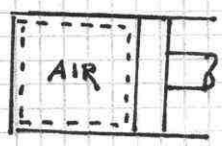
PROBLEM 3.140

KNOWN: Air within a piston-cylinder assembly undergoes a power cycle. State data are provided.

FIND: Evaluate the thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:



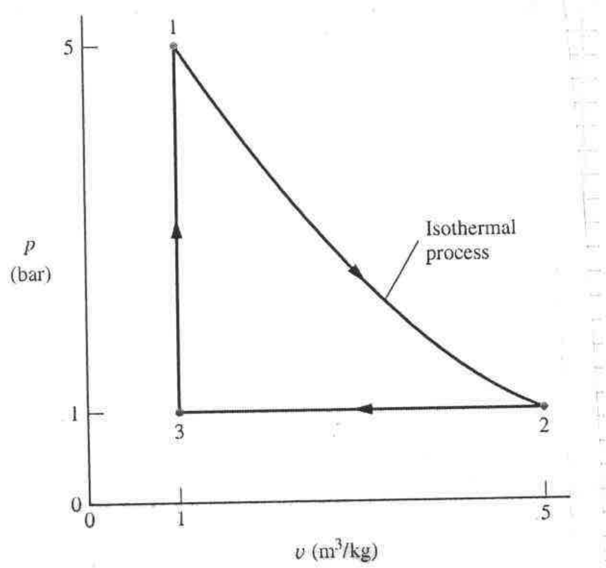


Fig. P3.140

ENGINEERING MODEL:

1. The air within the piston-cylinder is the closed system.
2. The air is modeled as an ideal gas.
3. Kinetic and potential energy effects are ignored.

ANALYSIS:

The thermal efficiency of a power cycle is given by Eq. 2.42 and Eq. 2.43:

$$\eta = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \qquad (1)$$

where the energy transfer terms are positive in the directions of the arrows on Fig. 2.17(a).

Next, we evaluate the work and heat transfer for each of the three processes of the cycle.

Process 1-2:

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT \ln\frac{V_2}{V_1} = P_1 V_1 \ln\frac{V_2}{V_1} \Rightarrow W_{12}/m = P_1 v_1 \ln\frac{v_2}{v_1}$$

constant ; Using $P_1V_1 = mRT$

$$\therefore \frac{W_{12}}{m} = \left(5\times10^5 \frac{N}{m^2}\right)\left(1\frac{m^3}{kg}\right)\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| \ln\left(\frac{5}{1}\right) = 804.7\ \frac{kJ}{kg}$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \Delta U = Q - W.$$

For process 1-2, temperature is constant. Thus with the ideal gas model $\Delta U = 0 \Rightarrow Q_{12}/m = W_{12}/m = 804.7\ kJ/kg$

Process 2-3:

$$W_{23} = \int_2^3 p\,dV = p\,[V_3 - V_2] \Rightarrow W_{23}/m = p(v_3 - v_2)$$

constant

$$\therefore \frac{W_{23}}{m} = \left(1\times10^5 \frac{N}{m^2}\right)(1-5)\frac{m^3}{kg}\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = -400\ \frac{kJ}{kg}$$

The energy balance gives, $\frac{Q_{23}}{m} = [u(T_3) - u(T_2)] + \frac{W_{23}}{m}$

PROBLEM 3.140 (Continued)

Using the ideal gas model equation of state, T_2 and T_3 can be determined:

$$T_2 = \frac{p_2 v_2}{R} = \frac{(10^5\,N/m^2)(5\,m^3/kg)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)} = 1742.2\ K$$

Table A-22 gives $u_2 = 1432.5\ kJ/kg$

$$T_3 = \frac{p_3 v_3}{R} = \frac{(10^5\,N/m^2)(1\,m^3/kg)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)} = 348.4\ K$$

Table A.22 gives $u_3 = 248.9\ kJ/kg$

Thus, $Q_{23}/m = (248.9 - 1432.5)\ kJ/kg + (-400\ kJ/kg) = -1583.6\ kJ/kg$

Process 3-1: Volume is constant.

$W_{31} = \int p\,\cancel{dV}^{\,0} = 0$. The energy balance reduces to $Q_{31}/m = u_1 - u_3$ $(= u_2)$

$\therefore\ \frac{Q_{31}}{m} = (1432.5 - 248.9)\,\frac{kJ}{kg} = 1183.6\ \frac{kJ}{kg}$

Collecting results,

Process	W/m	Q/m
1-2	804.7	804.7
2-3	−400	−1583.6
3-1	0	1183.6
TOTAL	404.7 kJ/kg	404.7 kJ/kg

Agrees with the cycle energy balance, Eq. 2.40

Returning to the expression for the thermal efficiency, Eq. (1),

$$W_{cycle}/m = 404.7\ kJ/kg$$

$$Q_{in}/m = Q_{12}/m + Q_{31}/m = (804.7 + 1183.6)\ kJ/kg = 1988.3\ kJ/kg$$

Thus,

$$\eta = \frac{404.7}{1988.3} = 0.204 \quad (20.4\%)$$