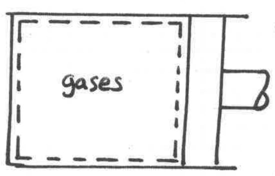
PROBLEM 2.24

KNOWN: Measured pressure-volume data for an expansion of gases within the cylinder of an internal combustion engine are given.

FIND: (a) Determine n for a fit of the data by pV^n = constant. (b) Use the result of part (a) to evaluate the work done in the expansion. (c) Evaluate the work done using graphical or numerical integration of the data. (d) Compare and discuss parts (c), (d).

SCHEMATIC & GIVEN DATA:



Data Point	p (bar)	V (cm^3)
1	15	300
2	12	361
3	9	459
4	6	644
5	4	903
6	2	1608

ENGR. MODEL: 1. As shown in the schematic, the gases within the piston-cylinder form the closed system. 2. The pressure values provided approximate the pressure at the piston face.

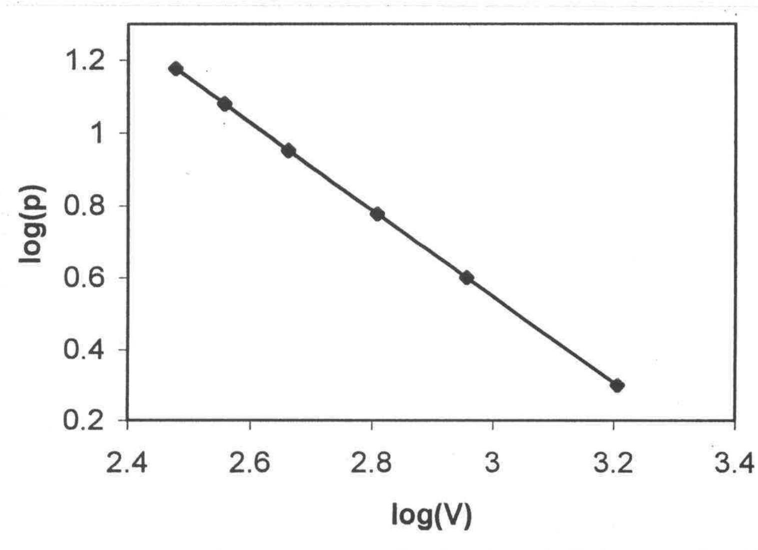
ANALYSIS: (a) One approach to find n is to begin with pV^n = constant. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

Thus, $(-n)$ corresponds to the slope of a plot of $\log p$ vs. $\log V$. Using
1 a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$(-n) = -1.1996$

or

$n = 1.1996$ ← n

Thus

$pV^{1.1996}$ = constant

PROBLEM 2.24 (Cont'd)

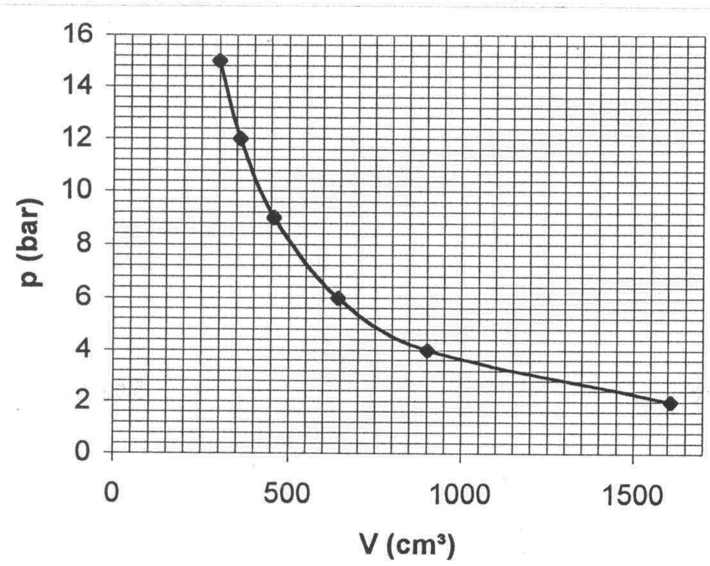
(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p\,dV = \frac{p_2V_2 - p_1V_1}{(1-n)}$$

$$= \frac{(2\text{ bar})(1608\text{ cm}^3) - (15)(300)}{(1-1.1996)} \left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$= 0.643$ kJ ← W

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$(.4\text{ bar})(50\text{ cm}^3) \left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 0.002\text{ kJ}$$

The number of rectangles is approximately 324, thus

$W \approx (324)(0.002) = 0.648$ kJ ← W

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented in Sec. 2.2.4 in the discussion of actual expansion and compression processes.

1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.