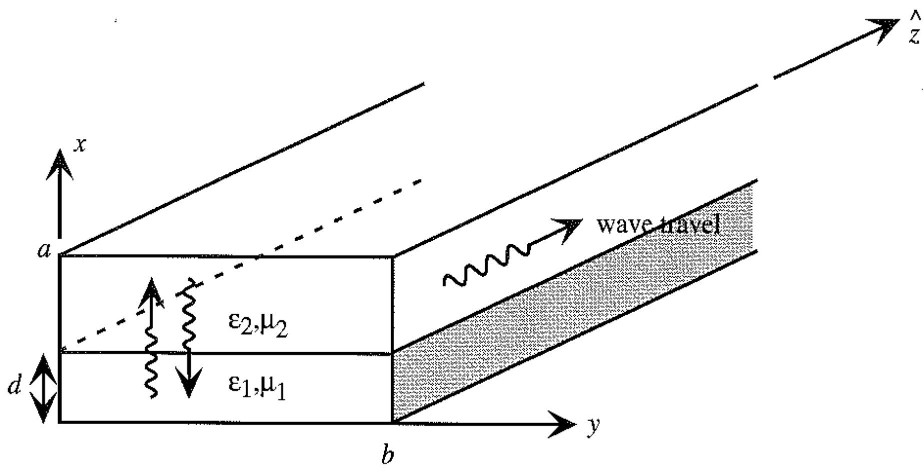
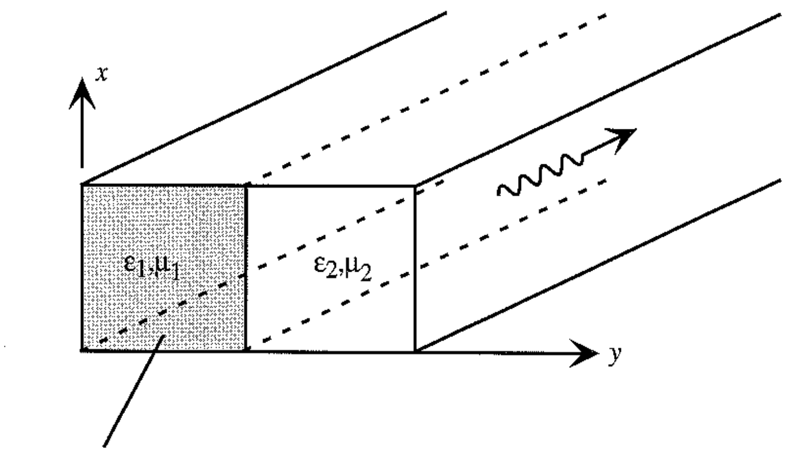
Partially filled guides



An attempt to satisfy BCs using TE^z or TM^z modes will fail. Instead, for this geometry, TE^x and TM^x modes will do the job. Similarly for the guide



TE^y and TM^y modes will do the job.

Let us consider the top geometry (Harrington p. 162, Balanis p. 394). To generate the TM^x modes, we choose the potential component.

$$\mathbf{A} = \hat{x} A_x$$

Then, the fields are given by (see p. 397 of Balanis)

$$E_x = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial x^2} + k^2\right) A_x \qquad H_x = 0$$

$$E_y = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2 A_x}{\partial x \partial y} \qquad H_y = \frac{\partial \psi}{\partial z}$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2 A_x}{\partial x \partial z} \qquad H_z = -\frac{\partial \psi}{\partial y}$$

Since the waves propagate along z, the wavefunction must be

$$\text{medium 1:}\quad \mathbf{A}_1 = \hat{x}\psi_1 = \hat{x}C_1 \left\{ \begin{array}{c} \cos k_{x1}x \\ \sin k_{x1}x \end{array} \right\} \left\{ \begin{array}{c} \cos k_{y1}y \\ \sin k_{y1}y \end{array} \right\} e^{-jk_z z}$$

$$\text{medium 2:}\quad \mathbf{A}_2 = \hat{x}\psi_2 = \hat{x}C_2 \left\{ \begin{array}{c} \cos k_{x2}x \\ \sin k_{x2}x \end{array} \right\} \left\{ \begin{array}{c} \cos k_{y2}y \\ \sin k_{y2}y \end{array} \right\} e^{-jk_z z}$$

Since $E_x = 0$ at $y = 0$ and $y = b$, and

$$E_{x_{\frac{1}{2}}} = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial x^2} + k^2\right) A_{x_{\frac{1}{2}}} \qquad \leftarrow \text{i.e., proportional to } A_x$$

$$E_{y_1} \sim \frac{\partial^2 A_x}{\partial x\,\partial y} = 0 \quad \text{at } x = 0,\ E_{y_2} = 0,\ x = a$$

it follows that

$$\mathbf{A}_1 = \hat{x}C_1 \cos k_{x_1}x \sin\left(\frac{n\pi}{b}y\right)\, e^{-jk_z z}$$

$$\mathbf{A}_2 = \hat{x}C_2 \cos[k_{x_2}(a-x)] \sin\left(\frac{n\pi}{b}y\right)\, e^{-jk_z z}$$

That is,

$$k_{y_1} = k_{y_2} = \frac{n\pi}{b}$$

We still need to find $k_{z_{1,2}}$ and $k_{x_{1,2}}$. These must of course satisfy the consistency or characteristic equations

$$k_{x_1}^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = k_1^2 \qquad \text{med. \#1}$$

$$k_{x_2}^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = k_2^2 \qquad \text{med. \#2}$$

We need two more equations to obtain (k_{x_1}, k_{x_2}) and (k_{z_1}, k_{z_2}). These will come by satisfying continuity of the fields at $x = d$.

1) Specifically,

$$E_{\tan_1} = E_{\tan_2} \qquad \text{or } E_{y_1} = E_{y_2} \text{ and } E_{z_1} = E_{z_2}$$

implying that

$$\boxed{\frac{1}{\varepsilon_1}\frac{C_1}{C_2} k_{x_1} \sin(k_{x_1}d) = -\frac{1}{\varepsilon_2} k_{x_2} \sin[k_{x_2}(a-d)]}$$

This equation can also be obtained by matching the wave impedances

$$Z_{+x} = \left(\frac{E_y}{H_z}\right)^{\text{med. 1}} = Z_{-x} = \left(\frac{E_y}{H_z}\right)^{\text{med. 2}}$$

This is referred to as impedance matching at the dielectric interface.

2) Also for the $\mathbf{H}$ field

$$H_{\tan_1} = H_{\tan_2} \qquad \text{or } H_{y_1} = H_{y_2} \text{ and } H_{z_1} = H_{z_2}$$

leading to

$$\boxed{\frac{C_1}{C_2}\cos(k_{x_1}d = \cos[k_{x_2}(a-d)]}$$

Similarly, this equation can be obtained by impedance matching at the dielectric interface.

$$Z_{+x} = \left(\frac{E_y}{H_z}\right)^{\text{med. 1}} = -Z_{-x} = \left(\frac{E_y}{H_z}\right)^{\text{med. 2}}$$

These two equations can be used to obtain C_1/C_2 and k_{x_1}, k_{x_1}. However, we can eliminate C_1/C_2 by dividing them to get

$$\boxed{\frac{k_{x1}}{\varepsilon_1}\tan k_{x_1}d = -\frac{k_{x2}}{\varepsilon_2}\tan[k_{x_2}(a-d)]}$$

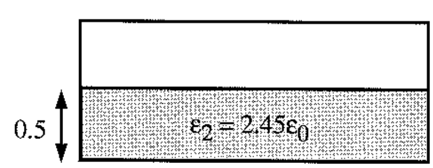
This is a transcendental equation which can be solved in conjunction with the characteristic equations. Note that $k_{x_1} = k_{x_2} = 0$ for $\varepsilon_2 = \varepsilon_1 = \varepsilon_0$ (empty or uniform guide). Thus, it is reasonable to assume that k_{x_1} and k_{x_2} are small. Consequently, the transcendental equation can be approximated by

$$\frac{(k_{x_1})^2}{\varepsilon_1}d = -\frac{(k_{x_2})^2}{\varepsilon_2}(a-d)$$
$$+k_{x_1}^2 + \left(\frac{\pi}{b}\right)^2 + k_z^2 = k_1^2$$
$$k_{x_2}^2 + \left(\frac{\pi}{b}\right)^2 + k_z^2 = k_2^2$$
$$k_{x_2}^2 = -\frac{\varepsilon_2}{\varepsilon_1}\left(\frac{d}{a-d}\right)k_{x_1}^2$$

which can be combined with the characteristic equations to solve for k_z. Also, we remark that for $b > a$, the dominant mode is

$$\mathrm{TM}_{01}^{x} \xrightarrow{\text{goes to}} \mathrm{TE}_{01} \quad \text{empty guide}$$

As an example, let us consider the guide where the lower half has $\varepsilon_2 = 2.45\varepsilon_0$ and the top half is empty. Also, $a = 1$ and $b = 2.22$ (i.e., $\frac{a}{b} = 0.45$).

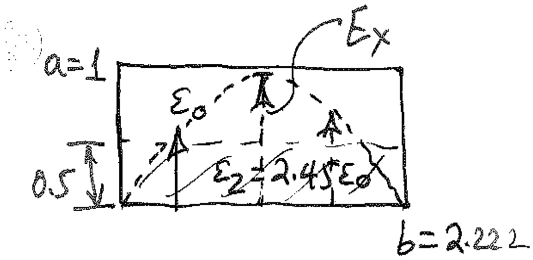


TE modes: For the TE modes, we follow the normal procedure and choose

$$\mathbf{F} = \hat{x}F_x = \hat{x}\psi_{1,2}$$
$$\psi_1 = C_1 \sin k_{x_1}x \cos\left(\frac{n\pi}{b}y\right) e^{-jk_z z}$$
$$\psi_2 = C_2 \sin[k_{x_2}(a-x)\cos\left(\frac{n\pi}{b}y\right) e^{-jk_z z}$$

Following the same approach as done for the TM modes, we obtain the transcendental equation

$$\frac{k_{x_1}}{\mu_1} \cot k_{x_1} d = \frac{-k_{x_2}}{\mu_2} \cot[k_{x_2}(a-d)]$$



$\mu_1 = \mu_2$

$$\omega_c \simeq \frac{\pi}{b}\sqrt{\frac{\varepsilon_1(a-d)+\varepsilon_2 d}{\mu_0 \varepsilon_1 \varepsilon_2 a}}$$

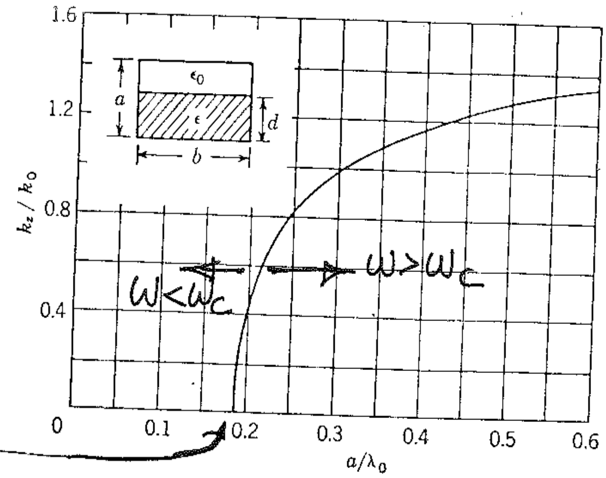


FIG. 4-7. Propagation constant for a rectangular waveguide partially filled with dielectric, $\epsilon = 2.45\epsilon_0$, $a/b = 0.45$, $d/a = 0.50$. (*After Frank.*)

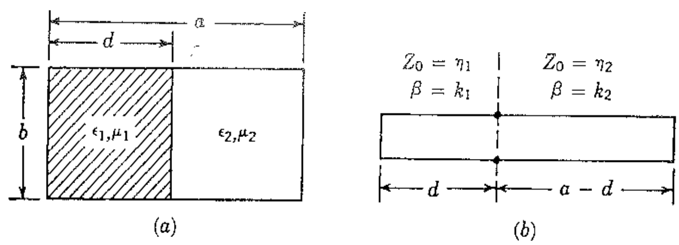


FIG. 4-8. (*a*) Partially filled waveguide; (*b*) transmission-line resonator. The cutoff frequency of the dominant mode of (*a*) is the resonant frequency of (*b*).

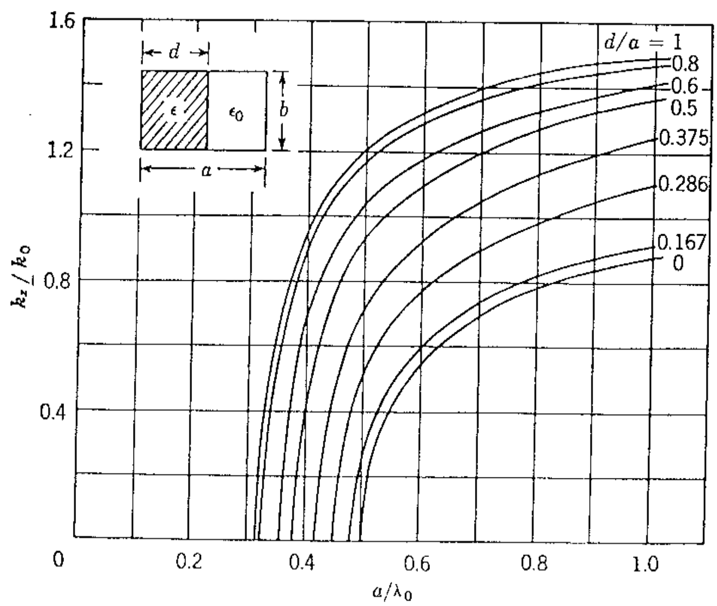


FIG. 4-9. Propagation constant for a rectangular waveguide partially filled with dielectric, $\varepsilon = 2.45\epsilon_0$. (*After Frank.*)