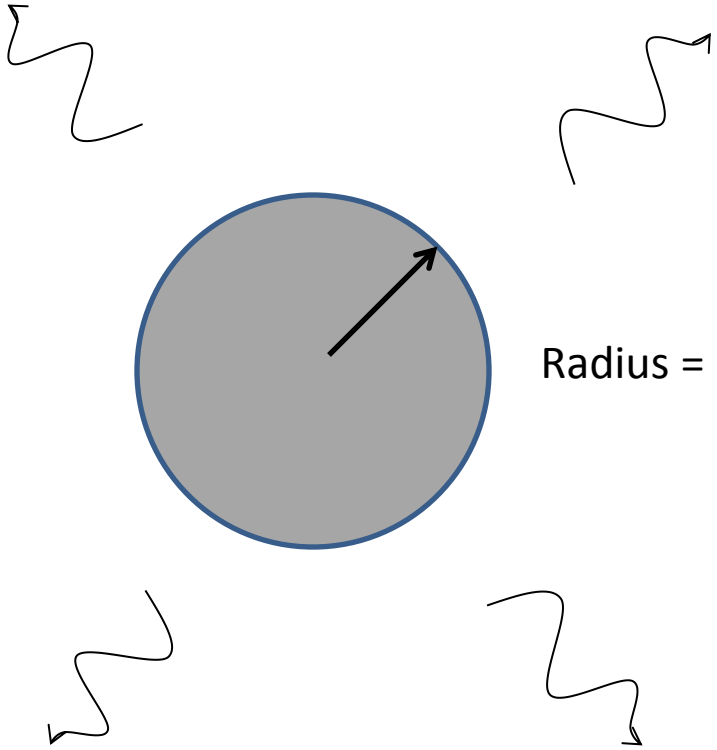
Scattering From A PEC Cylinder (TM Incidence)

Incident Plane Wave:

$$\boldsymbol{E_i} = \hat{\boldsymbol{z}} E_0 e^{-j\beta x}$$



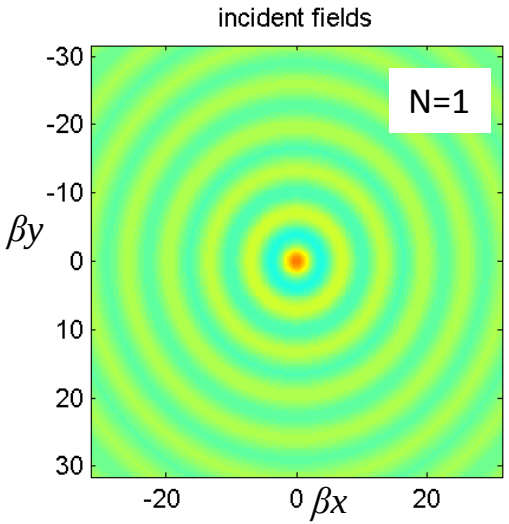
Represent incident plane wave as sum of Bessel Fns

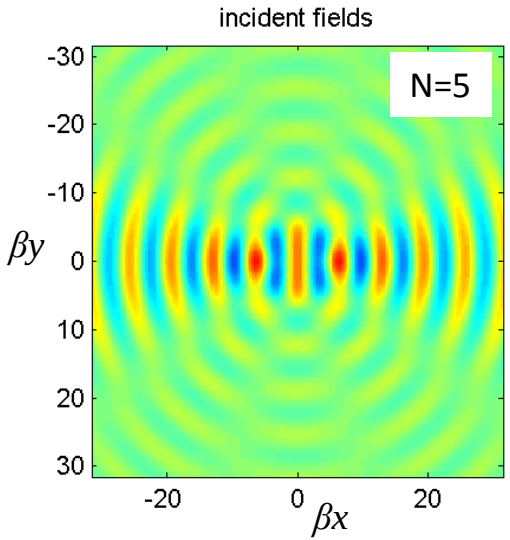
$$\boldsymbol{E_i} = \hat{\boldsymbol{z}} E_0 \sum_{n=-\infty}^{\infty} j^{-n} J_n(\beta\rho) e^{-jn\varphi}$$

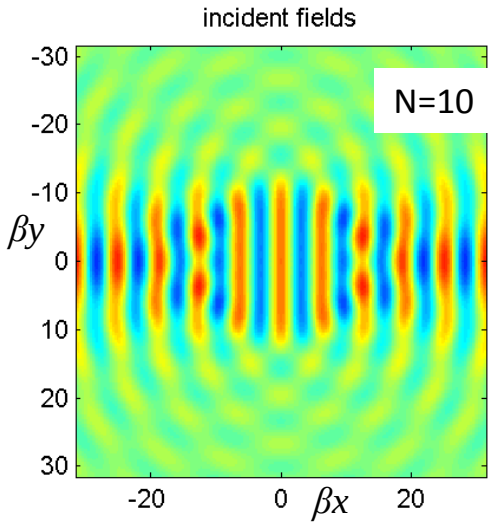
Represent scattered fields as sum of Hankel Fns of 2nd kind

$$\boldsymbol{E_s} = -E_0 \hat{\boldsymbol{z}} \sum_{n=-\infty}^{+\infty} j^{-n} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} H_n^{(2)}(\beta\rho) e^{-jn\varphi}$$

Approximating a Plane Wave with Finite Sum of Bessel Fns.





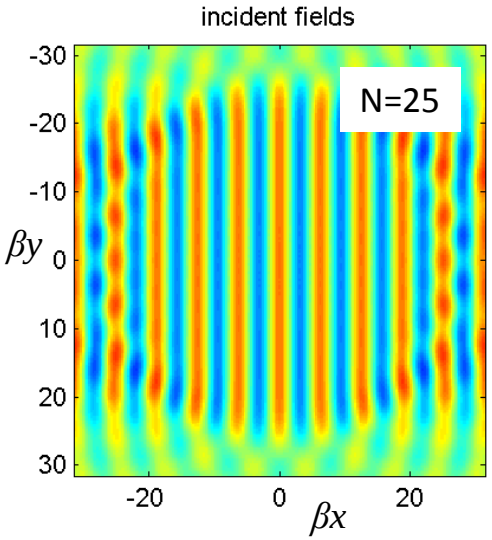


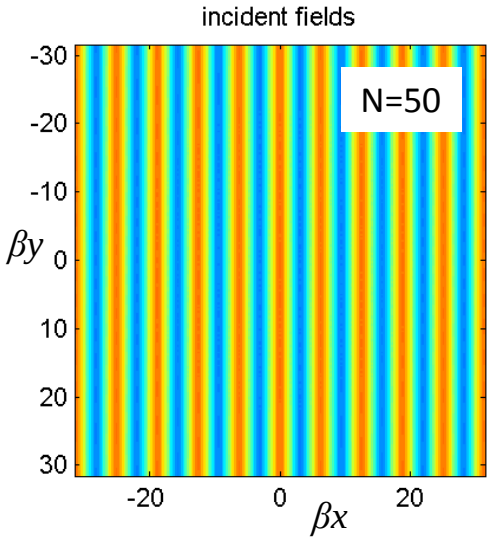
$$\boldsymbol{E_i} = \hat{\boldsymbol{z}} E_0 e^{-j\beta x} \approx \hat{\boldsymbol{z}} E_0 \sum_{n=-N}^{+N} j^{-n} J_n(\beta\rho) e^{-jn\varphi}$$

Approximation is good up to $n \sim \beta\rho$

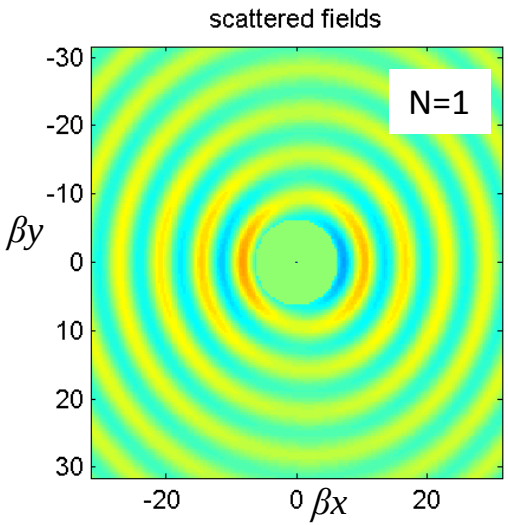
To compute the scattering from a cylinder of radius a, we would therefore need to use at least βa modes.

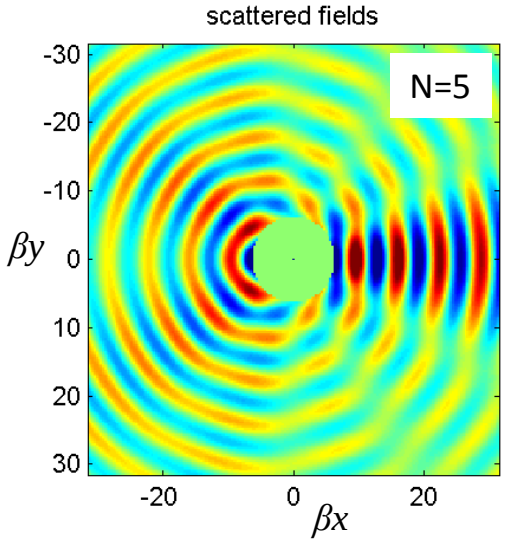
For $a=\lambda$, we need at least 6-7 modes. Modes above n=7 will not contribute significantly to the scattering result

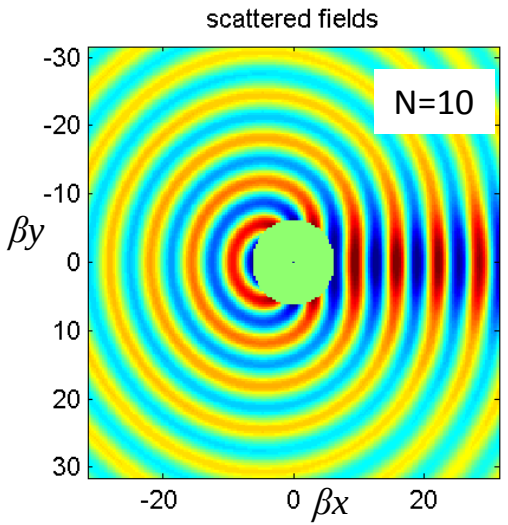




Scattered Fields, using finite number of modes





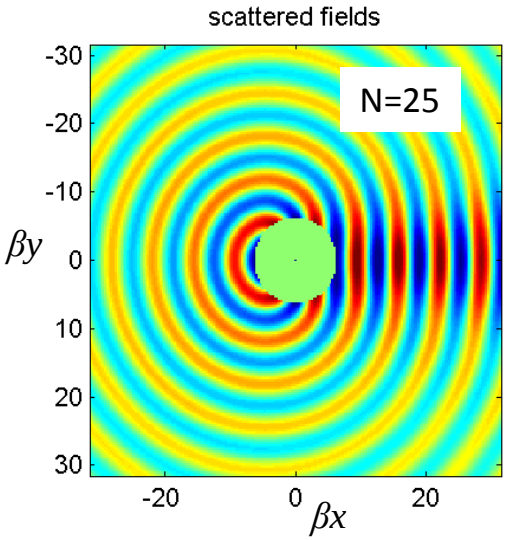


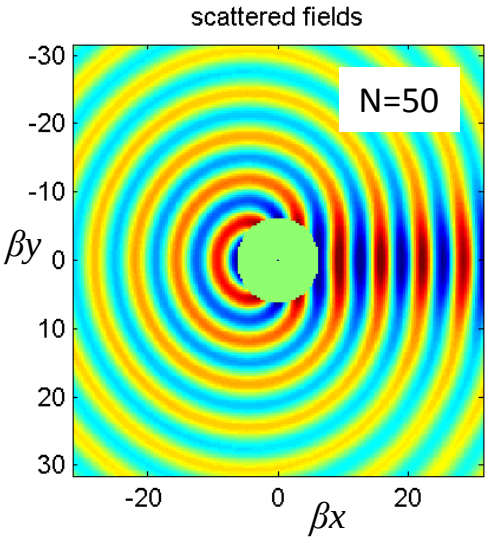
$$\boldsymbol{E_s} \approx -E_0\hat{\boldsymbol{z}} \sum_{n=-N}^{+N} j^{-n} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} H_n^{(2)}(\beta\rho) e^{-jn\varphi}$$

Scattered Fields computed for $a=\lambda$ ($\beta a = 2\pi$)

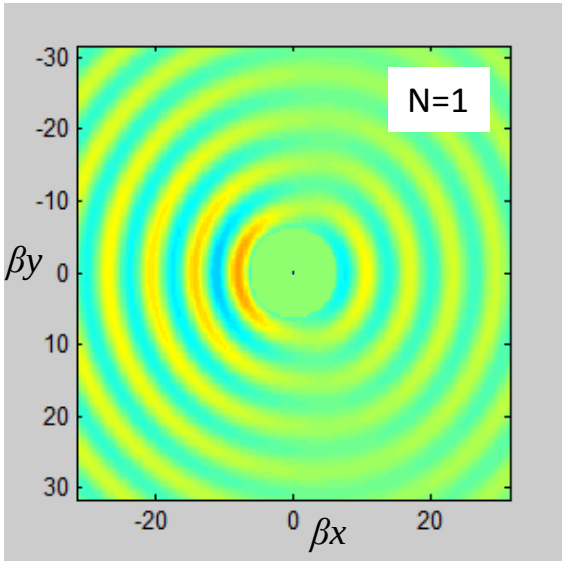
Note that N=1 is completely insufficient, N=5 is close, and the result is essentially identical for N≥10.

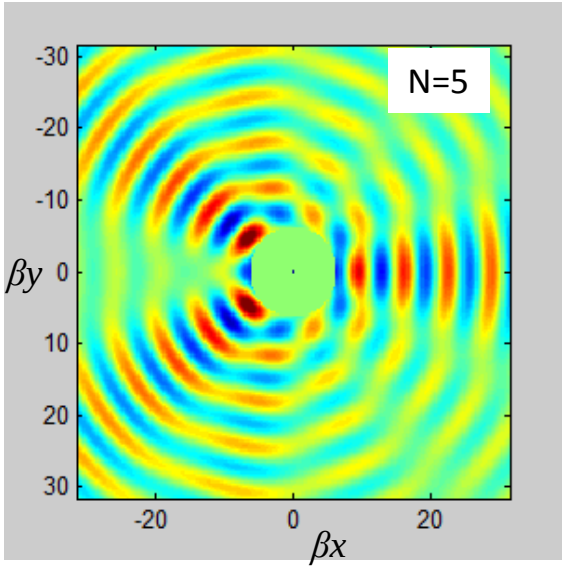
Why is scattered field strongest in the same direction as the incident wave?

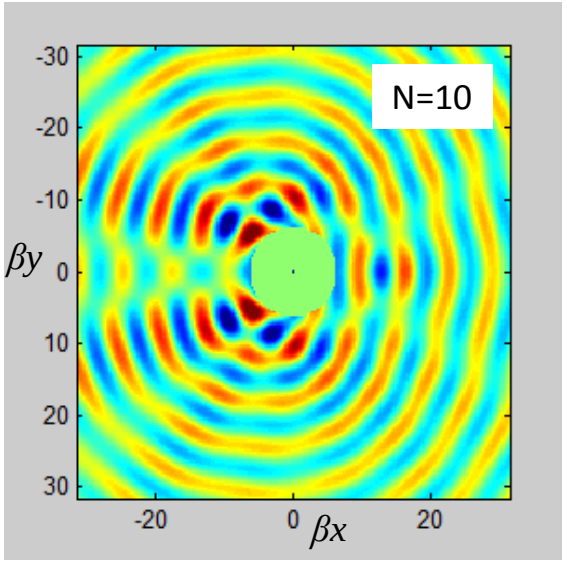




Total Fields, using finite number of modes







$$E_t = E_i + E_s$$

Scattering accounts not only for additional power radiating from the object in new directions, but also accounts for **shadowing**. Scattered fields also "remove" power from incident field.

"Forward Scattering": scattering in the same direction as the incident wave. This is almost always the peak of the scattered pattern, due to shadowing