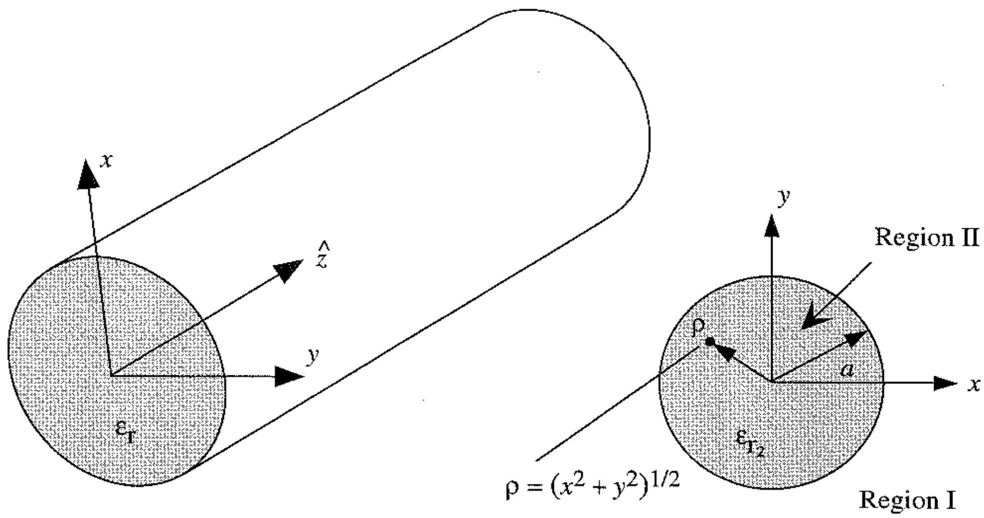
Propagation in Dielectric Rod

(see Balanis p. 506, Harrington p. 220)



Dielectric rod
of radius a

- Dielectric rod is the basic form of an optical fiber.
- Modes are found by satisfying wave equation in cylindrical coordinates and satisfying continuity conditions on $\hat{n} \times \mathbf{H}$ and $\hat{n} \times \mathbf{E}$ across the two regions (Regions I and II).
- BCs can only be satisfied when both TE^z and TM^z modes are present.
  - This gives rise to hybrid modes called Hybrid Electric/Magnetic modes or HEM.
  - HEM_{11} is the lowest order mode in the dielectric rod.

Proposed Modes

Interior Potentials (Region II):

$$\mathbf{A} = \hat{z}A\, J_m(k_{\rho_2}\rho)\cos(m\phi)\, e^{-jk_z z}$$
$$\mathbf{F} = \hat{z}B\, J_m(k_{\rho_2}\rho)\sin(m\phi)\, e^{-jk_z z}$$

where

$$(k_{\rho_2})^2 + (k_z)^2 = k_0^2 = \omega^2\mu_2\varepsilon_2 = k_0^2\mu_{r_2}\varepsilon_{r_2}$$

Exterior Region Potentials (Region I):

$$\mathbf{A} = \hat{z}A'\, H_m^{(2)}(k_{\rho_1}\rho)\cos(m\phi)\, e^{-jk_z z}$$
$$\mathbf{F} = \hat{z}B'\, H_m^{(2)}(k_{\rho_1}\rho)\sin(m\phi)\, e^{-jk_z z}$$

where

$$(k_{\rho_1})^2 + (k_z)^2 = k_1^2 = \omega^2 \mu_1 \varepsilon_1$$

- We chose J_m as the Bessel solution in region II (interior) because the field must be finite at $\rho = 0$.

- We chose $H_m^{(2)}$ as the Bessel equation solution in region I (exterior) because

$$H_m^{(2)}(k_{\rho_1}\rho)\Big|_{\rho\to\infty} \sim \frac{e^{-jk_{\rho_1}\rho}}{\sqrt{\rho}}$$

  i.e., $H_m^{(2)}$ represents outgoing waves.

  Also, we know that for a mode to exist, the field must decay as ρ becomes larger. Thus, we expect $k_{\rho_1} = -j\alpha_{\rho_1}$, so that

$$H_m^{(2)}(k_{\rho_1}\rho)\Big|_{\rho\to\infty} \sim \frac{e^{-jk_{\rho_1}\rho}}{\sqrt{\rho}} = \frac{e^{-\alpha_{\rho_1}\rho}}{\sqrt{\rho}}$$

  in which the last term is exponentially decaying.

- Note that by definition

$$-j^{m+1}\left(\frac{\pi}{2}\right) H_m^{(2)}(-j\alpha_{\rho_1}\rho) = K_m(\alpha_{\rho_1}\rho)$$

  where $K_m(\cdot)$ is the modified Bessel function of the second kind.

- Thus, for region I, it is appropriate to rewrite the potentials as

$$\left.\begin{array}{rcl} \mathbf{A} & = & \hat{z}A''\, K_m(\alpha_{\rho_1}\rho)\cos(m\phi)\, e^{-jk_z z} \\ \mathbf{B} & = & \hat{z}B''\, K_m(\alpha_{\rho_1}\rho)\sin(m\phi)\, e^{-jk_z z} \end{array}\right\} \text{Region I}$$

$$-\alpha_{\rho_1}^2 + k_z^2 = k_1^2 = k_0^2(\varepsilon_{r_1}\mu_{r_1})$$

- To find the appropriate values for α_{ρ_1} and k_{ρ_2}, we need to satisfy the BCs at $\rho = a$. For metallic guides we had set $E_\phi = 0$ and this resulted in $J_m(k_\rho\rho)|_{\rho=a} = 0$ or $J_m'(k_\rho\rho)|_{\rho=a} = 0$ leading to $k_\rho = \chi_{mn}/a$ and $k_\rho = \chi_{mn}'/a$ respectively.

- For the dielectric rod, the only BCs are those of *tangential field continuity* at $\rho = a$. Specifically, on setting $E_z^{\mathrm{I}} = E_z^{\mathrm{II}}$, $H_z^{\mathrm{I}} = H_z^{\mathrm{II}}$, $H_\phi^{\mathrm{I}} = H_\phi^{\mathrm{II}}$, and $E_\phi^{\mathrm{I}} = E_\phi^{\mathrm{II}}$, we get a set of four equations. Setting the determinant of the four equations to zero leads to a non-trivial solution (see Harrington p. 221, Balanis p. 511) for α_{ρ_1} and k_{ρ_2}. Namely, the equation satisfied by k_{ρ_2} and k_{ρ_1} is

$$\left[\frac{1}{u}\frac{J_m'(u)}{J_m(u)} - \frac{1}{\upsilon}\frac{H_m^{(2)\prime}(\upsilon)}{H_m^{(2)}(\upsilon)}\right]\left[\frac{\varepsilon_{r_1}\mu_{r_1}}{u}\frac{J_m'(u)}{J_m(u)} - \frac{\varepsilon_{r_2}\mu_{r_2}}{\upsilon}\frac{H_m^{(2)\prime}(\upsilon)}{H_m^{(2)}(\upsilon)}\right] = \left[\frac{mk_z}{k_0}\left(\frac{1}{u^2} - \frac{1}{\upsilon^2}\right)\right]^2$$

$$u = k_{\rho_1}a = -j\alpha_{\rho_1}a, \qquad \upsilon = k_{\rho_2}a$$

  For $m = 0$, this breaks to two equations to be solved for k_{ρ_1} and k_{ρ_2}.

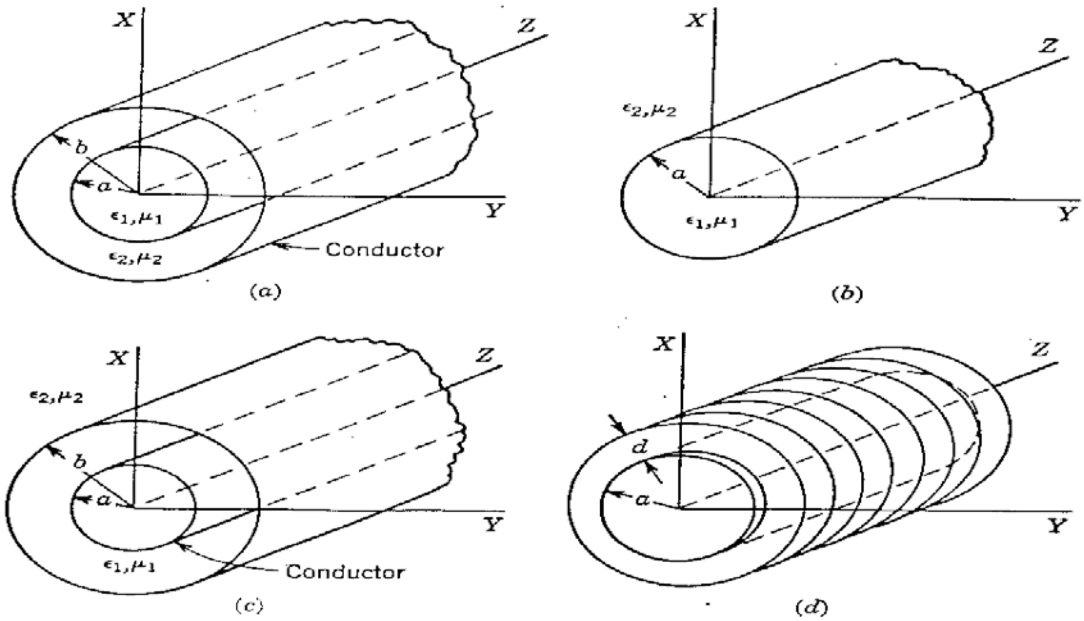
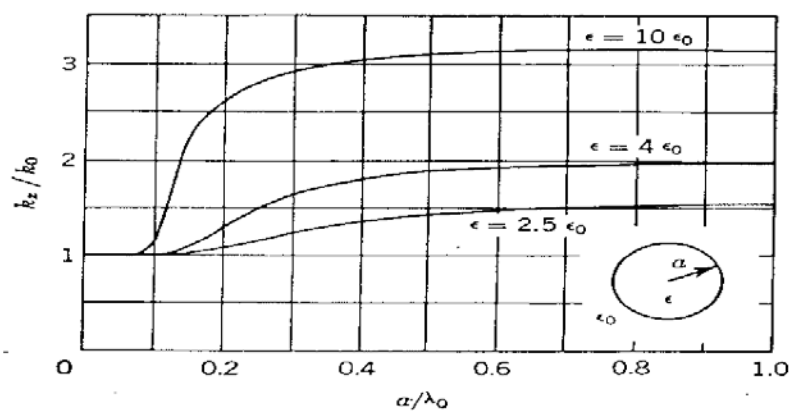


FIG. 5-10. Some circular waveguides. (a) Partially filled; (b) dielectric slab; (c) coated conductor; (d) corrugated conductor.



Phase constant for the circular dielectric rod. (*After M. C. Gray.*)

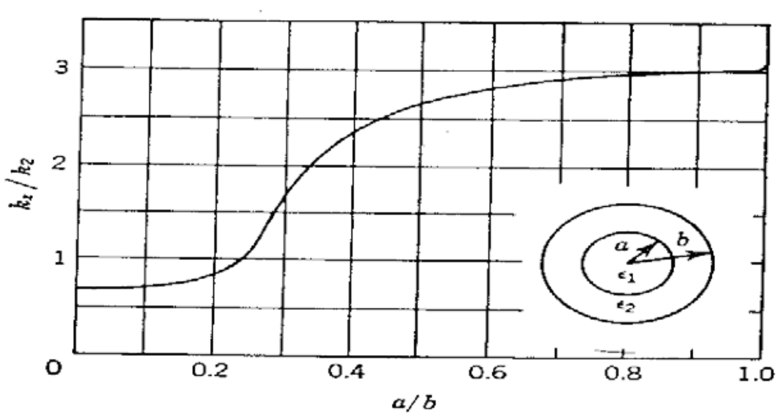


FIG. 5-11. Phase constant for the partially filled circular waveguide, $\epsilon_1 = 10\epsilon_2$, $b = 0.4\lambda_2$. (*After H. Seidel.*)