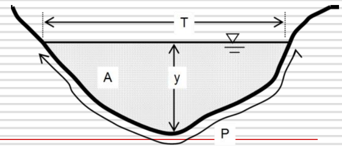
Hydraulic Efficiency in Open Channels



Recall Manning's Eq'n: $Q = AV = (k_M/n)AR_h^{2/3}S_e^{1/2}$

Based on this equation, how would we maximize Q for a given slope and "n" value? **Ans:**

Alternatively:

Which of the shapes below is most efficient?

Is that shape practical? Why? Note the best alternatives.

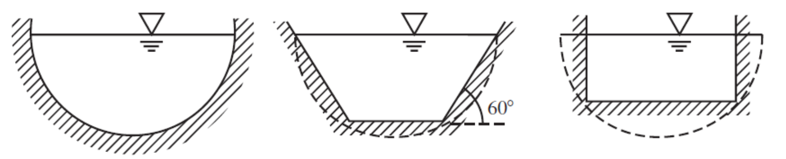
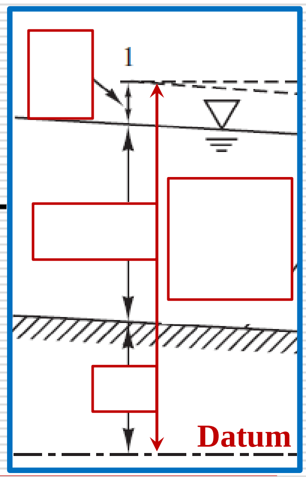


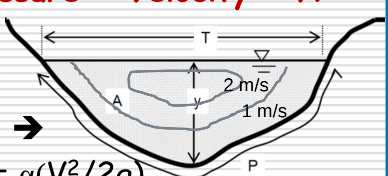
Figure 6.5 Hydraulically efficient sections

Energy Principles in Open Channels

(Three Forms of Energy per Unit Weight)

Like pipe flow, the energy forms are:
Potential, Pressure, and Kinetic
and expressed as energy head: ➔
Position + Pressure + Velocity = H





Since V varies across channel ➔
Avg "V" Head = $\alpha(V^2/2g)$
where α = energy coef. (1.05 to 1.20)
Also, p/γ can vary if bottom slope is not constant due to centrifugal force.

Specific Energy in Open Channels

(Interrelationships Between Energy Forms)

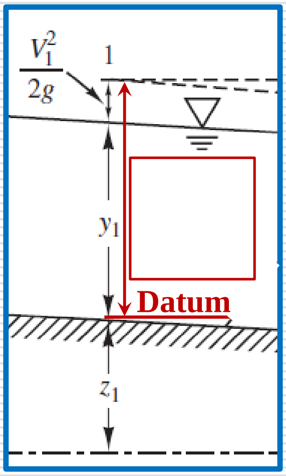
Total Energy Head in Open Channels:

$H = z + y + V^2/2g$ ➔ arbitrary datum

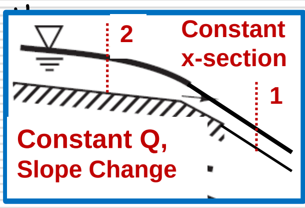
However, **specific energy** head is:

$E = y + V^2/2g = y + Q^2/(2gA^2)$ ➔

when the channel bottom is the datum.

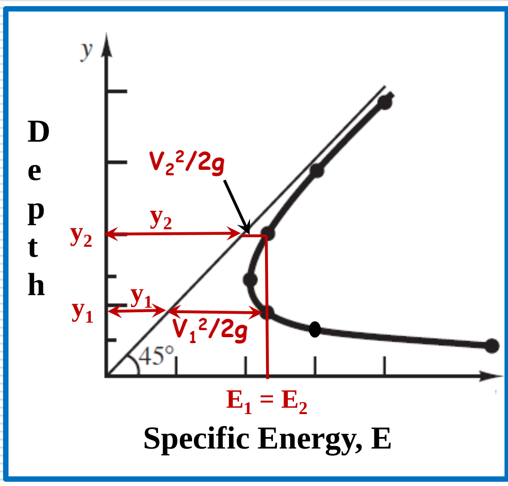


If $E_2 = E_1$ below (minimal losses), how do the specific energy components change from Section 2 to 1?



Specific Energy Curves

(Flow Regimes & Alternate Depths)



$E = y + Q^2/(2gA^2)$

For a constant Q, plotting "E" vs. a varying "y" (depth) of a given X-section yields:

🡐 Specific Energy Curve

Observe that 2 different flow conditions occur at most energy levels, $E_1 = E_2$

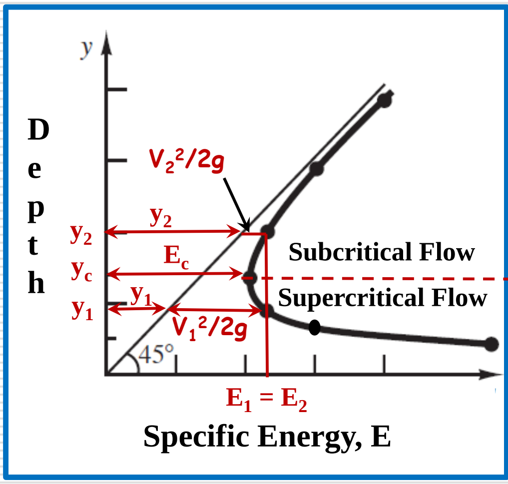
E_1 ➔ low depth, high V

E_2 ➔ high depth, low V

Alternate depths: y_1 & y_2

Minimum Energy and Critical Depth

(Subcritical and Supercritical Flow)



At one location, the energy is a minimum (E_c ➔ **critical flow**) and the depth is called **critical depth** (y_c).

It separates flow regimes:

Supercritical Flow:

E_1 ➔ low depth, high V

Subcritical Flow:

E_2 ➔ high depth, low V

Steep channel slopes will produce supercritical flow.

Critical Depth & the Froude Number

(Minimizing Specific Energy in a Channel)

To find y_c, set the 1st derivative of E equal to 0: **$dE/dy = 0$**

$dE/dy = d/dy\,[y + Q^2/2gA^2] = 1 - [2Q^2/2gA^3](dA/dy) = 0$

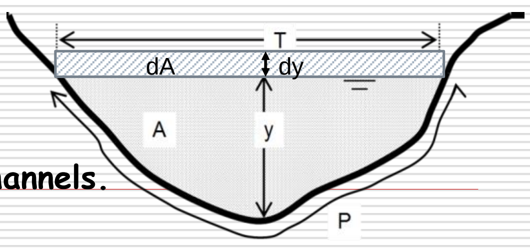
Note from the figure that $dA/dy = T$. Substituting yields,

$1 = Q^2T/gA^3$ ➔ Also, $A/T = D$ (hydraulic depth). Thus,

$1 = Q^2/(gDA^2) = V^2/gD$ or $1 = V/(gD)^{1/2} = N_F$ ➔ **Froude Number**

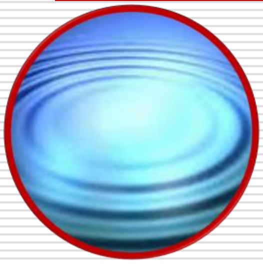
Rectangular channels:
$y_c = (q^2/g)^{1/3}$; $q = Q/b$
and b = channel width.

Use: $Q^2/g = A^3/T = DA^2$
to find **y_c** for **all other channels.**



Froude Number and Critical Depth

(Rectangular and Non-rectangular Channels)



N_F ➔ **Ratio of inertial to gravity force.**

Alternatively, the ratio of **flow velocity** to the **velocity of a disturbance wave**.

🡐 Disturbance wave (throw stone in pond)

$N_F = V/(gD)^{1/2}$ **= 1 (critical flow).** Thus the channel velocity equals the wave speed.

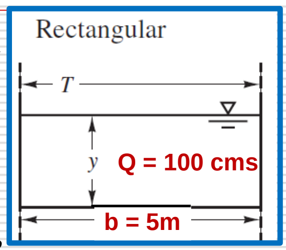
N_F < 1 (subcritical flow) ➔ The channel velocity (V) is less than the wave speed $(gD)^{1/2}$ (i.e., throw a stone into channel and the disturbance wave will propagate upstream).

N_F > 1 (supercritical flow) Disturbance wave washes away.

Critical Depth & Froude Number

(Example Problem – Rectangular Channel)

Given: Concrete channel with $S_o = 0.01$ ➔



Find normal & critical depths & N_F.

From Table 6.2: n = 0.013, & Table 6.1:

A = P = Find d_n:

$Q = (1/n)AR_h^{2/3}S_o^{1/2} = (1/n)(A^{5/3}/P^{2/3})S_o^{1/2}$

Hence, $Qn/S_o^{1/2} = (100*0.013)/(0.01)^{1/2} = (5y_n)^{5/3}/[5+2y_n]^{2/3}$

Solving iteratively (or w/charts or software), **y_n = 2.31m**

V = Q/A = D = A/T = $N_F = V/(gD)^{1/2}$ =

Flow is supercritical since N_F > 1. q = Q/b =

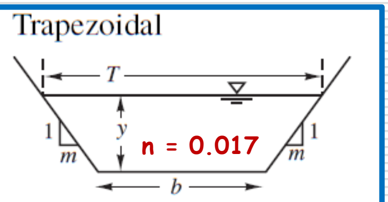
$y_c = (q^2/g)^{1/3} = (20^2/9.81)^{1/3}$ = **Note: $y_n < y_c$**

Critical Depth & Froude Number

(Example Problem ➔ Trapezoidal Channel)

Given: Channel w/Q = 1510 cfs

S_o = 0.00088, m = 1.5, b = 25 ft



Solution: From Table 6.1:

A = ☐ T = ☐ Thus,

A = y(25 + 1.5y) T = 25 + 3y. At critical depth: $Q^2/g = A^3/T$

$Q^2/g = (1510)^2/32.2 = 70{,}800 = [y_c(25 + 1.5y_c)]^3/[25 + 3y_c]$

Solving iteratively (or w/charts or software), **y_c = 4.41 ft**

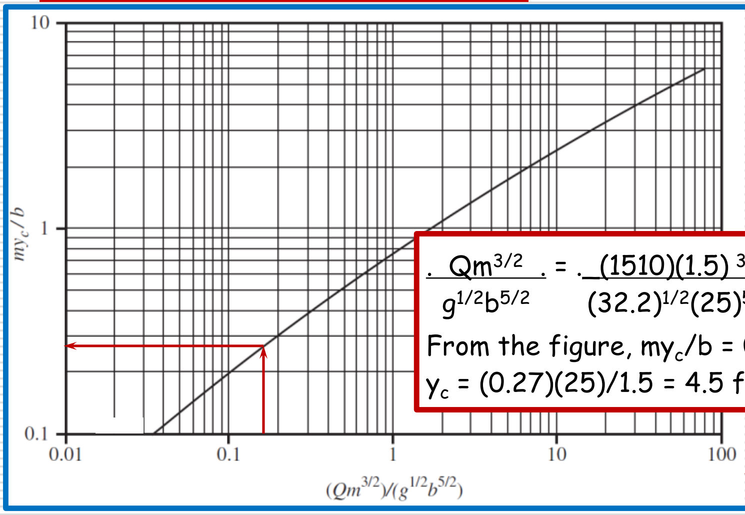
Recall from previous class for this channel: y_n = 6.25 ft

Since $y_n > y_c$ ➔ **Flow is** ☐

Alternative Solution

(Trapezoidal Channel, Fig. 6.9a)

Homework Problems:



$$\frac{Qm^{3/2}}{g^{1/2}b^{5/2}} = \frac{(1510)(1.5)^{3/2}}{(32.2)^{1/2}(25)^{5/2}} = 0.16$$

From the figure, my_c/b = 0.27

y_c = (0.27)(25)/1.5 = 4.5 ft