Laws and Theorems of Boolean Algebra

Operations with 0 and 1:

1. $X + 0 = X$
2. $X + 1 = 1$

1D. $X \cdot 1 = X$
2D. $X \cdot 0 = 0$

Idempotent laws:

3. $X + X = X$

3D. $X \cdot X = X$

Involution law:

4. $(X')' = X$

Laws of complementarity:

5. $X + X' = 1$

5D. $X \cdot X' = 0$

Commutative laws:

6. $X + Y = Y + X$

6D. $XY = YX$

Associative laws:

7. $(X + Y) + Z = X + (Y + Z) = X + Y + Z$

7D. $(XY)Z = X(YZ) = XYZ$

Distributive laws:

8. $X(Y + Z) = XY + XZ$

8D. $X + YZ = (X + Y)(X + Z)$

Simplification theorems:

9. $XY + XY' = X$
10. $X + XY = X$
11. $(X + Y')Y = XY$

9D. $(X + Y)(X + Y') = X$
10D. $X(X + Y) = X$
11D. $XY' + Y = X + Y$

DeMorgan's laws:

12. $(X + Y + Z + \ldots)' = X'Y'Z' \ldots$

12D. $(XYZ \ldots)' = X' + Y' + Z' + \ldots$

Duality:

13. $(X + Y + Z + \ldots)^D = XYZ \ldots$

13D. $(XYZ \ldots)^D = X + Y + Z + \ldots$

Theorem for multiplying out and factoring:

14. $(X + Y)(X' + Z) = XZ + X'Y$

14D. $XY + X'Z = (X + Z)(X' + Y)$

Consensus theorem:

15. $XY + YZ + X'Z = XY + X'Z$

15D. $(X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z)$