IDENTITIES $\left(\begin{matrix}\bar{A}, \bar{B}, \bar{C}, \bar{D} : \text{Vectors} \\ \psi, \phi : \text{Scalars}\end{matrix}\right)$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times \nabla\psi = 0$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$$

$$\nabla \cdot (\psi\mathbf{A}) = \mathbf{A} \cdot \nabla\psi + \psi\nabla \cdot \mathbf{A}$$

$$\nabla \times (\psi\mathbf{A}) = \nabla\psi \times \mathbf{A} + \psi\nabla \times \mathbf{A}$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}\nabla \cdot \mathbf{B} - \mathbf{B}\nabla \cdot \mathbf{A} + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla \cdot \nabla\psi = \nabla^2\psi$$

$\nabla[\phi(J)] = \phi'(J) \cdot \nabla J$, J is a variable

$$\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi \cdot \nabla\psi + \psi\nabla^2\phi$$

$$\nabla(\nabla \cdot \psi\bar{A}) = \nabla\psi\,\nabla \cdot \bar{A} + \psi\nabla\nabla \cdot \bar{A} + \nabla\psi \times (\nabla \times \bar{A}) + (\bar{A} \cdot \nabla)\nabla\psi + (\nabla\psi \cdot \nabla)\bar{A}$$

$$\nabla \times \nabla \times (\psi\bar{A}) = \nabla\psi \times (\nabla \times \bar{A}) - \bar{A}\,\nabla^2\psi + (\bar{A} \cdot \nabla)\nabla\psi + \psi\nabla \times \nabla \times \bar{A} + \nabla\psi\,\nabla \cdot \bar{A} - (\nabla\psi \cdot \nabla)\bar{A}$$

$$\bar{A} \cdot \bar{B} \times \bar{C} = \bar{B} \cdot \bar{C} \times \bar{A} = \bar{C} \cdot \bar{A} \times \bar{B}$$

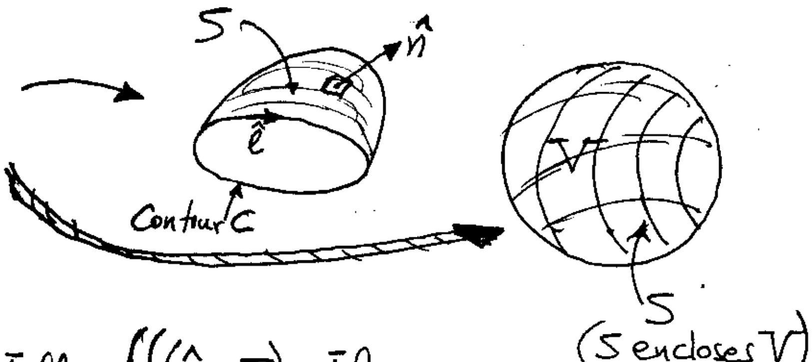
$$\bar{A} \times (\bar{B} \times \bar{C}) = (\bar{A} \cdot \bar{C})\bar{B} - (\bar{A} \cdot \bar{B})\bar{C}$$

$$(\bar{A} \times \bar{B}) \cdot (\bar{C} \times \bar{D}) = \bar{A} \cdot \bar{B} \times (\bar{C} \times \bar{D}) = (\bar{A} \cdot \bar{C})(\bar{B} \cdot \bar{D}) - (\bar{A} \cdot \bar{D})(\bar{B} \cdot \bar{C})$$

$$(\bar{A} \times \bar{B}) \times (\bar{C} \times \bar{D}) = (\bar{A} \times \bar{B} \cdot \bar{D})\bar{C} - (\bar{A} \times \bar{B} \cdot \bar{C})\bar{D}$$

$\oint_C \mathbf{A} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$ Stokes' theorem

$\oiint_S \mathbf{A} \cdot d\mathbf{s} = \iiint_V (\nabla \cdot \mathbf{A})\, dv$ divergence theorem



$$\oiint_S (\hat{n} \times \mathbf{A})\, ds = \iiint_V (\nabla \times \mathbf{A})\, dv$$

$$\oiint_S \psi\, d\mathbf{s} = \iiint_V \nabla\psi\, dv$$

$\oint_C \psi\, d\mathbf{l} = \iint_S \hat{n} \times \nabla\psi\, ds$ or $\oint_C \hat{\ell} \times \bar{A}\, d\ell = \iint_S (\hat{n} \times \nabla) \times \bar{A}\, ds$

$\oiint \psi \frac{\partial\phi}{\partial n}\, ds = \iiint (\psi\nabla^2\phi + \nabla\psi \cdot \nabla\phi)\, d\tau$ Green's 1st Identity

$\oiint (\mathbf{A} \times \nabla \times \mathbf{B}) \cdot d\mathbf{s} = \iiint (\nabla \times \mathbf{A} \cdot \nabla \times \mathbf{B} - \mathbf{A} \cdot \nabla \times \nabla \times \mathbf{B})\, dv$ Vector Analog of Green's 1st Identity

$\oiint \left(\psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\psi}{\partial n}\right) ds = \iiint (\psi\nabla^2\phi - \phi\nabla^2\psi)\, dv$ Green's Theorem (or Green's 2nd Identity)

$\oiint (\mathbf{A} \times \nabla \times \mathbf{B} - \mathbf{B} \times \nabla \times \mathbf{A}) \cdot d\mathbf{s}$
$= \iiint (\mathbf{B} \cdot \nabla \times \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \nabla \times \mathbf{B})\, dv$ Vector Analog of Green's theorem

Maxwell's Equations for Time Harmonic Fields

$$\mathcal{E}(x, y, z; t) = \mathrm{Re}[\mathbf{E}(x, y, z)e^{j\omega t}]$$
$$= \hat{x}E_{x0}\cos(\omega t + \phi_x) + \hat{y}E_{y0}\cos(\omega t + \phi_y) + \hat{z}E_{z0}\cos(\omega t + \phi_z)$$

where the complex vector

$$\mathbf{E}(x, y, z) = \hat{x}E_{x0}e^{j\phi_x} + \hat{y}E_{y0}e^{j\phi_y} + \hat{z}E_{z0}e^{j\phi_z}$$

Differential Forms

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mathbf{E}$$ Maxwell-Ampere Law

$$\nabla \times \mathbf{E} = -\mathbf{M} - j\omega\mu\mathbf{H}$$ Faraday's Law

$$\nabla \cdot (\mu\mathbf{H}) = \rho_m$$

$$\nabla \cdot (\epsilon\mathbf{E}) = \rho$$

Continuity equations

$$\nabla \cdot \mathbf{J} + j\omega\rho = 0$$

$$\nabla \cdot \mathbf{M} + j\omega\rho_m = 0$$

Duality Relations

$\mathbf{J} \to \mathbf{M}$
$\mathbf{M} \to -\mathbf{J}$
$\mathbf{E} \to \mathbf{H}$
$\mathbf{H} \to -\mathbf{E}$
$\mu \to \epsilon$
$\epsilon \to \mu$

For Isotropic Media

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\epsilon_r\mathbf{E}$$
$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mu_r\mathbf{H}$$
$$\mathbf{J} = \sigma\mathbf{E}$$
$$\mathbf{M} = \sigma_m\mathbf{H}$$

$$\dot{\epsilon}_r = \epsilon_r - j\frac{\sigma}{\omega\epsilon_0} = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta)$$

$$\dot{\mu}_r = \mu_r - j\frac{\sigma_m}{\omega\mu_0} = \mu' - j\mu'' = \mu'(1 - j\tan\delta_m)$$

Loss tangents

$$\tan\delta = \frac{\epsilon''}{\epsilon'}$$

$$\tan\delta_m = \frac{\mu''}{\mu'}$$

For Anisotropic Media: $\bar{D} = \bar{\bar{\epsilon}}\cdot\bar{E}$, $\bar{B} = \bar{\bar{\mu}}\cdot\bar{H}$

For Bianisotropic Media: $\bar{D} = \bar{\bar{\epsilon}}\cdot\bar{E} + \bar{\bar{\zeta}}\cdot\bar{B}$, $\bar{B} = \bar{\bar{\mu}}\cdot\bar{H} + \bar{\bar{\xi}}\cdot\bar{E}$

Integral Forms

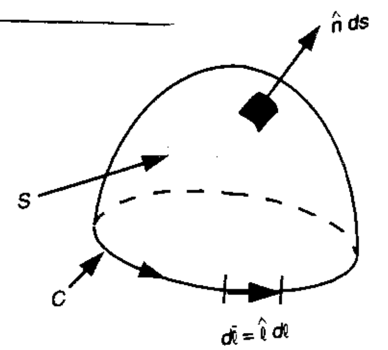
$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \iint_S (\mathbf{J}_i + j\omega\epsilon\mathbf{E})\cdot d\mathbf{S}$$

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\iint_S (\mathbf{M}_i + j\omega\mu\mathbf{H})\cdot d\mathbf{S}$$

$$\oiint_{S_c} \mu\mathbf{H}\cdot d\mathbf{S} = -\iiint_V \frac{\nabla\cdot\mathbf{M}_i}{j\omega}\,dV = \iiint_V \rho_m\,dV$$

$$\oiint_{S_c} \epsilon\mathbf{E}\cdot d\mathbf{S} = -\iiint_V \frac{\nabla\cdot\mathbf{J}_i}{j\omega}\,dV = \iiint_V \rho\,dV$$

S_c encloses V



UNITS...UNITS

$\mathbf{E}$ = electric field intensity in volts/meter (V/m)
$\mathbf{H}$ = magnetic field intensity in amperes/meter (A/m)
$\mathbf{J}$ = electric current density in amperes/meter^2 (A/m^2)
$\mathbf{M}$ = magnetic current density in volts/meter^2 (V/m^2)
$\mathbf{D}$ = electric flux density in coulombs/meter^2 (C/m^2)
$\mathbf{B}$ = magnetic field density in webers/meter^2 (Wb/m^2)
ρ = electric charge density in coulombs/meter^3 (C/m^3)
ρ_m = magnetic charge density in webers/meter^3 (Wb/m^3)

ϵ_0 = free space permittivity = 8.854×10^{-12} farads/meter (F/m)
μ_0 = free space permeability = $4\pi \times 10^{-7}$ henrys/meter (H/m)
ϵ_r = medium's relative permittivity constant
μ_r = medium's relative permeability constant
σ = electric current conductivity in mhos/m ($\frac{1}{\Omega}$/m) or S/m
σ_m = magnetic current conductivity in ohms/m (Ω/m)

RADIATION CONDITION

$$\lim_{r\to\infty} r\left[\nabla \times \begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} + jk_0\hat{r} \times \begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix}\right] = 0$$

$$k_0 = \frac{2\pi}{\lambda_0} = \omega\sqrt{\mu_0\epsilon_0}$$