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Problems

Flat Plate in Parallel Flow

7.1 Consider the following fluids at a film temperature of 300 K in parallel flow over a flat plate with velocity of 1 m/s: atmospheric air, water, engine oil, and mercury.

(a) For each fluid, determine the velocity and thermal boundary layer thicknesses at a distance of 40 mm from the leading edge.

(b) For each of the prescribed fluids and on the same coordinates, plot the boundary layer thicknesses as a function of distance from the leading edge to a plate length of 40 mm.

7.2 Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1-m-long flat plate maintained at 20°C. Determine:

(a) The velocity and thermal boundary layer thicknesses at the trailing edge.

(b) The local heat flux and surface shear stress at the trailing edge.

(c) The total drag force and heat transfer per unit width of the plate.

(d) Plot the boundary layer thicknesses and local values of the surface shear stress, convection coefficient, and heat flux as a function of x for $0 \leq x \leq 1$ m.

7.3 Consider steady, parallel flow of atmospheric air over a flat plate. The air has a temperature and free stream velocity of 300 K and 25 m/s.

(a) Evaluate the boundary layer thickness at distances of $x = 1$, 10, and 100 mm from the leading edge. If a second plate were installed parallel to and at a distance of 3 mm from the first plate, what is the distance from the leading edge at which boundary layer merger would occur?

(b) Evaluate the surface shear stress and the y-velocity component at the outer edge of the boundary layer for the single plate at $x = 1$, 10, and 100 mm.

(c) Comment on the validity of the boundary layer approximations.

7.4 Consider a liquid metal ($Pr \ll 1$), with free stream conditions u_∞ and T_∞, in parallel flow over an isothermal flat plate at T_s. Assuming that $u = u_\infty$ throughout the thermal boundary layer, write the corresponding form of the boundary layer energy equation. Applying appropriate initial ($x = 0$) and boundary conditions, solve this equation for the boundary layer temperature field, $T(x, y)$. Use the result to obtain an expression for the local Nusselt number Nu_x. *Hint:* This problem is analogous to one-dimensional heat transfer in a semi-infinite medium with a sudden change in surface temperature.

7.5 Consider the velocity boundary layer profile for flow over a flat plate to be of the form $u = C_1 + C_2 y$. Applying appropriate boundary conditions, obtain an expression for the velocity profile in terms of the boundary layer thickness δ and the free stream velocity u_∞. Using the integral form of the boundary layer momentum equation (Appendix F), obtain expressions for the boundary layer thickness and the local friction coefficient, expressing your result in terms of the local Reynolds number. Compare your results with those obtained from the exact solution (Section 7.2.1) and the integral solution with a cubic profile (Appendix F).

7.6 Consider a steady, turbulent boundary layer on an isothermal flat plate of temperature T_s. The boundary layer is "tripped" at the leading edge $x = 0$ by a fine wire. Assume constant physical properties and velocity and temperature profiles of the form

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \frac{T - T_\infty}{T_s - T_\infty} = 1 - \left(\frac{y}{\delta_t}\right)^{1/7}$$

(a) From experiment it is known that the surface shear stress is related to the boundary layer thickness by an expression of the form

$$\tau_s = 0.0228 \rho u_\infty^2 \left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

5×10^5, for what values of Re_L would the total heat transfer be independent of orientation?

7.14 In fuel cell stacks, it is desirable to operate under conditions that promote uniform surface temperatures for the electrolytic membranes. This is especially true in high-temperature fuel cells where the membrane is constructed of a brittle ceramic material. Electrochemical reactions in the electrolytic membranes generate thermal energy, while gases flowing above and below the membranes cool it. The stack designer may specify top and bottom flows that are in the same, opposite, or orthogonal directions. A preliminary study of the effect of the relative flow directions is conducted whereby a 150 mm $\times$ 150 mm *thin* sheet of material, producing a uniform heat flux of 100 W/m^2, is cooled (top and bottom) by air with a free stream temperature and velocity of 25°C and 2 m/s, respectively.

(a) Determine the minimum and maximum local membrane temperatures for top and bottom flows that are in the same, opposite, and orthogonal directions. Which flow configuration minimizes the membrane temperature? *Hint*: For the opposite and orthogonal flow cases, the boundary layers are subject to boundary conditions that are neither uniform temperature nor uniform heat flux. It is, however, reasonable to expect that the resulting temperatures would be *bracketed* by your answers based on the constant heat flux and constant temperature boundary conditions.

(b) Plot the surface temperature distribution $T(x)$ for the cases involving flow in the opposite and same directions. Thermal stresses are undesirable and are related to the spatial temperature gradient along the membrane. Which configuration minimizes spatial temperature gradients?

7.15 Air at a pressure of 1 atm and a temperature of 50°C is in parallel flow over the top surface of a flat plate that is heated to a uniform temperature of 100°C. The plate has a length of 0.20 m (in the flow direction) and a width of 0.10 m. The Reynolds number based on the plate length is 40,000. What is the rate of heat transfer from the plate to the air? If the free stream velocity of the air is doubled and the pressure is increased to 10 atm, what is the rate of heat transfer?

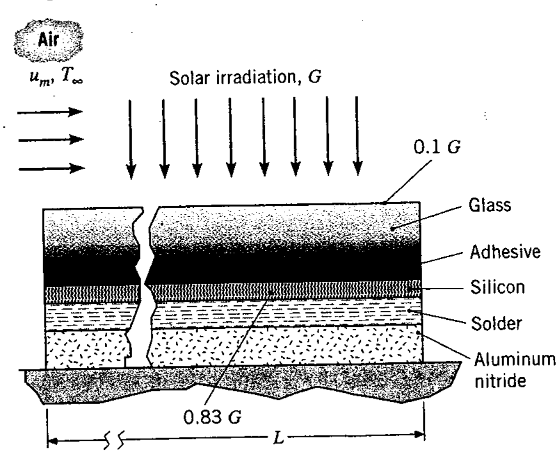
7.16 Consider a rectangular fin that is used to cool a motorcycle engine. The fin is 0.15 m long and at a temperature of 250°C, while the motorcycle is moving at 80 km/h in air at 27°C. The air is in parallel flow over both surfaces of the fin, and turbulent flow conditions may be assumed to exist throughout.

(a) What is the rate of heat removal per unit width of the fin?

(b) Generate a plot of the heat removal rate per unit width of the fin for motorcycle speeds ranging from 10 to 100 km/h.

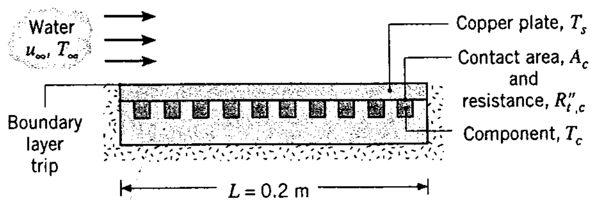
7.17 The Weather channel reports that it is a hot, muggy day with an air temperature of 90°F, a 10 mph breeze out of the southwest, and bright sunshine with a solar insolation of 400 W/m^2. Consider the wall of a metal building over which the prevailing wind blows. The length of the wall in the wind direction is 10 m, and the emissivity is 0.93. Assume that all the solar irradiation is absorbed, that irradiation from the sky is negligible, and that flow is fully turbulent over the wall. Estimate the average wall temperature.

7.18 A photovoltaic solar panel consists of a sandwich of (top to bottom) a 3-mm-thick ceria-doped glass (k_g = 1.4 W/m · K), a 0.1-mm-thick optical grade adhesive (k_a = 145 W/m · K), a *very thin* silicon semiconducting material, a 0.1-mm-thick solder layer (k_s = 50 W/m · K) and a 2-mm-thick aluminum nitride substrate (k_{an} = 120 W/m · K). The solar-to-electrical conversion efficiency within the semiconductor depends on the silicon temperature, T_{si}, and is described by the expression $\eta = 0.28 - 0.001T_{si}$, where T_{si} is in °C, for 25°C $\leq T_{si} \leq$ 250°C. Ten percent of the solar irradiation is absorbed at the top surface of the glass, while 83% of the solar irradiation is transmitted to and absorbed by the silicon (the remaining 7% is reflected away from the cell). The glass has an emissivity of 0.90.



(a) Consider an L = 1 m long, w = 0.1 m wide solar cell that is placed on an insulated surface. Determine the silicon temperature and the electric power produced by the solar cell for an air velocity of 4 m/s parallel to the long direction, with air and surroundings temperatures of 25°C. The solar irradiation is 700 W/m^2. The boundary layer is tripped to a turbulent condition at the leading edge of the panel.

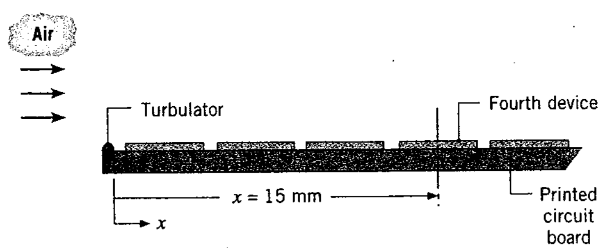
A protuberance at the leading edge of the plate acts to *trip* the boundary layer, and the plate itself may be assumed to be isothermal. The water velocity and temperature are $u_\infty = 2$ m/s and $T_\infty = 17°C$, and the water's thermophysical properties may be approximated as $\nu = 0.96 \times 10^{-6}$ m²/s, $k = 0.620$ W/m · K, and $Pr = 5.2$.



(a) What is the temperature of the copper plate?

(b) If each component has a plate contact surface area of 1 cm² and the corresponding contact resistance is 2×10^{-4} m² · K/W, what is the component temperature? Neglect the temperature variation across the thickness of the copper plate.

7.33 Air at 27°C with a free stream velocity of 10 m/s is used to cool electronic devices mounted on a printed circuit board. Each device, 4 mm by 4 mm, dissipates 40 mW, which is removed from the top surface. A turbulator is located at the leading edge of the board, causing the boundary layer to be turbulent.

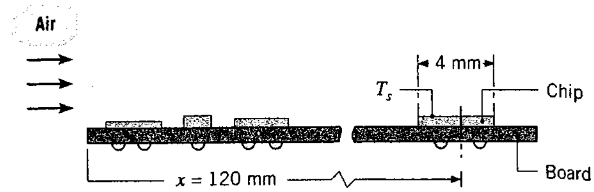


(a) Estimate the surface temperature of the fourth device located 15 mm from the leading edge of the board.

(b) Generate a plot of the surface temperature of the first four devices as a function of the free stream velocity for $5 \le u_\infty \le 15$ m/s.

(c) What is the minimum free stream velocity if the surface temperature of the hottest device is not to exceed 80°C?

7.34 Forced air at 25°C and 10 m/s is used to cool electronic elements mounted on a circuit board. Consider a chip of length 4 mm and width 4 mm located 120 mm from the leading edge. Because the board surface is irregular, the flow is disturbed and the appropriate convection correlation is of the form $Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$.



Estimate the surface temperature of the chip, T_s, if its heat dissipation rate is 30 mW.

7.35 Air at atmospheric pressure and a temperature of 25°C is in parallel flow at a velocity of 5 m/s over a 1-m-long flat plate that is heated with a uniform heat flux of 1250 W/m². Assume the flow is fully turbulent over the length of the plate.

(a) Calculate the plate surface temperature, $T_s(L)$, and the local convection coefficient, $h_x(L)$, at the trailing edge, $x = L$.

(b) Calculate the average temperature of the plate surface, $\bar{T}_s$.

(c) Plot the variation of the surface temperature, $T_s(x)$, and the convection coefficient, $h_x(x)$, with distance on the same graph. Explain the key features of these distributions.

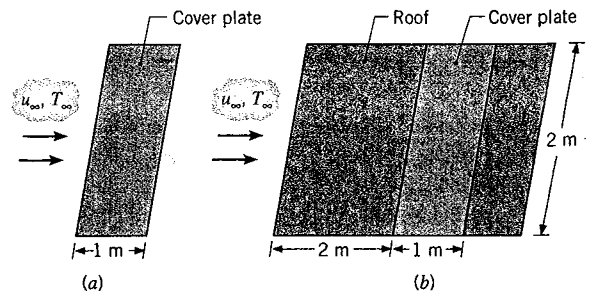
7.36 Consider atmospheric air at $u_\infty = 2$ m/s and $T_\infty = 300$ K in parallel flow over an isothermal flat plate of length $L = 1$ m and temperature $T_s = 350$ K.

(a) Compute the local convection coefficient at the leading and trailing edges of the heated plate with and without an unheated starting length of $\xi = 1$ m.

(b) Compute the average convection coefficient for the plate for the same conditions as part (a).

(c) Plot the variation of the local convection coefficient over the plate with and without an unheated starting length.

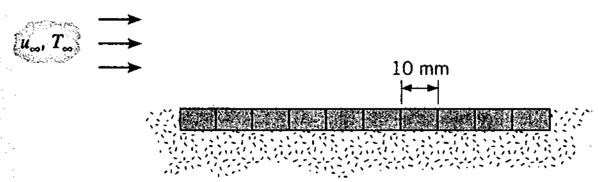
7.37 The cover plate of a flat-plate solar collector is at 15°C, while ambient air at 10°C is in parallel flow over the plate, with $u_\infty = 2$ m/s.



(a) What is the rate of convective heat loss from the plate?

(b) If the plate is installed 2 m from the leading edge of a roof and flush with the roof surface, what is the rate of convective heat loss?

7.38 An array of 10 silicon chips, each of length $L = 10$ mm on a side, is insulated on one surface and cooled on the opposite surface by atmospheric air in parallel flow with $T_\infty = 24°C$ and $u_\infty = 40$ m/s. When in use, the same electrical power is dissipated in each chip, maintaining a uniform heat flux over the entire cooled surface.

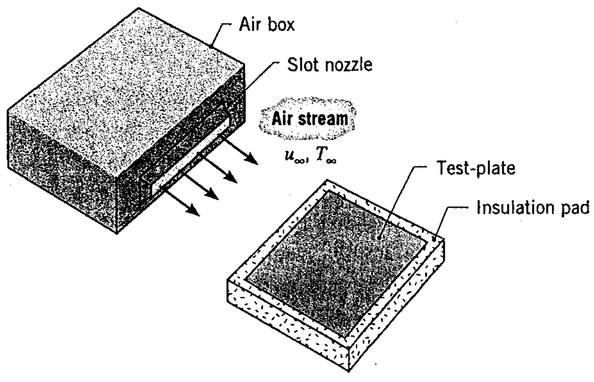


If the temperature of each chip may not exceed 80°C, what is the maximum allowable power per chip? What is the maximum allowable power if a turbulence promoter is used to trip the boundary layer at the leading edge? Would it be preferable to orient the array normal, instead of parallel, to the airflow?

7.39 A square (10 mm × 10 mm) silicon chip is insulated on one side and cooled on the opposite side by atmospheric air in parallel flow at $u_\infty = 20$ m/s and $T_\infty = 24°C$. When in use, electrical power dissipation within the chip maintains a uniform heat flux at the cooled surface. If the chip temperature may not exceed 80°C at any point on its surface, what is the maximum allowable power? What is the maximum allowable power if the chip is flush mounted in a substrate that provides for an unheated starting length of 20 mm?

7.40 Working in groups of two, our students design and perform experiments on forced convection phenomena using the general arrangement shown schematically. The air box consists of two muffin fans, a plenum chamber, and flow straighteners discharging a nearly uniform airstream over the flat *test-plate.* The objectives of one experiment were to measure the heat transfer coefficient and to compare the results with standard convection correlations. The velocity of the airstream was measured using a thermistor-based anemometer, and thermocouples were used to determine the temperatures of the airstream and the *test-plate.*

With the airstream from the box fully stabilized at $T_\infty = 20°C$, an aluminum plate was preheated in a convection oven and quickly mounted in the test-plate holder. The subsequent temperature history of the plate was determined from thermocouple measurements, and histories obtained for airstream velocities of 3 and 9 m/s were fitted by the following polynomial:



$$T(t) = a + bt + ct^2 + dt^3 + et^4$$

The temperature T and time t have units of °C and s, respectively, and values of the coefficients appropriate for the time interval of the experiments are tabulated as follows:

Velocity (m/s)	**3**	**9**
Elapsed Time (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	−0.1472	−0.2641
c (°C/s²)	3×10^{-4}	9×10^{-4}
d (°C/s³)	-4×10^{-7}	-2×10^{-6}
e (°C/s⁴)	2×10^{-10}	1×10^{-9}

The plate is square, 133 mm to a side, with a thickness of 3.2 mm, and is made from a highly polished aluminum alloy ($\rho = 2770$ kg/m³, $c = 875$ J/kg·K, $k = 177$ W/m·K).

(a) Determine the heat transfer coefficients for the two cases, assuming the plate behaves as a spacewise isothermal object.

(b) Evaluate the coefficients C and m for a correlation of the form

$$\overline{Nu}_L = C\, Re^m\, Pr^{1/3}$$

Compare this result with a standard flat-plate correlation. Comment on the *goodness* of the comparison and explain any differences.

Cylinder in Cross Flow

7.41 Consider the following fluids, each with a velocity of $V = 5$ m/s and a temperature of $T_\infty = 20°C$, in cross flow over a 10-mm diameter cylinder maintained at 50°C: atmospheric air, saturated water, and engine oil.

(a) Calculate the rate of heat transfer per unit length, q', using the Churchill–Bernstein correlation.

2024 aluminum pin fin through the wall separating the two fluids. The pin is inserted to a depth of d into fluid 1. Fluid 1 is air with a mean temperature of 10°C and nearly uniform velocity of 10 m/s. Fluid 2 is air with a mean temperature of 40°C and mean velocity of 3 m/s.

(a) Determine the rate of heat transfer from the warm air to the cool air through the pin fin for d = 50 mm.

(b) Plot the variation of the heat transfer rate with the insertion distance, d. Does an optimal insertion distance exist?

7.55 Repeat Problem 7.54, except now fluid 1 is ethylene glycol with a mean temperature of 10°C and nearly uniform velocity of 10 m/s. Fluid 2 is water with a mean temperature of 40°C and mean velocity of 3 m/s.

7.56 Hot water at 50°C is routed from one building in which it is generated to an adjoining building in which it is used for space heating. Transfer between the buildings occurs in a steel pipe (k = 60 W/m · K) of 100-mm outside diameter and 8-mm wall thickness. During the winter, representative environmental conditions involve air at T_∞ = −5°C and V = 3 m/s in cross flow over the pipe.

(a) If the cost of producing the hot water is $0.05 per kW · h, what is the representative daily cost of heat loss from an uninsulated pipe to the air per meter of pipe length? The convection resistance associated with water flow in the pipe may be neglected.

(b) Determine the savings associated with application of a 10-mm-thick coating of urethane insulation (k = 0.026 W/m · K) to the outer surface of the pipe.

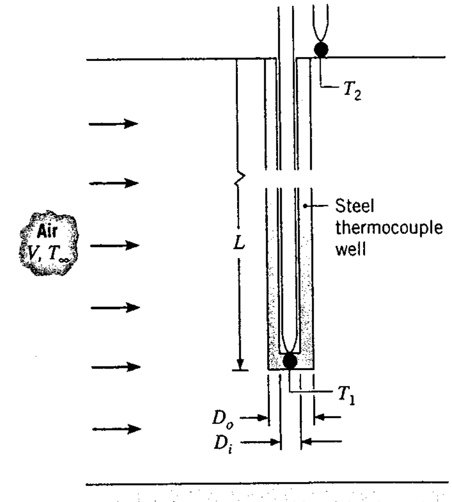
7.57 An uninsulated steam pipe is used to transport high-temperature steam from one building to another. The pipe is of 0.5-m diameter, has a surface temperature of 150°C, and is exposed to ambient air at −10°C. The air moves in cross flow over the pipe with a velocity of 5 m/s.

(a) What is the heat loss per unit length of pipe?

(b) Consider the effect of insulating the pipe with a rigid urethane foam (k = 0.026 W/m · K). Evaluate and plot the heat loss as a function of the thickness δ of the insulation layer for $0 \le \delta \le 50$ mm.

7.58 A thermocouple is inserted into a hot air duct to measure the air temperature. The thermocouple (T_1) is soldered to the tip of a steel *thermocouple well* of length L = 0.15 m and inner and outer diameters of D_i = 5 mm and D_o = 10 mm. A second thermocouple (T_2) is used to measure the duct wall temperature.

Consider conditions for which the air velocity in the duct is V = 3 m/s and the two thermocouples register temperatures of T_1 = 450 K and T_2 = 375 K. Neglecting radiation, determine the air temperature T_∞. Assume that,



for steel, k = 35 W/m · K, and, for air, ρ = 0.774 kg/m³, $\mu = 251 \times 10^{-7}$ N · s/m², k = 0.0373 W/m · K, and Pr = 0.686.

7.59 Consider conditions for which a mercury-in-glass thermometer of 4-mm diameter is inserted to a length L through the wall of a duct in which air at 77°C is flowing. If the stem of the thermometer at the duct wall is at the wall temperature T_w = 15°C, conduction heat transfer through the glass causes the bulb temperature to be lower than that of the airstream.

(a) Develop a relationship for the *immersion* error, $\Delta T_i = T(L) - T_\infty$, as a function of air velocity, thermometer diameter, and insertion length L.

(b) To what length L must the thermometer be inserted if the immersion error is not to exceed 0.25°C when the air velocity is 10 m/s?

(c) Using the insertion length determined in part (b), calculate and plot the immersion error as a function of air velocity for the range 2 to 20 m/s.

(d) For a given insertion length, will the immersion error increase or decrease if the diameter of the thermometer is increased? Is the immersion error more sensitive to the diameter or air velocity?

7.60 Fluid velocities can be measured using hot-film sensors, and a common design is one for which the sensing element forms a thin film about the circumference of a quartz rod. The film is typically comprised of a thin (~100 nm) layer of platinum, whose electrical resistance is proportional to its temperature. Hence, when submerged in a fluid stream, an electric current may be passed through the film to maintain its temperature above that of the fluid. The temperature of the film is

(a) The stress test begins with the components at ambient temperature ($T_i = 20°C$) and proceeds with heating by the fluid at $T_\infty = 80°C$. If the fluid velocity is $V = 0.2$ m/s, estimate the ratio of the time constant of the chip to that of a solder ball. Which component responds more rapidly to the heating process?

(b) The thermal stress acting on the solder joint is proportional to the chip-to-solder temperature difference. What is this temperature difference 0.25 s after the start of heating?

Tube Banks

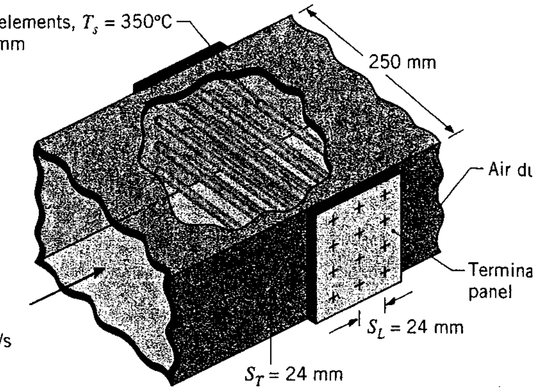
7.83 Repeat Example 7.7 for a more compact tube bank in which the longitudinal and transverse pitches are $S_L = S_T = 20.5$ mm. All other conditions remain the same.

7.84 A preheater involves the use of condensing steam at 100°C on the inside of a bank of tubes to heat air that enters at 1 atm and 25°C. The air moves at 5 m/s in cross flow over the tubes. Each tube is 1 m long and has an outside diameter of 10 mm. The bank consists of 196 tubes in a square, aligned array for which $S_T = S_L = 15$ mm. What is the total rate of heat transfer to the air? What is the pressure drop associated with the airflow?

7.85 Consider the in-line tube bank of Problem 7.84 ($D = 10$ mm, $L = 1$ m, and $S_T = S_L = 15$ mm), with condensing steam used to heat atmospheric air entering the tube bank at $T_i = 25°C$ and $V = 5$ m/s. In this case, however, the desired outlet temperature, not the number of tube rows, is known. What is the minimum value of N_L needed to achieve an outlet temperature of $T_o \geq 75°C$? What is the corresponding pressure drop across the tube bank?

7.86 A tube bank uses an aligned arrangement of 10-mm-diameter tubes with $S_T = S_L = 20$ mm. There are 10 rows of tubes with 50 tubes in each row. Consider an application for which cold water flows through the tubes, maintaining the outer surface temperature at 27°C, while flue gases at 427°C and a velocity of 5 m/s are in cross flow over the tubes. The properties of the flue gas may be approximated as those of atmospheric air at 427°C. What is the total rate of heat transfer per unit length of the tubes in the bank?

7.87 An air duct heater consists of an aligned array of electrical heating elements in which the longitudinal and transverse pitches are $S_L = S_T = 24$ mm. There are 3 rows of elements in the flow direction ($N_L = 3$) and 4 elements per row ($N_T = 4$). Atmospheric air with an upstream velocity of 12 m/s and a temperature of 25°C moves in cross flow over the elements, which have a diameter of 12 mm, a length of 250 mm, and are maintained at a surface temperature of 350°C.



(a) Determine the total heat transfer to the air and the temperature of the air leaving the duct heater.

(b) Determine the pressure drop across the element bank and the fan power requirement.

(c) Compare the average convection coefficient obtained in your analysis with the value for an isolated (single) element. Explain the difference between the results.

(d) What effect would increasing the longitudinal and transverse pitches to 30 mm have on the exit temperature of the air, the total heat rate, and the pressure drop?

7.88 A tube bank uses an aligned arrangement of 30-mm-diameter tubes with $S_T = S_L = 60$ mm and a tube length of 1 m. There are 10 tube rows in the flow direction ($N_L = 10$) and 7 tubes per row ($N_T = 7$). Air with upstream conditions of $T_\infty = 27°C$ and $V = 15$ m/s is in cross flow over the tubes, while a tube wall temperature of 100°C is maintained by steam condensation inside the tubes. Determine the temperature of air leaving the tube bank, the pressure drop across the bank, and the fan power requirement.

7.89 Electrical components mounted to each of two isothermal plates are cooled by passing atmospheric air between the plates, and an in-line array of aluminum pin fins is used to enhance heat transfer to the air.

The pins are of diameter $D = 2$ mm, length $L = 100$ mm, and thermal conductivity $k = 240$ W/m · K. The longitudinal and transverse pitches are $S_L = S_T = 4$ mm, with a square array of 625 pins ($N_T = N_L = 25$) mounted to square plates that are each of width $W = 100$ mm on a side. Air enters the pin array with a velocity of 10 m/s and a temperature of 300 K.

(a) Evaluating air properties at 300 K, estimate the average convection coefficient for the array of pin fins.