Media characterization

The relationship between D and E, B and H, and J and E depends on the type of media under consideration. For simple media (linear and isotropic, non-dispersible), the relation

$$
\begin{array}{ll}
\mathrm{D} = \epsilon \mathrm{E} & \epsilon = \epsilon_0 \epsilon_r = \text{permittivity of medium} \\
\mathrm{B} = \mu \mathrm{H} & \mu = \mu_0 \mu_r = \text{permeability of medium} \\
\mathrm{J} = \sigma \mathrm{E} & \sigma = \text{conductivity of medium}
\end{array}
$$

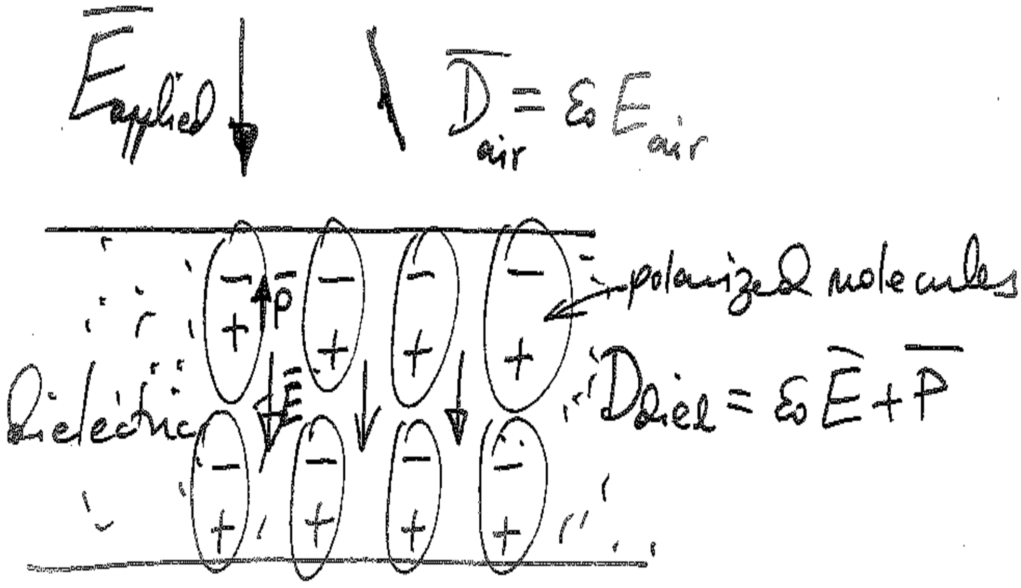
$\epsilon_r > 1$	dielectric
$\epsilon_r = 4$	glass
$\epsilon_r = 4.2$	FR-4 substrate (used for printed circuit boards)
$\epsilon_r = 10.2$	silica
$\epsilon_r \approx 18$	GaAs

For $\mu_r > 1$ we have a magnetic type of medium like iron. Copper has $\sigma \approx 10^7$ S/m. For free space and insulators $\sigma = 1$ (non-conducting).

A more precise characterization of the various media carries the following nomenclature:

- *linear:* constitutive parameters are independent of field intensity (intensity of E, H, etc.) (otherwise *non-linear*).
- *homogeneous:* constitutive parameters are independent of location (otherwise medium is called *inhomogeneous*).
- *dispersive:* parameters are dependent on frequency (otherwise *non-dispersive*).
- *isotropic:* parameters independent of field polarization (otherwise *anisotropic*). Crystals and some biological media cannot support linearly polarized waves, i.e., they are anisotropic.

Explanation of $\mathbf{D} = \epsilon \mathbf{E}$



Molecules contain bound charges that are "polarized," i.e., stretched by the applied field as shown.
Representing the additional flux in dielectric by $\mathbf{P}$, we have

$$\mathbf{D}_{\text{diel}} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

which can be rewritten as (assuming $\mathbf{P}$ is along the same direction as $\mathbf{E}$)

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \chi_{\text{e}} \epsilon_0 \mathbf{E} = \epsilon_0 (1 + \chi_{\text{e}}) \mathbf{E} = \epsilon_0 \epsilon_{\text{r}} \mathbf{E}$$

$$\Rightarrow \qquad \epsilon_{\text{r}} = 1 + \chi_{\text{e}} \quad \text{or} \quad \epsilon_{\text{r}} = 1 + \left| \frac{P}{E} \right|$$

where χ_{e} is the electric susceptibility.

Similarly

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} = \mu_0 \underbrace{(1 + \chi_{\text{m}})}_{\mu_{\text{r}}} \mathbf{H}$$

where χ_{m} = magnetic susceptibility.

Constitutive relations of anisotropic media

$\mathbf{D} = \bar{\bar{\epsilon}}\mathbf{E}$ (dyadic notation, borrowed from tensors) where

$$\bar{\bar{\epsilon}} = [\epsilon] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

Here $\mathbf{D} = \bar{\bar{\epsilon}}\mathbf{E}$ means the matrix equation

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}$$

Example: $D_x = \epsilon_{xx}E_x + \epsilon_{xy}E_y + \epsilon_{yz}E_z$.

Similarly

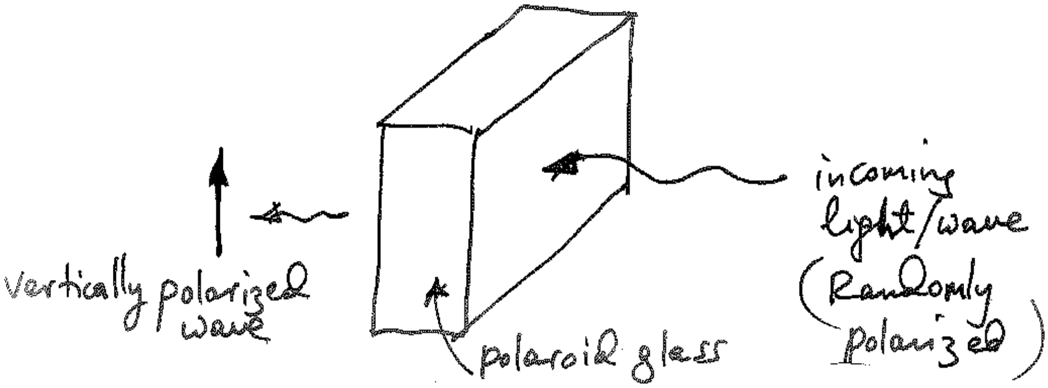
$$\mathbf{B} = \bar{\bar{\mu}}\mathbf{H} \qquad \bar{\bar{\mu}} \to 3 \times 3 \text{ matrix}$$
$$\mathbf{J} = \bar{\bar{\sigma}}\mathbf{E} \qquad \bar{\bar{\sigma}} \to 3 \times 3 \text{ matrix}$$

Most common examples of anisotropic media are those called biaxial and uniaxial.

$$\textit{biaxial} \longrightarrow \qquad \bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

$$\textit{uniaxial}\ (\epsilon_x = \epsilon_y) \longrightarrow \qquad \epsilon = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

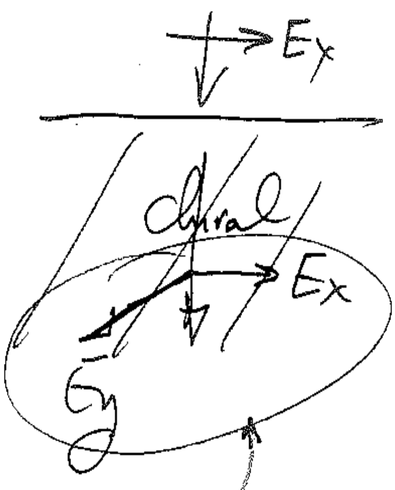
where $\epsilon_x = \epsilon_y$ for uniaxial, and the z-axis is the optics axis. Note that the usual polaroid glass is such a medium.



General bianisotropic media

$$\mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} + \overline{\overline{\zeta}} \cdot \mathbf{B}$$
$$\mathbf{B} = \overline{\overline{\mu}} \cdot \mathbf{H} + \overline{\overline{\xi}} \cdot \mathbf{E}$$

A special case of this medium is the so-called "chiral medium." *Chira* is the Greek word for "hand." That is, the medium exhibits a "handedness effect."



$$\mathbf{E} = \hat{x}E_x + \hat{y}E_y$$
$$E_x = E_y e^{j\pi/2} \qquad \text{for circularly polarized waves}$$

The chiral medium supports both E_x and E_y in quadrature (i.e., 90° phase difference exists between the E_x and E_y).

Math relation of a chiral medium

$$\overline{\overline{\epsilon}} \to \epsilon, \qquad \overline{\overline{\mu}} \to \mu, \qquad \zeta = -\frac{\xi}{\mu} = -j\chi, \qquad \overline{\overline{\zeta}} \to j\mu\chi$$

$$\chi = \text{chirality parameter}$$

$$\boxed{\begin{aligned} \mathbf{D} &= \epsilon\mathbf{E} - j\chi\mathbf{B} \\ \mathbf{B} &= \mu\mathbf{H} + j\mu\chi\mathbf{E} \end{aligned}}$$

The above relations refer to time harmonic (phasor quantities) fields.