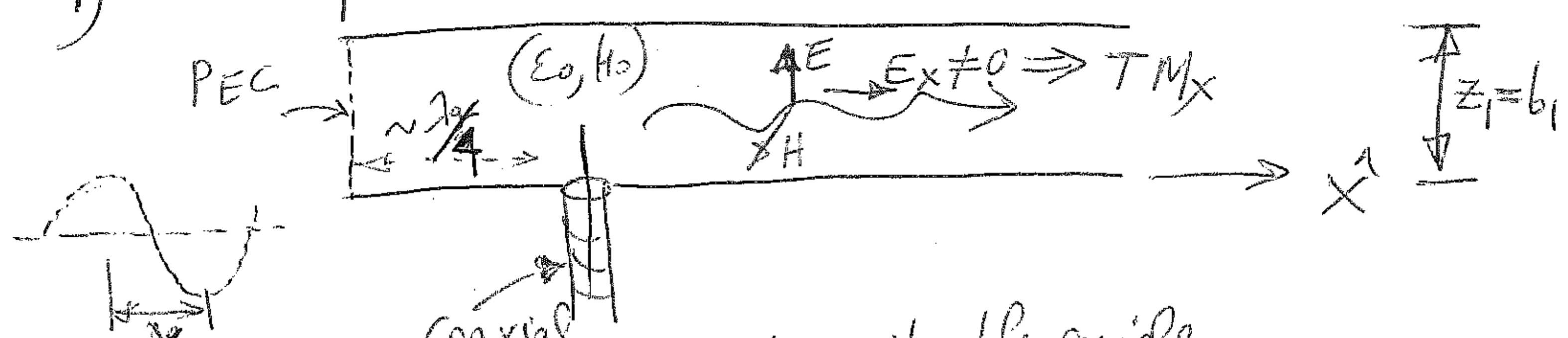
Some questions answered (1

1) How are modes excited

2) How is cut-off frequency (or cut-off wavelength) in a waveguide determined

3) velocity computation

1) parallel plate guide



Coaxial cable is used to excite the guide

TM Modes

$$E_x = \sum_{n=0}^{\infty} A_n \sin\left(\frac{n\pi}{z_1} z\right) e^{-\gamma x} = \sum_{n=0}^{\infty} A_n E_{nx}$$

nth Mode

$$k_z = \frac{n\pi}{z_1}, \quad \gamma = j k_x$$

From

$$\nabla^2 E_x + k_0^2 E_x = 0 \Rightarrow \boxed{k_x^2 + k_z^2 = k_0^2}$$

Characteristic Equ.

$$k_0 = \frac{2\pi}{\lambda_0} = \omega_0 \sqrt{\mu_0 \varepsilon_0} = 2\pi f_0 \sqrt{\mu_0 \varepsilon_0} \quad \text{or} \quad -\gamma^2 + \left(\frac{n\pi}{z_1}\right)^2 = k_0^2$$

freespace wavelength

2) Cut-off frequency & wavelength for the mode

The modes propagate when $\gamma < 0$ or when k_x is real

Noting that $\rightarrow$ $k_x = \sqrt{k_0^2 - k_z^2} = \sqrt{k_0^2 - \left(\frac{n\pi}{z_1}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{n\pi}{z_1}\right)^2}$

the mode begins to propagate when $k_x = 0^+$, i.e. as soon as k_x exceeds zero.

Setting $\gamma = k_x = 0$, we get

$$\frac{2\pi}{\lambda_0} = \frac{n\pi}{z_1} \Rightarrow \boxed{\lambda_0 = \frac{2z_1}{n}}$$

or

$$\omega_0\sqrt{\mu_0\varepsilon_0} = \frac{n\pi}{z_1} \Rightarrow \omega_0 = \frac{n\pi}{z_1}\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Replace with $\frac{1}{\sqrt{\mu\varepsilon}}$ if the guide is filled with dielectric

$$\boxed{f_0 = \frac{n}{2z_1}\frac{1}{\sqrt{\mu_0\varepsilon_0}}}$$

$v_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$ = speed of light

We refer to the above wavelength and frequency as the cut-off wavelengths/frequencies

$$\boxed{\lambda_{0c} = \frac{2z_1}{n}} \quad \text{or} \quad \boxed{z_1 = n\frac{\lambda_{0c}}{2}}$$

$$\boxed{f_{0c} = \frac{n}{2z_1}\frac{1}{\sqrt{\mu_0\varepsilon_0}}} \rightarrow \frac{n}{2z_1}\frac{1}{\sqrt{\mu\varepsilon}}$$

if the guide is filled with another material

~~If the~~ operating frequency is

$$f_0 > f_{0c} \quad \text{or} \quad \lambda_0 < \lambda_{0c} \qquad \left(\lambda = \frac{0.3}{f_{GHz}}\right)$$

then the mode will propagate. Else the mode will have $\gamma > 0$ ~~and~~ will be attenuated and "die-off" soon after it leaves ~~the~~ excitation probe

3) Note that $\gamma = jk_x = j\beta \rightarrow j\beta_{guided} = j\beta_g = j\sqrt{\left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{2\pi}{\lambda_{0c}}\right)^2}$

$$= j\frac{2\pi}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{\lambda_{0c}}\right)^2}$$

$$V_{guided} = V_{phase} = \frac{\omega_0}{\beta} = \frac{\cancel{(2\pi/\lambda_0)}/\sqrt{\mu\varepsilon}}{\cancel{\left(\frac{2\pi}{\lambda_0}\right)}\sqrt{1-\left(\frac{\lambda_0}{\lambda_{0c}}\right)^2}} \rightarrow \boxed{V_g = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\lambda_0/\lambda_{0c}\right)^2}}}$$