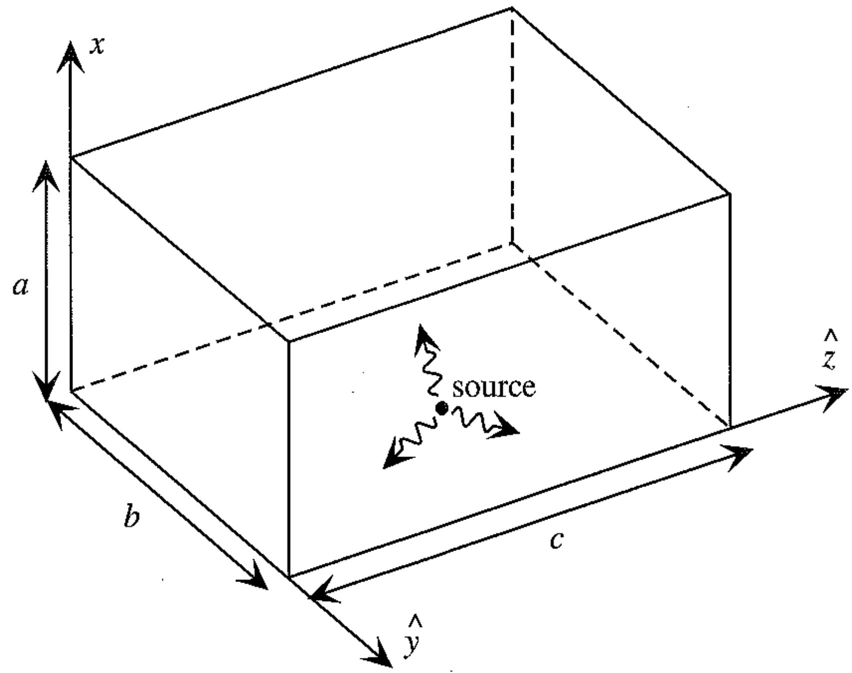
Cavities



If the waveguide is closed at $z = 0$ and at $z = c$, then the wave functions must be revised to include propagation along the $+z$ and $-z$ directions.

TM modes

$$A_z = \Psi^{\mathrm{TM}}(x, y, z) = B_{mn}^{\mathrm{TM}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)\ h(z)$$

To determine the sinusoidal form of $h(z)$, note the boundary conditions

1) $E_x = 0,\ \ Z = 0,\ \ Z = b$

2) $E_y = 0,\ \ Z = 0,\ \ Z = b$

Further, since

$$E_x = -\frac{j}{\omega\mu\epsilon}\frac{\partial^2 A_z}{\partial x\,\partial z}$$

and

$$E_y = -\frac{j}{\omega\mu\epsilon}\frac{\partial^2 A_z}{\partial x\,\partial z}$$

it follows that

$$\frac{dh(z)}{dz}, \qquad z = 0, \qquad z = c$$

Thus

$$h(z) = \cos\left(\frac{p\pi}{c}z\right), \qquad p = 0, 1, 2, 3, 4, \ldots$$

and

$$A_z = \Psi^{\mathrm{TM}}(x, y, z) = B_{mnp}^{\mathrm{TM}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{c}z\right)$$

subject to the characteristic equation

$$\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = k^2 = \omega^2\mu\epsilon$$

so that the wave equation is satisfied. This condition implies that only certain frequencies can excite fields/waves within the cavity. These are the resonant frequencies of the cavity and are given by

$$\omega_{mnp} = \frac{\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}}{\sqrt{\mu\epsilon}}$$

Your microwave oven is designed to resonate at one of these frequencies, usually the lowest possible frequency.

TE modes

$$F_z = \Psi^{\mathrm{TE}}(x,y,z) = A_{mn}^{\mathrm{TE}} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)\ h(z)$$

Since E_x and E_y is proportional to $h(z)$, i.e., no derivative with respect to z is involved, it follows that

$$h(z) = \sin\left(\frac{p\pi}{c}z\right), \qquad p = 0, 1, 2, \ldots$$

so that $E_x = E_y = 0$ at $z = 0$ and at $z = b$. Thus

$$F_z = \Psi_{mnp}^{\mathrm{TE}} = A_{mnp}^{\mathrm{TE}} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{p\pi}{c}z\right)$$

where again (m, n, p) are subject to the characteristic relation.

We remark that for $a < b < c$ the dominant or lowest order mode is TE_{011}. Also for m, n, p all nonzero, the TE_{mnp} and TM_{mnp} are degenerate. That is, the TE and TM modes exist concurrently at the corresponding frequencies ω_{mnp}.

TABLE 4-3. $\frac{(f_r)_{mnp}}{(f_r)_{011}}$ FOR THE RECTANGULAR CAVITY, $a \le b \le c$

$\frac{b}{a}$	$\frac{c}{a}$	TE_{011}	TE_{101}	TM_{110}	TM_{111} TE_{111}	TE_{012}	TE_{021}	TE_{201}	TE_{102}	TM_{120}	TM_{210}	TM_{112} TE_{112}
1	1	1	1	1	1.22	1.58	1.58	1.58	1.58	1.58	1.58	1.73
1	2	1	1	1.26	1.34	1.26	1.84	1.84	1.26	2.00	2.00	1.55
2	2	1	1.58	1.58	1.73	1.58	1.58	2.91	2.00	2.00	2.91	2.12
2	4	1	1.84	2.00	2.05	1.26	1.84	3.60	2.00	2.53	3.68	2.19
4	4	1	2.91	2.91	3.00	1.58	1.58	5.71	3.16	3.16	5.71	3.24
4	8	1	3.62	3.65	3.66	1.26	1.84	7.20	3.65	4.03	7.25	3.82
4	16	1	3.88	4.00	4.01	1.08	1.96	7.76	3.91	4.35	7.83	4.13

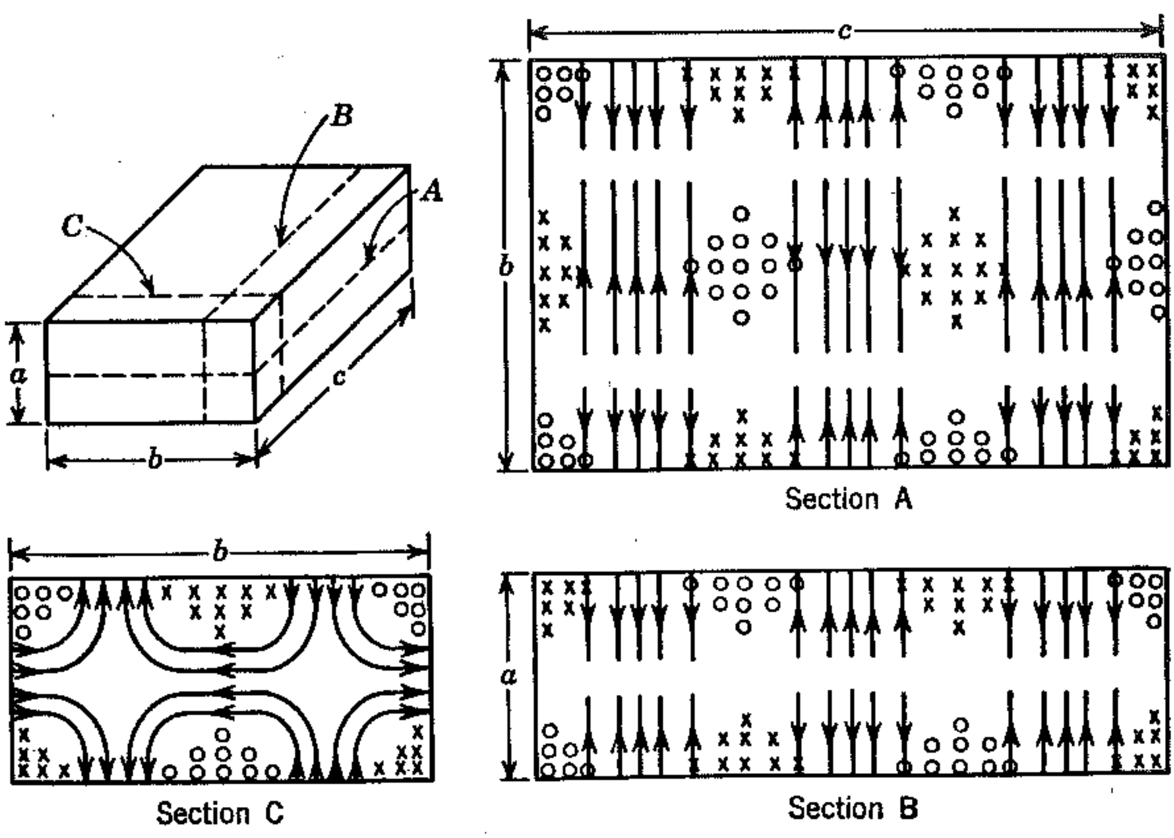


FIG. 4-5. Rectangular cavity mode pattern for the TE_{123} mode.

TABLE 4-3. $\frac{(f_r)_{mnp}}{(f_r)_{011}}$ FOR THE RECTANGULAR CAVITY, $a \leq b \leq c$

$\frac{b}{a}$	$\frac{c}{a}$	TE_{011}	TE_{101}	TM_{110}	TM_{111} TE_{111}	TE_{012}	TE_{021}	TE_{201}	TE_{102}	TM_{120}	TM_{210}	TM_{112} TE_{112}
1	1	1	1	1	1.22	1.58	1.58	1.58	1.58	1.58	1.58	1.73
1	2	1	1	1.26	1.34	1.26	1.84	1.84	1.26	2.00	2.00	1.55
2	2	1	1.58	1.58	1.73	1.58	1.58	2.91	2.00	2.00	2.91	2.12
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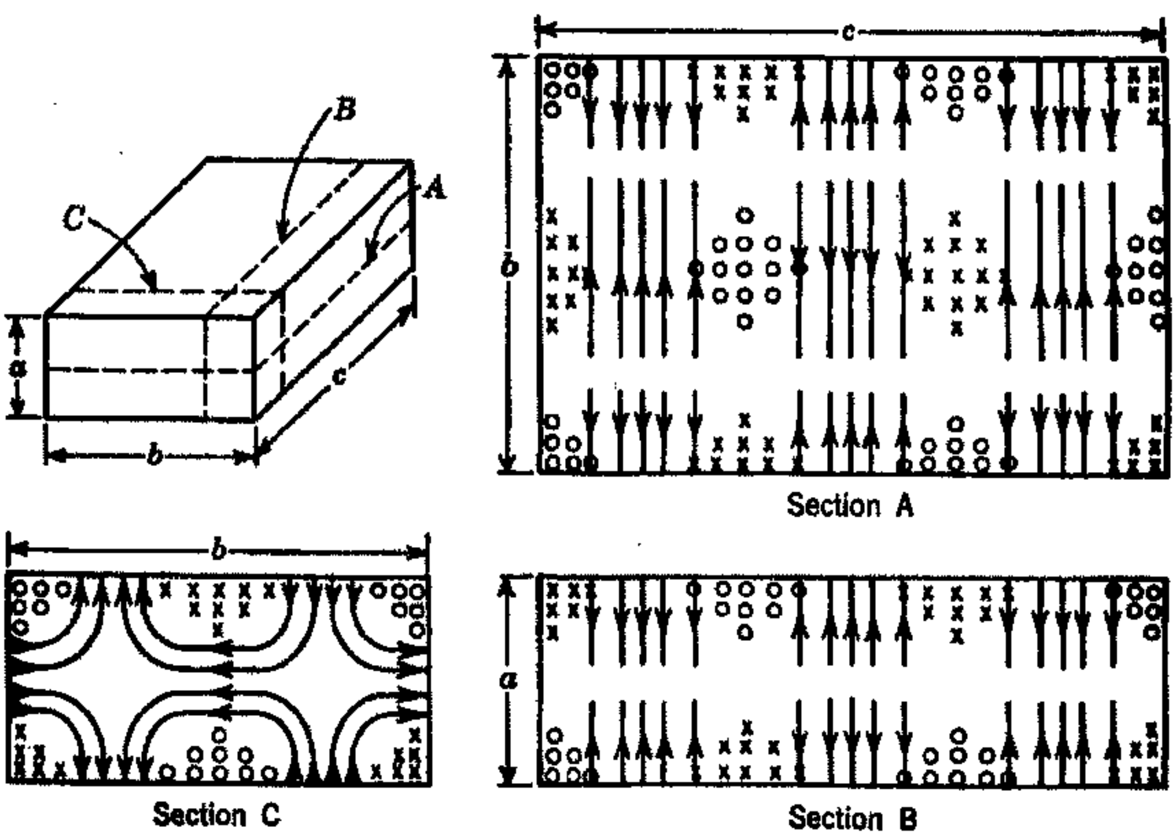

FIG. 4-5. Rectangular cavity mode pattern for the TE_{123} mode.