CWR-5535C: AMAWRE

Handout in Support of Chapter 1:
Basic Hydraulic Principles
(CAHE, Bentley, 9th Ed.)

Reference: Wurbs & James, 2002
(Prentice-Hall).

4.1.2 Energy Equation

Figure 4.2 represents a pumped-storage hydroelectric plant where water is pumped to the upper reservoir during the off-peak power period and used to generate electricity during the peak power period. In Fig. 4.2a, water is being pumped from the lower supply reservoir through a pipeline to an upper storage reservoir. Water discharges from the upper reservoir in Fig. 4.2b through a pipeline and turbine into the lower reservoir.

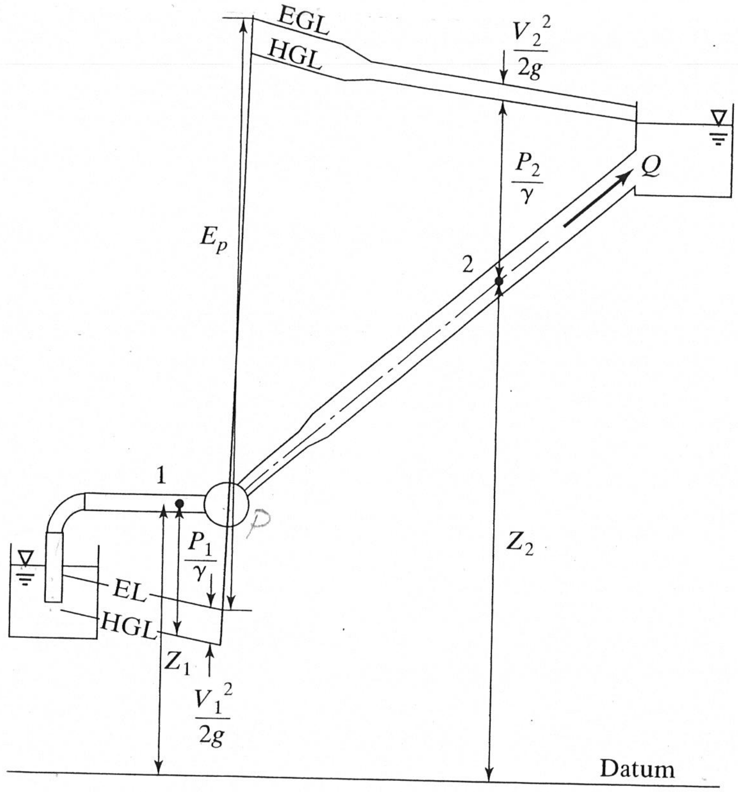


Figure 4.2a Pump system.

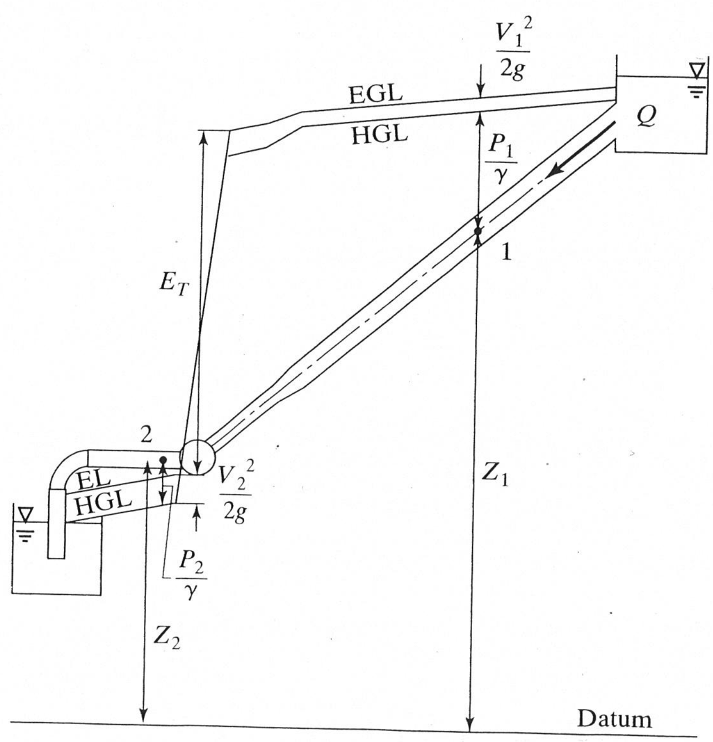


Figure 4.2b Turbine system.

The energy grade line (EGL) and hydraulic grade line (HGL) are shown in Fig. 4.2. The HGL is located one velocity head ($V^2/2g$) below the EGL. The EGL and HGL are parallel when the pipe size is uniform. The EGL slopes downward in the direction of flow because of energy loss. The vertical distance between the center of the pipe and the HGL is the pressure head (P/γ). If the HGL is above the pipe, the pressure is positive, and if the HGL is below the pipe, the pressure is negative. Z is the vertical distance above the datum (usually mean sea level—msl).

The energy equation written for flow in a pipeline (Fig. 4.3) is

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 + E_P = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + H_L \qquad \textbf{(4.4)}$$

where z is the elevation of the pipe, P/γ is the pressure head, $V^2/2g$ is the velocity head, E_P is the energy head added by the pump, and H_L is the total headloss between points 1 and 2. Each term in the energy equation has units of length and represents energy per unit weight of fluid (Newton-meters per Newton of fluid flowing or ft-lbs per lb).

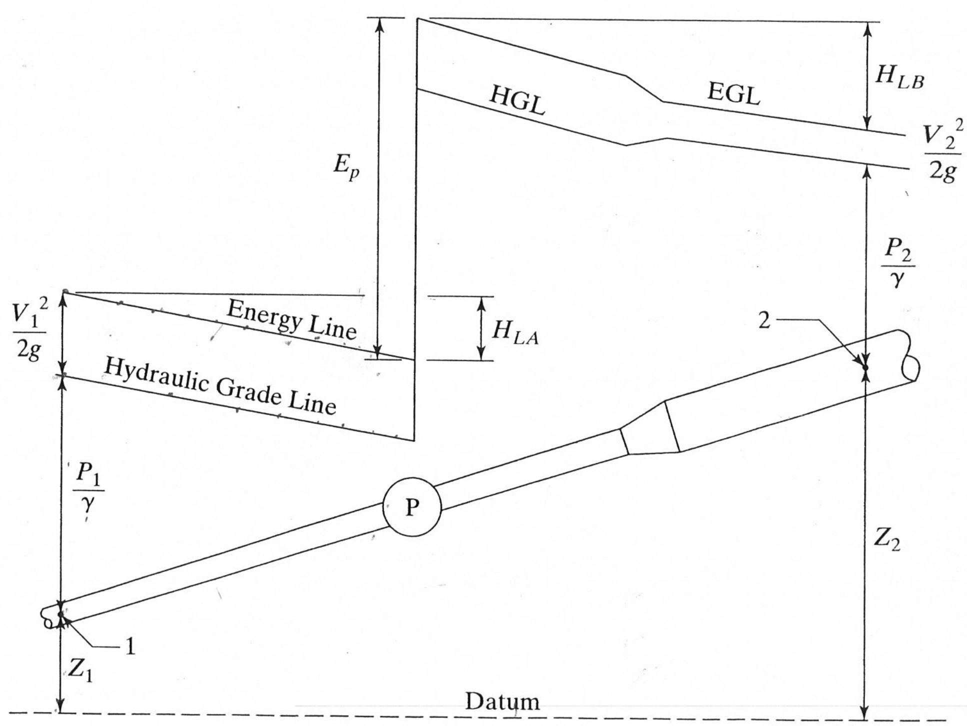
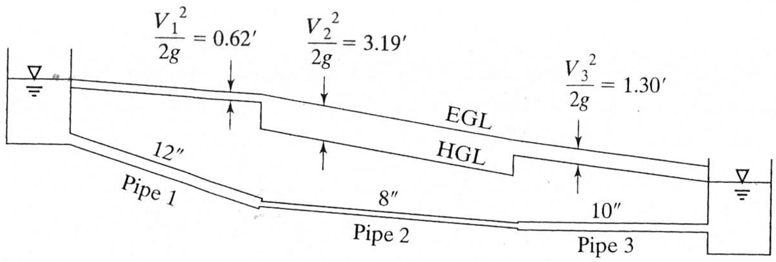


Figure 4.3 Energy equation for pipeline flow.

Example 4.1 Continuity Equation

If 5 cfs of water is flowing in the pipeline from the upper reservoir to the lower reservoir, determine the velocity in each line. Sketch the energy grade line (EGL) and hydraulic grade line (HGL) on the figure.



$$V_1 = \frac{Q}{A_1} = \frac{5}{0.785} = 6.34 \text{ fps}$$

$$\frac{V_1^2}{2g} = 0.62 \text{ ft}$$

$$V_2 = \frac{Q}{A_2} = \frac{5}{0.349} = 14.3 \text{ fps}$$

$$\frac{V_2^2}{2g} = 3.19 \text{ ft}$$

$$V_3 = \frac{Q}{A_3} = \frac{5}{0.545} = 9.16 \text{ fps}$$

$$\frac{V_3^2}{2g} = 1.30 \text{ ft}$$

4.1.3 Headloss

Headlosses in pipelines are caused by pipe friction, transitions, valves, bends, and fittings. For long pipelines, pipe friction is generally the major component of headloss and the other components are often neglected. Headlosses caused by transitions, valves, bends, and fittings are referred to as minor losses and in short pipelines such as highway culverts cannot be neglected.

4.1.3.1 Pipe friction headloss. Headloss caused by pipe friction can be estimated using the Darcy–Weisbach equation, Hazen–Williams equation, or the Manning equation. From Section 3.6, the Darcy–Weisbach equation is

$$H_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{2gD}\times\frac{1}{A^2}\times Q^2 \quad (4.5)$$

or $Q = A\left(\frac{2gD}{Lf}H_L\right)^{1/2}$ where $\frac{H_L}{L} = S$ ← EGL slope

where f is the friction factor, L is the length of the pipe, and D is the diameter of the pipe. The friction factor can be estimated from the Moody diagram in Fig. 4.4 and is a function of the Reynolds number ($Re = DV/\nu$) of the flow and the relative roughness of the pipe (ε/D). In most pipelines, the flow will be fully turbulent.

The hydraulic radius (R) is defined as the area (A) of the flow cross-section divided by the wetted perimeter (P). For a circular pipe flowing full

$$R = \frac{A}{P} = \frac{\frac{1}{4}\pi D^2}{\pi D} = \frac{1}{4}D \quad (4.6)$$

For noncircular pipes, the headloss can be estimated using the equations for circular pipe with $4R$ substituted for D. Pipe roughness values are listed in Table 4.1 for common pipe materials.

After a pipe has been in service for some time, the diameter and roughness of the pipe may change, and it may be difficult to estimate the roughness of the pipe. The Hazen–Williams equation is often used in pipe network analysis. Tables are available relating the Hazen–Williams coefficient (C_H) to the age of the pipe.

The Hazen–Williams equation is

$$Q = C_w C_H A R^{0.63} S^{0.54} \quad (4.7)$$

where $C_w = 0.85$ for International System (SI) units [1.318 for British Gravitational (BG) units] and S is the slope of energy line. Writing the Hazen–Williams equation for headloss gives

$$H_L = S \times L = L\left(\frac{4}{D}\right)^{1.17}\left(\frac{V}{C_w C_H}\right)^{1.85} \quad (4.8)$$

The Manning equation is commonly used to estimate the friction headloss in culverts and storm sewers. The Manning equation is

$$Q = \frac{C_m}{n} A R^{2/3} S^{1/2} \quad (4.9)$$

where $C_m = 1.00$ for SI units (1.49 for BG units) and n is the Manning roughness coefficient. Writing the Manning equation for headloss gives

$$H_L = S \times L = \frac{n^2 V^2 L}{C_m^2 R^{4/3}} = \frac{n^2 L}{C_m^2 R^{4/3} A^2} Q^2 \quad (4.10)$$

In general,

$$H_L = K Q^n$$

$n = 2$ in D-W and Manning; $n = 1.85$ in H-W

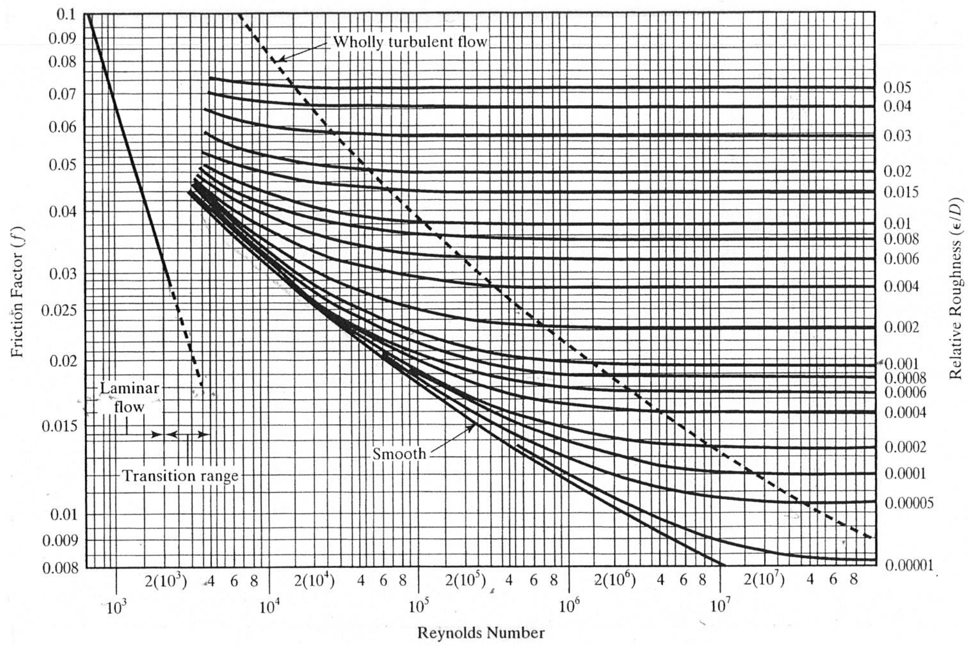


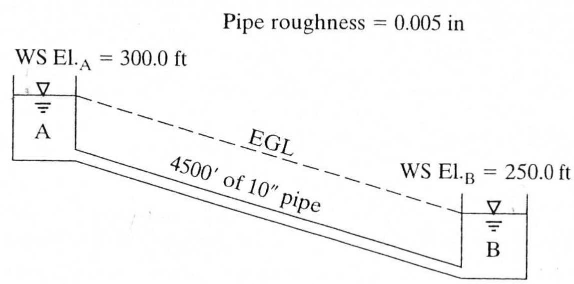
Figure 4.4 Moody diagram of Darcy-Weisbach friction factors.

TABLE 4.1 PIPE ROUGHNESS VALUES

Material	ε mm	C_H Hazen–Williams	n Manning
Plastic, PVC	0.001	150	0.009
Asbestos cement	—	140	0.011
Welded steel	0.045	120	0.012
Riveted steel	0.9–9	110	0.015
Concrete	0.3–3	130	0.012
Asphalted iron	0.12	—	0.013
Galvanized iron	0.15	—	0.016
Cast iron (new)	0.25	130	0.013
Cast iron (old)	—	100	0.025
Corrugated metal	—	—	0.025

Example 4.2 Pipe Friction

Two reservoirs are connected with a 10-inch diameter pipeline 4500 ft long. If the pipe roughness is 0.005 inches, determine the discharge rate in the pipeline. Neglect minor losses.



$$\text{Relative roughness } \frac{\varepsilon}{D} = 0.0005$$

from the Moody diagram $f = 0.017$ for $R_e > 10^6$

$$\text{headloss } H_L = \frac{fL}{D}\frac{V^2}{2g}$$

$$300.0 - 250.0 = \frac{0.017 \times 4500}{10/12}\frac{V^2}{2g} = 91.8\frac{V^2}{2g}$$

$$\frac{V^2}{2g} = 0.545 \text{ ft}$$

$$V = (2g \times 0.545)^{1/2} = 5.90 \text{ fps}$$

$$R_e = \frac{DV}{\nu} = \frac{0.833 \times 5.9}{10^{-5}} = 4.9 \times 10^5$$

From the Moody diagram $f = 0.018$

$$H_L = 97.2\frac{V^2}{2g}$$

$$\frac{V^2}{2g} = 0.514 \text{ ft}$$

$$V = 5.75 \text{ fps}$$

$$R_e = 4.8 \times 10^5$$

$$Q = AV = 0.545 \times 5.75 = 3.14 \text{ cfs}$$

4.1.3.2 Minor losses. Minor losses are caused by excessive turbulence generated by a change in flow geometry. They represent the headloss that is in excess of the normal pipe friction at transitions, bends, valves, and other fittings. The coefficient (K) is used to give the minor headloss (H_M) as a function of the velocity head

$$H_M = K\frac{V^2}{2g} \tag{4.11}$$

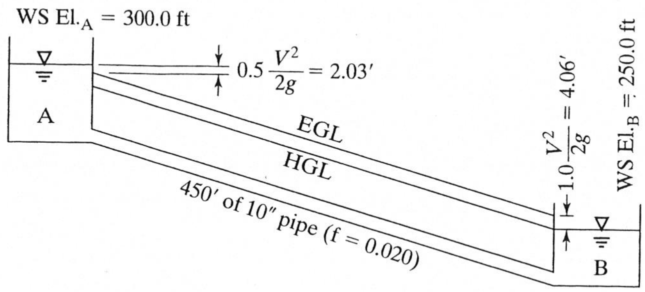
At transitions V is the velocity in the smaller pipe. Minor loss coefficients are listed in Table 4.2.

TABLE 4.2 MINOR LOSS COEFFICIENTS (K)

Transitions		
Diameter ratio	Expansion	Contraction
0	1.0	0.5
0.2	0.92	0.45
0.4	0.70	0.38
0.6	0.40	0.29
0.8	0.12	0.12
1.0	0.0	0.0
Entrance		
Pipe projection	0.8	
Square edge	0.5	
Rounded	0.1	
Exit	1.0	
Bends		
Radius/diameter	90°	45°
1	0.5	0.37
2	0.3	0.22
4	0.25	0.19
6	0.15	0.11
Valves		
Globe (open)	10	
Swing check	2.0	
Gates (open)	0.2	
Gate (1/2 open)	5.6	
Butterfly (open)	1.2	
Ball (open)	0.05	

Example 4.3 Short Pipe Problem

The two reservoirs are connected with 450 ft of 10-in. diameter pipe ($f = 0.020$). The entrance loss coefficient is 0.5 at the upper reservoir, and the exit loss coefficient is 1.0 at the lower reservoir. Determine the discharge rate in the pipe. Draw the *EGL* and *HGL* on the sketch



$$H_L = 0.5\frac{V^2}{2g} + \frac{fL}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$300.0 - 250.0 = \left(1.5 + \frac{0.02 \times 450}{0.833}\right)\frac{V^2}{2g}$$

$$50.0 = 12.3\frac{V^2}{2g}$$

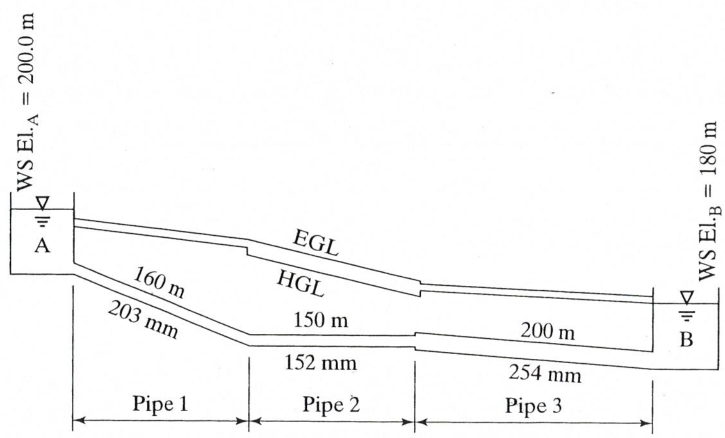
$$\frac{V^2}{2g} = 4.06 \text{ ft}$$

$$V = 16.2 \text{ fps}$$

$$Q = AV = 0.545 \times 16.2 = 8.83 \text{ cfs}$$

Example 4.4 Minor Losses

A pipeline consisting of three pipes in series ($f = 0.02$) extends from an upper reservoir (Elevation 200.0 m) to a lower reservoir (Elevation 180.0 m). Compute the discharge rate in the pipeline using minor loss coefficients of 0.5 for entrance, 0.15 for contraction at junction 1, 0.40 for expansion at junction 2, and 1.0 for exit. The minor loss coefficients at the two pipe junctions are based on the velocity in the smaller pipe ($D = 152$ mm). The headloss between A and B is the difference in water surface elevation ($WSEL$).



$$H_L = WSEL_A - WSEL_B$$

$$= 0.5\frac{V_1^2}{2g} + \frac{fL_1}{D_1}\frac{V_1^2}{2g} + 0.15\frac{V_2^2}{2g} + \frac{fL_2}{D_2}\frac{V_2^2}{2g} + 0.40\frac{V_2^2}{2g} + \frac{fL_3}{D_3}\frac{V_3^2}{2g} + 1.0\frac{V_3^2}{2g}$$

$$= (0.5 + 15.8)\frac{V_1^2}{2g} + (0.15 + 19.74 + 0.40)\frac{V_2^2}{2g} + (15.7 + 1.0)\frac{V_3^2}{2g}$$

$$= 16.3\frac{V_1^2}{2g} + 20.3\frac{V_2^2}{2g} + 16.7\frac{V_3^2}{2g}$$

Based on the continuity equation

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = 0.56V_2$$

$$V_3 = \left(\frac{D_2}{D_3}\right)^2 V_2 = 0.36V_2$$

$$H_L = 20 \text{ m} = \left[16.3(0.56)^2 + 20.3 + 16.7(0.36)^2\right]\frac{V_2^2}{2g} = 27.6\frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{20}{27.6} = 0.72 \text{ m}$$

$$V_2 = (2g \times 0.72)^{1/2} = (2.0 \times 9.81 \times 0.72)^{1/2} = 3.76 \text{ mps}$$

$$Q = A_2V_2 = 0.0181 \times 3.76 = 0.0682 \text{ cms}$$