Near-zone field of time-harmonic electric line source

Consider an electric line source with arbitrary current distribution $I(z')$. This source is located on the z axis, extends from z_1 to z_2 and radiates in unbounded free space. The vector potential A has only a z component, and thus the fields are given by

$$E_\rho = \frac{1}{j\omega\mu\varepsilon} \frac{\partial^2 A_z}{\partial\rho\,\partial z} \tag{1}$$

$$E_z = \frac{1}{j\omega\mu\varepsilon} \left(k^2 A_z + \frac{\partial^2 A_z}{\partial z^2} \right) \tag{2}$$

where

$$A(x,y,z) = \frac{\mu}{4\pi} \int \frac{I(z')\, e^{-jkR}}{R}\, dz' \tag{3}$$

Interchanging the order of integration and differentiation, we obtain

$$Z\text{: medium impedance} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_z = \frac{-jZ}{4\pi k} \int I(z')\, \frac{e^{-jkR}}{R^5} \left[2(1+jkR)R^2 - (3+3jkR-k^2R^2)\rho^2 \right] dz' \qquad \rho = \sqrt{x^2+y^2} \tag{4}$$

$$E_z = \frac{-jZ}{4\pi k} \int I(z')\, \frac{e^{-jkR}}{R^3} \left[(3\cos^2\theta - 1)(1+jkR) + k^2R^2 \sin^2\theta \right] dz' \tag{5}$$

The charge density is given by

$$\rho_\upsilon(z) = -\frac{-I'(z)}{j\omega} \tag{6}$$

where

$$I'(z) = \frac{dI(z)}{dz}$$

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi \qquad \mathbf{\Phi}\text{: scalar potential} \tag{7}$$

From eqs. 6 and 7,

$$\begin{aligned} E_z = {} & \frac{jZ}{4\pi k} \int I'(z)\, \frac{e^{-jkR}}{R^2}\, (1+jkR)\cos\theta\, dz' - \frac{jkZ}{4\pi} \int I(z')\, \frac{e^{-jkR}}{R}\, dz' \\ & - \frac{jkZ}{4\pi} \left[I(z)\, e^{-jkR} \frac{1+jkR}{k^2R^2} \cos\theta \right]_{z_1}^{z_2} \end{aligned} \tag{8}$$

Equation 8 can also be obtained from eq. 2 by integrating once by parts. Integrating (8) once by parts again, we obtain

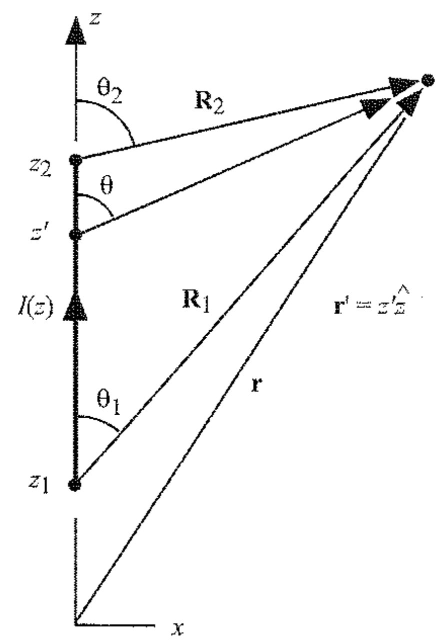
$$\begin{aligned} E_z = {} & \frac{jZ}{4\pi k} \left[I'(z')\, \frac{e^{-jkR}}{R} - I(z')\, \frac{e^{-jkR}}{R^2}\, (1+jkR)\cos\theta \right]_{z_1}^{z_2} \\ & - \frac{jZ}{4\pi k} \int_{z_1}^{z_2} \left[I''(z') + k^2 I(z') \right] \frac{e^{-jkR}}{R}\, dz' \end{aligned} \tag{9}$$

where I'' denotes the second derivative and $R = \sqrt{x^2+y^2+(z-z')^2}$. For sinusoidal currents, the integral of the second line reduces to

$$\left[\frac{-jZ}{4\pi}\int_{z_1}^{z_2}\frac{d}{dz'}\left\{I(z')\left[\delta(z'-z_1)-\delta(z'-z_2)\right]\right\}\frac{e^{-jkR}}{R}\,dz'\right]$$

This can be readily integrated by transferring the derivative with respect to z to operate on e^{-jkR}/R via integration by parts. The resulting expression will cancel the second term of (9). Thus, for sinusoidal currents

$$E_z = \frac{jZ}{4\pi k}\left[I'(z')\,\frac{e^{-jk\,R(z')}}{R(z')}\right]_{z_1}^{z_2}$$



$$\cos\theta_{\substack{1\\2}} = \frac{z - z_{\substack{1\\2}}}{R_{\substack{1\\2}}}$$