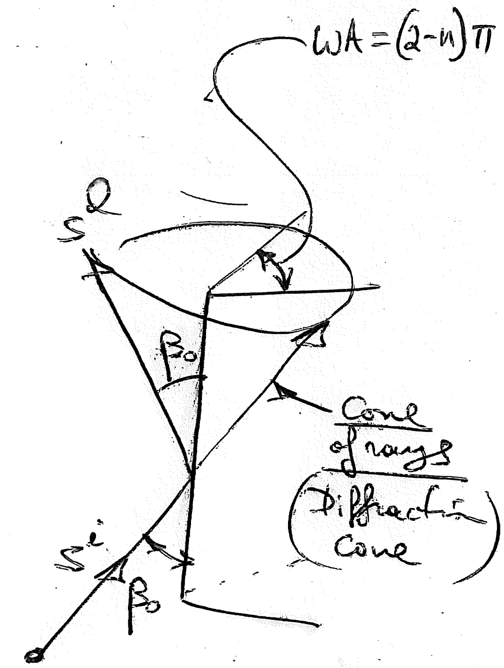
$$\bar{E}^i(Q_E) = \hat{\beta}_0' E^i_{\beta_0'} + \hat{\phi}' E^i_{\phi'}$$

and

$$\bar{E}^d(P) = \hat{\beta}_0 E^d_{\beta_0} + \hat{\phi} E^d_{\phi} \,.$$

Then

$$\bar{\bar{D}}^k_e = -\hat{\beta}_0' \hat{\beta}_0 D^k_{es} - \hat{\phi}' \hat{\phi} D^k_{eh}$$



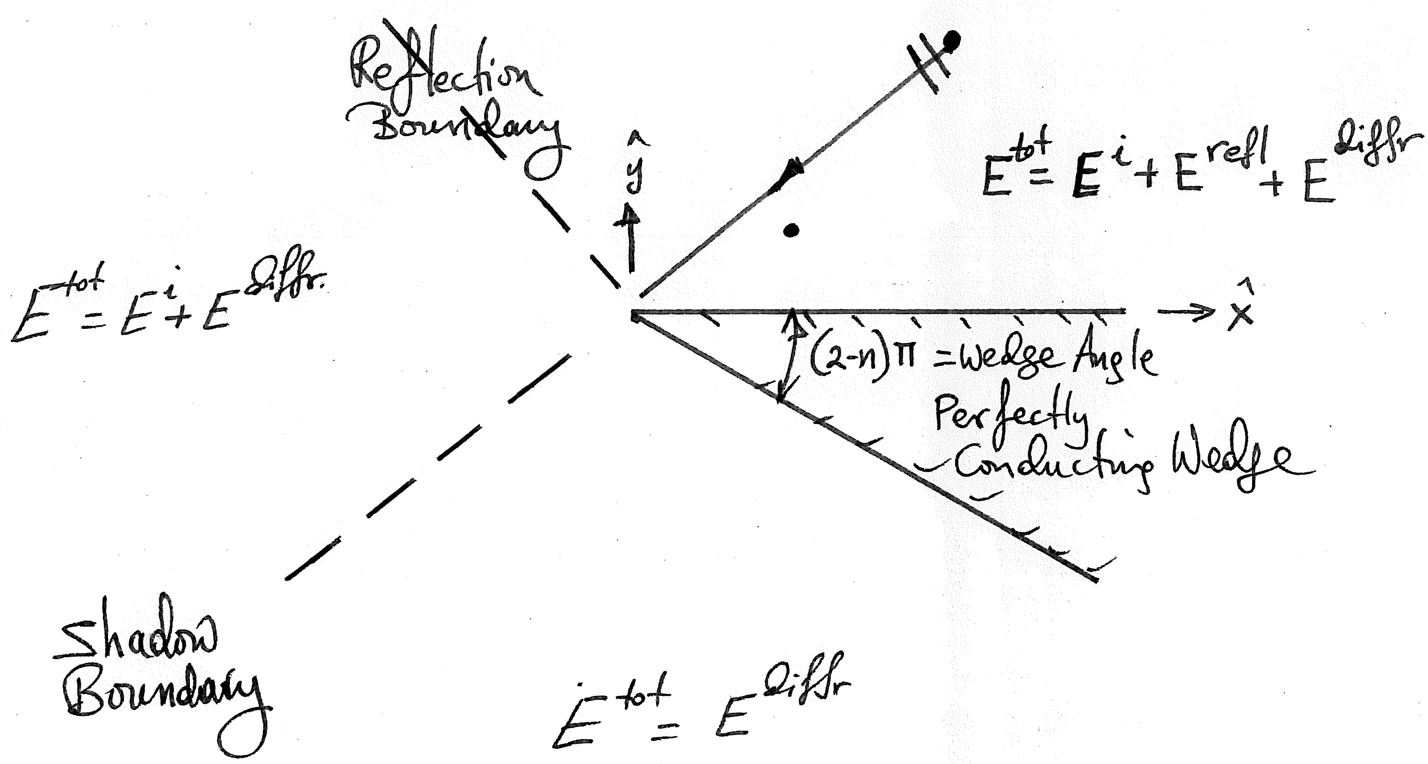
Keller's (Non-uniform edge Diffraction coefficient)

$$D^k_{\substack{es\\eh}}(\phi,\phi';\beta_0) = \frac{e^{-j\frac{\pi}{4}} \sin\frac{\pi}{n}}{n\sqrt{2\pi k}\sin\beta_0} \cdot \left[\frac{1}{\cos\frac{\pi}{n} - \cos\left(\frac{\phi-\phi'}{n}\right)} \mp \frac{1}{\cos\frac{\pi}{n} - \cos\left(\frac{\phi+\phi'}{n}\right)} \right]$$

$$\bar{E}^d(P) = \bar{E}^i(Q_E) \cdot \bar{\bar{D}}_e(\phi, \phi', \beta_0; k) \sqrt{\frac{\rho_e}{s^d(\rho_e+s^d)}}\, e^{-jks^d} \,.$$

ρ_e = edge caustic distance

Uniform Solutions



- A Uniform diffracted field is required to maintain continuity at the SB and RB boundary.

- Keller's Diffraction Coefficient is infinite at RB and SB and thus Invalid in the nearby regions of the SB and RB boundaries (Transition Regions)

- Uniform Diffraction Coefficient can be Derived through a More accurate evaluation of the exact integral representing the Scattered fields by the wedge.

- The Uniform Diffraction Coefficient gives $\pm\frac{1}{2}E^i$ at the SB so that

$$E^i D_{\substack{es\\eh}} \sqrt{\frac{\rho_e}{(\rho_e+s)s}}$$

(lit region) $E^{tot} = E^i + E^{diffr} = E^i - \frac{1}{2}E^i = \frac{1}{2}E^i$

(shadowed region) $E^{tot} = E^{diffr} = \frac{1}{2}E^i$

Continuous (uniform) at the SB

$$E^{diffr} = E^i D_{\substack{es\\eh}} \sqrt{\frac{\rho_e}{(\rho_e+s^d)s^d}}\, e^{-jk_o s^d}$$

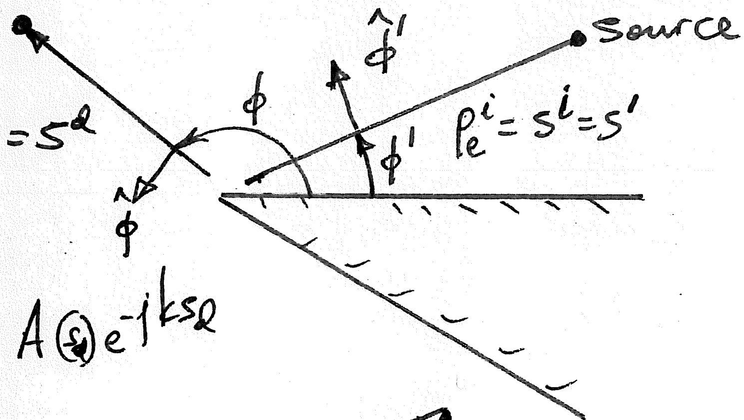
- The Uniform Diffraction Coefficient gives $\pm\frac{1}{2}E^{refl.}$ at the RB so that

(lit region of reflected field)
$$E^{tot} = E^i + E^{refl} + E^{diffr}$$
$$= E^i + E^{refl} - \frac{1}{2}E^{refl} = E^i + \frac{1}{2}E^{refl}$$

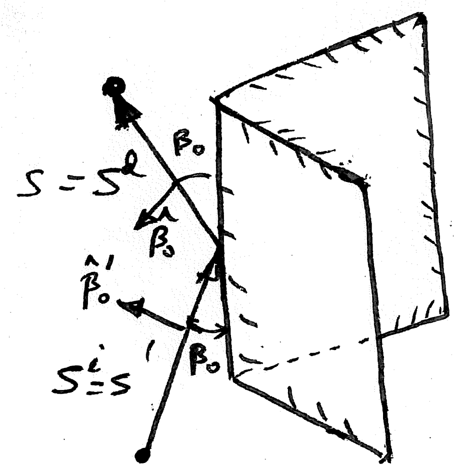
Continuous

(shadowed region of reflected field)
$$E^{tot} = E^i + E^{diffr} = E^i + \frac{1}{2}E^{refl}$$

Uniform Diffraction Coefficient for a Straight Wedge



$$E^d_{\substack{\phi\\ \beta}} = E^i_{\substack{\phi'\\ \beta_o'}} \, D_{\substack{eh\\ es}} \, A(s_d) \, e^{-jks_d}$$

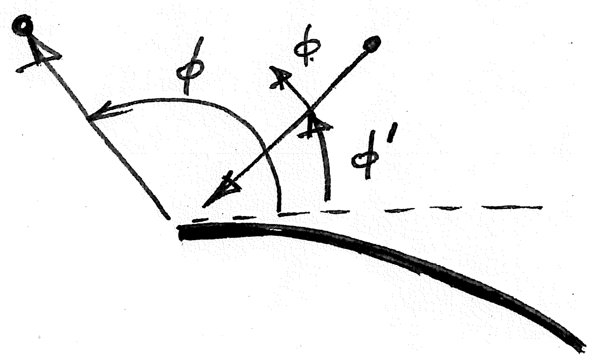


$$A(s_d) = \sqrt{\frac{\rho_e}{s^d(\rho_e + s^d)}} = \begin{cases} \dfrac{1}{\sqrt{s^i}} & \text{(line source illumination)} \\[2ex] \sqrt{\dfrac{s^i}{s^d(s^i + s^d)}} & \text{(point source illumination)} \end{cases}$$

divergence factor

Diffraction Coefficient for a Curved Half Plane

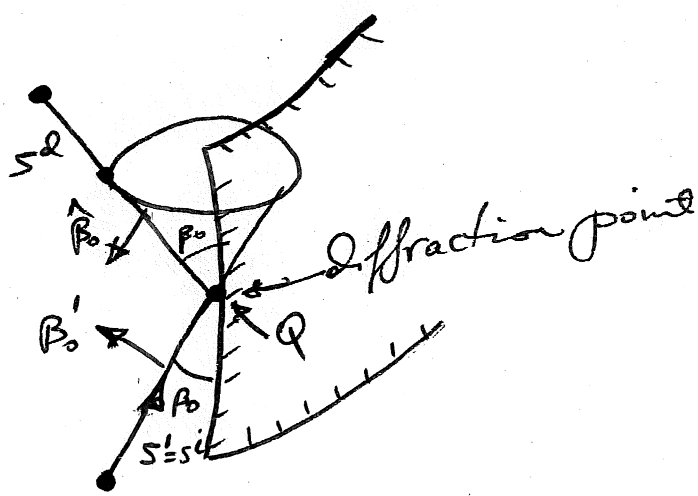
D



$$E^{diffr}_{\substack{\phi \\ \beta_0}} = E^{i}_{\substack{\phi' \\ \beta_0'}} \, D_{\substack{eh \\ es}} \sqrt{\frac{\rho^d}{s^d(\rho^d + s^d)}}$$

$$\overline{H}^{diffr} = Y_0 \, \hat{s}^d \times \overline{E}^{diffr}$$

same as the plane wave or far zone relation



$$D_{\substack{es \\ eh}} = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k}} \left\{ \frac{F\left(2k_0 L^i \cos^2\left(\frac{\phi - \phi'}{2}\right)\right)}{\cos \frac{\phi - \phi'}{2}} \mp \frac{F\left[2k_0 L^r \cos^2\left(\frac{\phi + \phi'}{2}\right)\right]}{\cos\left(\frac{\phi + \phi'}{2}\right)} \right\}$$

$$F(x) = 2j \left|\sqrt{x}\right| e^{jx} \int_{|\sqrt{x}|}^{\infty} e^{-j\tau^2} d\tau = \text{Transition Function}$$

$$L^{i,r} = \frac{s^d(\rho_e + s^d)\rho_1 \rho_2}{\rho_e(\rho_1 + s^d)(\rho_2 + s^d)} \sin^2 \beta_0$$

$\rho_{1,2}$: principal radii of curvature of the incident or reflected wavefront at Q

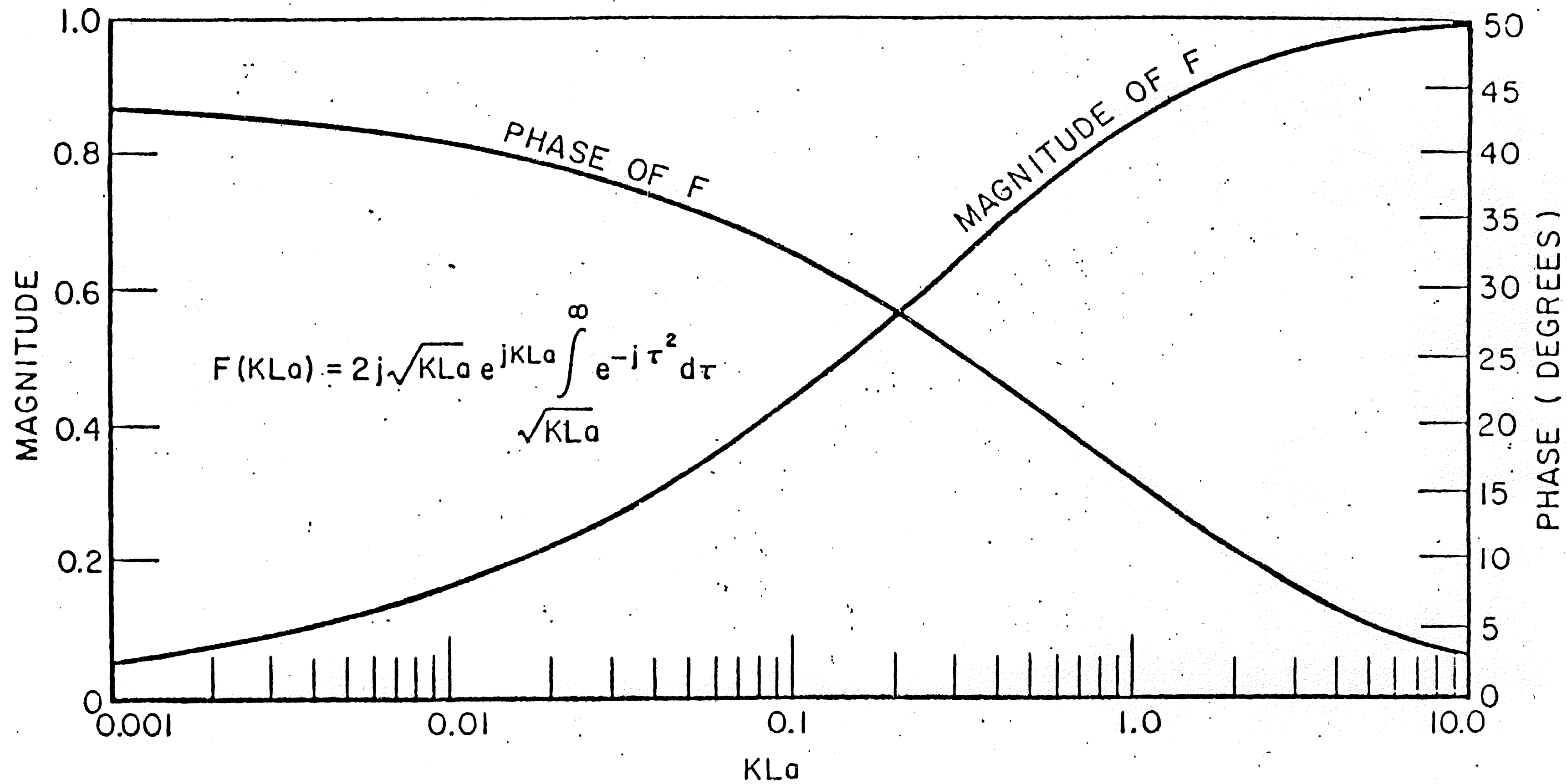
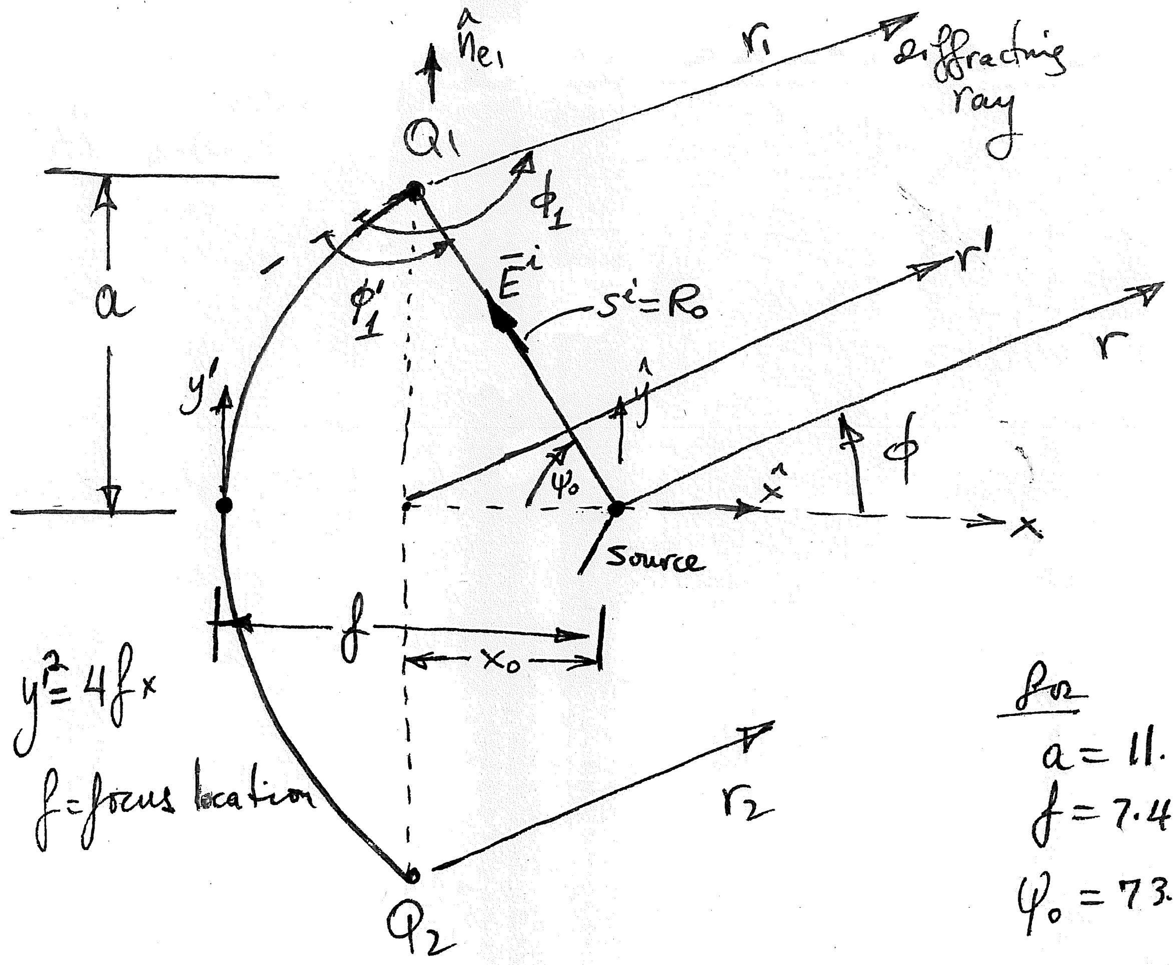


Figure 2.8. Transition function.



<u>for</u>

$a = 11.18\lambda$

$f = 7.45\lambda$

$\psi_0 = 73.76^\circ$

$R_0 = \dfrac{a}{\sin\psi_0} = 11.644\lambda$

<u>Also</u>

$\phi_1' = \phi_2' = 53.12^\circ$

$\phi_1 = \pi/2 + \phi_1' - (\pi/2 - \psi_0) \underbrace{}_{+\phi} = 126.88^\circ + \phi$

$\phi_2 = 126.88^\circ - \phi$

<u>Total Field</u>

$$E^{tot} = E_1^d + E_2^d + E^i$$

P.A.J. Ratnasiri

(P. H. Pathak).

$$E_1^d(\phi, r_1) = E^i(\varphi_1)\, D_{es}(\phi_1, \phi_1'; \beta_0, L^i, L^r) \sqrt{\frac{\rho_{e1}}{(\rho_{e1}+r_1)r_1}}\; e^{-jk_0 r_1}$$

$$= E^i(\varphi_1)\, D_{es}(\phi_1, \phi_1'; \beta_0, L^i, L^r)\, \frac{\sqrt{\rho_{e1}}}{r'}\; e^{-jk_0(r' - a\sin\phi)}$$

$$E_2^d(\phi, r_2) = E^i(\varphi_2)\, D_{es}(\phi_2, \phi_2'; \beta_0, L^i, L^r)\, \frac{\sqrt{\rho_{e2}}}{r'}\; e^{-jk_0(r' + a\sin\phi)}$$

$$r = r + (f - x_0)\cos\phi$$

$$D_{es}(\phi_i, \phi_i'; \beta_0 = \tfrac{\pi}{2}, L^i, L^r) = -\frac{e^{-j\pi/4}}{2\sqrt{2\pi k_0}} \cdot \left\{ \frac{F\left[2k_0 L^i \cos^2\left(\frac{\phi_i - \phi_i'}{2}\right)\right]}{\cos\left(\frac{\phi_i - \phi_i'}{2}\right)} - \frac{F\left[2k_0 L^r \cos^2\left(\frac{\phi_i + \phi_i'}{2}\right)\right]}{\cos\left(\frac{\phi_i + \phi_i'}{2}\right)} \right\}$$

$$L^i = \frac{s^d(\rho_e^i + s^d)\,\rho_1^i \rho_2^i}{\rho_e^i(\rho_1^i + s^d)(\rho_2^i + s^d)} = \frac{r'(R_0 + r')R_0^2}{R_0(R_0 + r')^2}\Bigg|_{r' \to \infty} = R_0$$

$$L^r = \frac{s^d(\rho_e^r + s^d)\,\rho_1^r \rho_2^r}{\rho_e^r(\rho^r + s^d)(\rho_2^r + s^d)}\Bigg|_{r' \to \infty} \to R_0$$

$$\frac{1}{\rho_{e1}} = \frac{1}{\rho_e^i} - \frac{\hat{n}_{e1}\cdot(\hat{s}^i - \hat{r}_1)}{a_e} = \frac{1}{R_0} - \frac{\hat{n}_{e1}\cdot(\hat{s}^i - \hat{r}_1)}{a}.$$

$$\hat{n}_{e1}\cdot\hat{s}^i = \cos(\tfrac{\pi}{2}-\varphi_0) = \sin\varphi_0$$

$$\hat{n}_{e1}\cdot\underset{\substack{\parallel \\ \hat{r}_1}}{\hat{s}^{\ell}} = \cos(\tfrac{\pi}{2}-\phi) = \sin\phi$$

Thus,

$$\frac{1}{\rho_{e1}} = \frac{1}{R_0} - \frac{\sin\varphi_0 - \sin\phi}{\underbrace{R_0\sin\varphi_0}_{=a}} \Rightarrow \boxed{\rho_{e1} = \frac{a}{\sin\phi}}$$

Similarly

$$\boxed{\rho_{e2} = -\frac{a}{\sin\phi} = -\rho_{e1}}$$

$$E_1^{\ell} = E^i(\varphi_1)\,\frac{e^{-jk_0 r'}}{r'}\sqrt{\frac{a}{\sin\phi}}\;D_{es}\left(\phi_1,\phi_1';\tfrac{\pi}{2}, L^i=R_0, L^r=R_0\right)$$

$e^{+jk_0 a\sin\phi}$

$$E_2^{\ell} = E^i(\varphi_2)\,\frac{e^{-jk_0 r'}}{r'}\sqrt{\frac{a}{\sin\phi}}\,e^{j\pi}\;D_{es}\left(\phi_2,\phi_2';\tfrac{\pi}{2}, L^i=R_0, L^r=R_0\right)$$

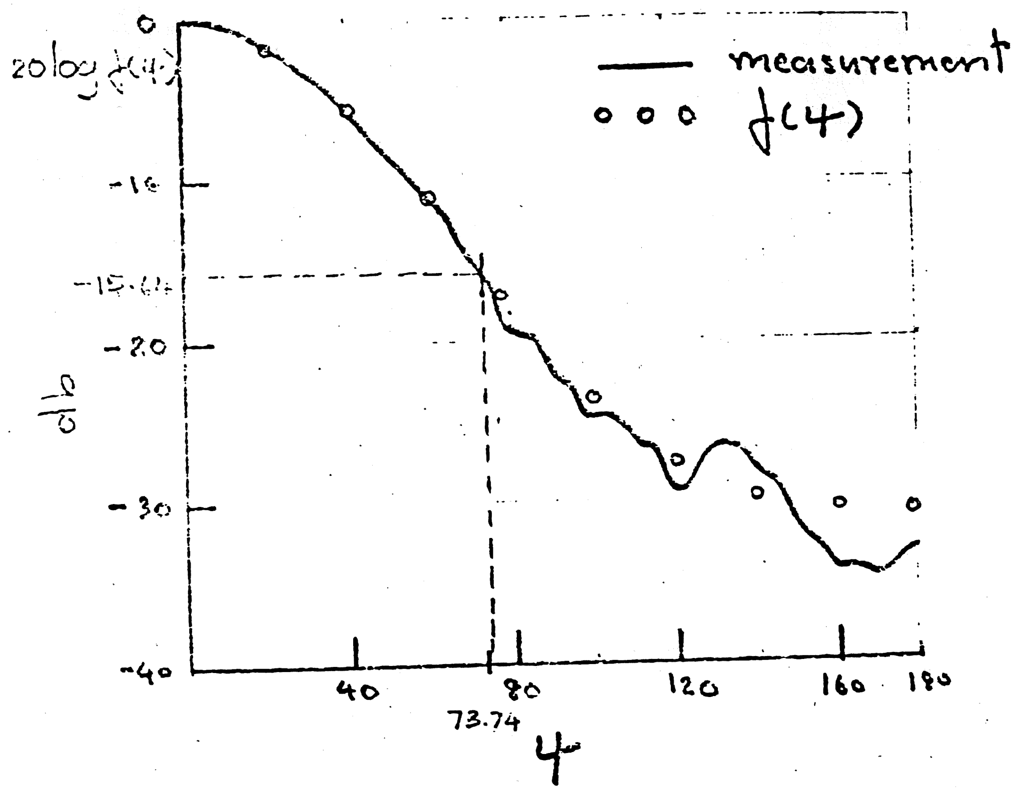
$e^{-jk_0 a\sin\phi}$

In the following plots

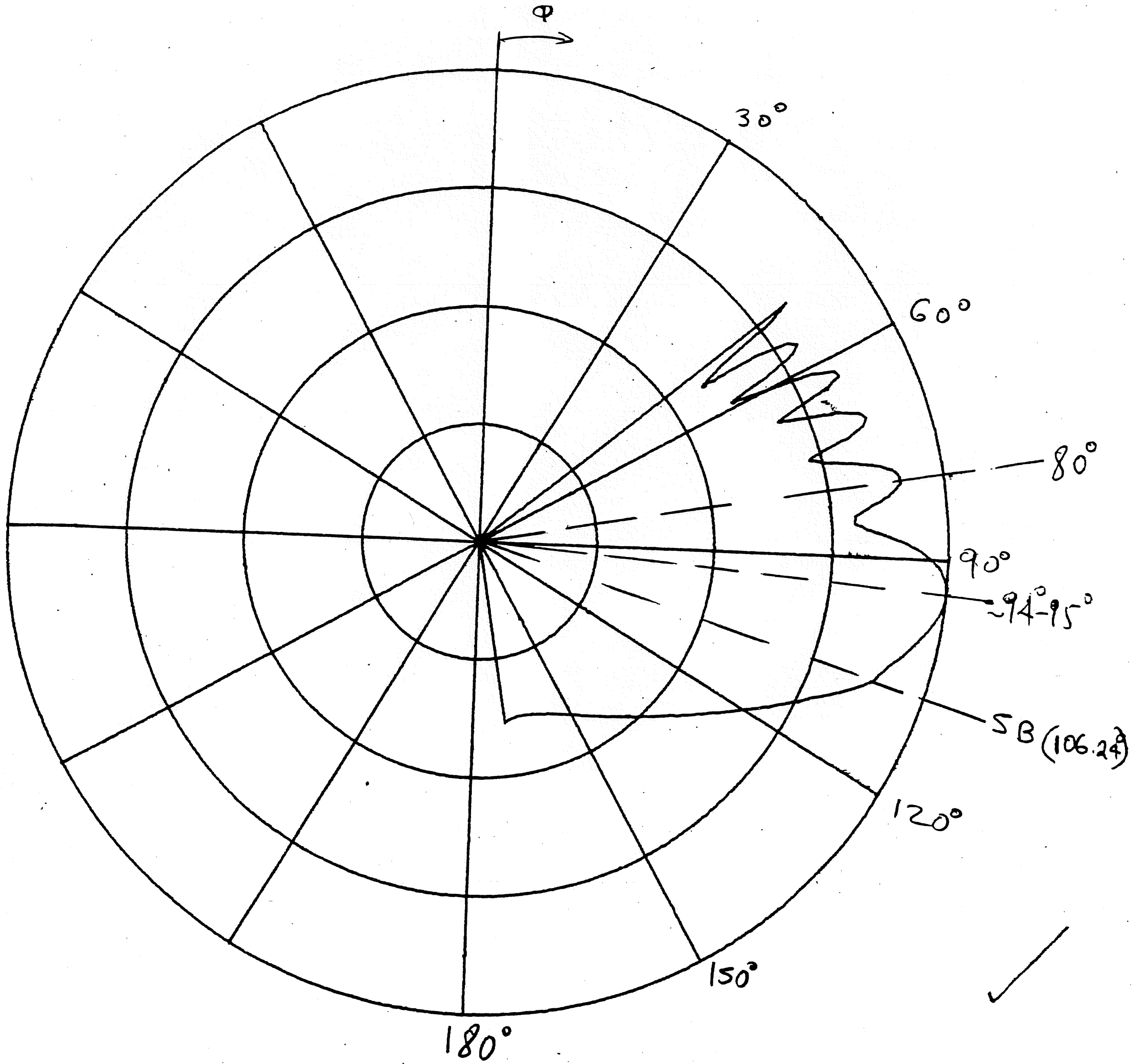
$$E^i(s^i;\varphi_0) = \hat{z}\,A\,\frac{e^{-jk_0 s^i}}{s^i}\,f(\varphi_0);\quad f(\varphi_0) = \begin{cases} e^{-4.4\times10^{-4}\varphi_0^2 + 2\times10^{-8}\varphi_0^4} & 0 < \varphi_0 < 78^\circ \\ 0.028 + 0.116\,e^{-.05(\varphi_0 - 78^\circ)} & 78^\circ < \varphi_0 < 180^\circ \end{cases}$$

$$U^i(\psi) = f(\psi)\,\frac{e^{-jkR_f}}{R_f}$$

$$f(\psi) = \begin{cases} f_1(\psi) = e^{-4.4\times10^{-4}\psi^2 + 2\times10^{-8}\psi^4} & ;\ \psi \le 78^\circ \\ d_2(\psi) = 0.028 + 0.116\,e^{-0.05(\psi - 78^\circ)} & ;\ \psi \ge 78^\circ \end{cases}$$



Pattern of a Flanged Waveguide Feed (H-plane)

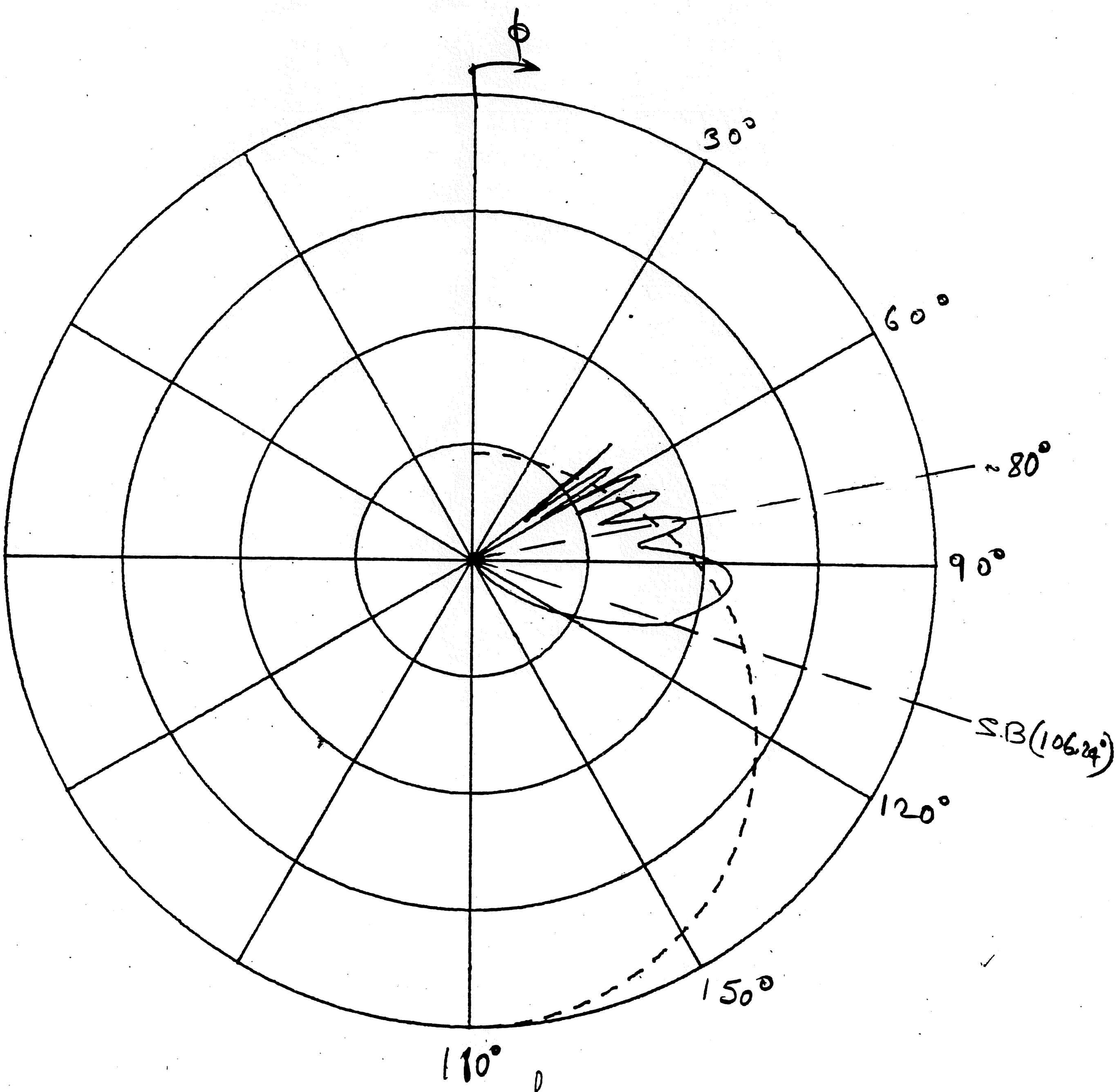


DB Plot of Scattered far field only

{Normalized to $E^s = A\frac{e^{-jkR}}{R} f(\varphi_0) = A\frac{e^{-jk(R' - d\cos\varphi)}}{R'} f(\varphi_0)$

$R, R' \to \infty$

DB PLOT
NORMALIZED TO
15.8506 DB



DB Plot of Scattered far field & Incident field
$\left\{ \text{Normalized to } E^{i} = A\frac{e^{-jkR}}{R} f(\psi_0) = \frac{A e^{-jk(R' - d\cos\phi)}}{R'} f(\psi_0) \right\}$