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Problems

Boundary Layer Profiles

6.1 In flow over a surface, velocity and temperature profiles are of the forms

$$u(y) = Ay + By^2 - Cy^3 \qquad \text{and}$$
$$T(y) = D + Ey + Fy^2 - Gy^3$$

where the coefficients A through G are constants. Obtain expressions for the friction coefficient C_f and the convection coefficient h in terms of u_∞, T_∞, and appropriate profile coefficients and fluid properties.

6.2 Water at a temperature of $T_\infty = 25°\text{C}$ flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is $T_{s,1} = 40°\text{C}$. The wall is 0.35 m thick, and its other surface temperature is $T_{s,2} = 100°\text{C}$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

6.3 In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as

$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp\left(-Pr\frac{u_\infty y}{\nu}\right)$$

where y is the distance normal to the surface and the Prandtl number, $Pr = c_p\mu/k = 0.7$, is a dimensionless fluid property. If $T_\infty = 400$ K, $T_s = 300$ K, and $u_\infty/\nu = 5000\ \text{m}^{-1}$, what is the surface heat flux?

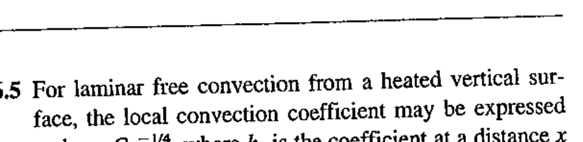
Heat Transfer Coefficients

6.4 For laminar flow over a flat plate, the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge ($x = 0$) of the plate. What is the ratio of the average coefficient between the leading edge and some location x on the plate to the local coefficient at x?

6.5 For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as $h_x = Cx^{-1/4}$, where h_x is the coefficient at a distance x from the leading edge of the surface and the quantity C, which depends on the fluid properties, is independent of x. Obtain an expression for the ratio $\bar{h}_x/h_x$, where $\bar{h}_x$ is the average coefficient between the leading edge ($x = 0$) and the x location. Sketch the variation of h_x and $\bar{h}_x$ with x.

6.6 A circular, hot gas jet at T_∞ is directed normal to a circular plate that has radius r_o and is maintained at a uniform temperature T_s. Gas flow over the plate is axisymmetric, causing the local convection coefficient to have a radial dependence of the form $h(r) = a + br^n$, where a, b, and n are constants. Determine the rate of heat transfer to the plate, expressing your result in terms of T_∞, T_s, r_o, a, b, and n.

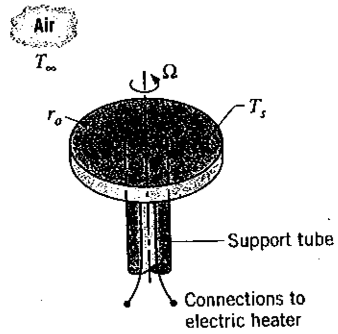
6.7 Parallel flow of atmospheric air over a flat plate of length $L = 3$ m is disrupted by an array of stationary rods placed in the flow path over the plate.



Laboratory measurements of the local convection coefficient at the surface of the plate are made for a prescribed value of V and $T_s > T_\infty$. The results are correlated by an expression of the form $h_x = 0.7 + 13.6x - 3.4x^2$, where h_x has units of $\text{W/m}^2 \cdot \text{K}$ and x is in meters. Evaluate the average convection coefficient $\bar{h}_L$ for the entire plate and the ratio $\bar{h}_L/h_L$ at the trailing edge.

6.8 Air at a free stream temperature of $T_\infty = 20°\text{C}$ is in parallel flow over a flat plate of length $L = 5$ m and temperature $T_s = 90°\text{C}$. However, obstacles placed in the flow intensify mixing with increasing distance x from the leading edge, and the spatial variation of temperatures the disk has an emissivity of $\varepsilon = 0.8$ and is exposed to large, isothermal surroundings for which $T_{sur} = T_\infty$. The remaining surfaces of the disk are well insulated, and heat transfer through the supporting rod may be neglected. Using results from the preceding problem, compute and plot temperature histories corresponding to air velocities of $V = 4$, 20, and 50 m/s. Constant properties may be assumed for the copper ($\rho = 8933\ \text{kg/m}^3$, $c_p = 425\ \text{J/kg}\cdot\text{K}$, $k = 386\ \text{W/m}\cdot\text{K}$) and air ($\nu = 38.8 \times 10^{-6}\ \text{m}^2\text{/s}$, $k = 0.0407\ \text{W/m}\cdot\text{K}$, $Pr = 0.684$).

6.13 If laminar flow is induced at the surface of a disk due to rotation about its axis, the local convection coefficient is known to be a constant, $h = C$, independent of radius. Consider conditions for which a disk of radius $r_o = 100$ mm is rotating in stagnant air at $T_\infty = 20°\text{C}$ and a value of $C = 20\ \text{W/m}^2\cdot\text{K}$ is maintained.



If an embedded electric heater maintains a surface temperature of $T_s = 50°\text{C}$, what is the local heat flux at the surface of the disk? What is the total electric power requirement? What can you say about the nature of boundary layer development on the disk?

Boundary Layer Transition

6.14 Consider airflow over a flat plate of length $L = 1$ m under conditions for which transition occurs at $x_c = 0.5$ m based on the critical Reynolds number, $Re_{x,c} = 5 \times 10^5$.

(a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.

(b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{\text{lam}}(x) = C_{\text{lam}}\,x^{-0.5} \quad \text{and} \quad h_{\text{turb}} = C_{\text{turb}}\,x^{-0.2}$$

where, at $T = 350$ K, $C_{\text{lam}} = 8.845\ \text{W/m}^{3/2}\cdot\text{K}$, $C_{\text{turb}} = 49.75\ \text{W/m}^{1.8}\cdot\text{K}$, and x has units of m. Develop an expression for the average convection coefficient, $\overline{h}_{\text{lam}}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \le x \le x_c$.

(c) Develop an expression for the average convection coefficient, $\overline{h}_{\text{turb}}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \le x \le L$.

(d) On the same coordinates, plot the local and average convection coefficients, h_x and $\overline{h}_x$, respectively, as a function of x for $0 \le x \le L$.

6.15 A fan that can provide air speeds up to 50 m/s is to be used in a low-speed wind tunnel with atmospheric air at 25°C. If one wishes to use the wind tunnel to study flat-plate boundary layer behavior up to Reynolds numbers of $Re_x = 10^8$, what is the minimum plate length that should be used? At what distance from the leading edge would transition occur if the critical Reynolds number were $Re_{x,c} = 5 \times 10^5$?

6.16 Assuming a transition Reynolds number of 5×10^5, determine the distance from the leading edge of a flat plate at which transition will occur for each of the following fluids when $u_\infty = 1$ m/s: atmospheric air, engine oil, and mercury. In each case, calculate the transition location for fluid temperatures of 27°C and 77°C.

6.17 To a good approximation, the dynamic viscosity μ, the thermal conductivity k, and the specific heat c_p are independent of pressure. In what manner do the kinematic viscosity ν and thermal diffusivity α vary with pressure for an incompressible liquid and an ideal gas? Determine α of air at 350 K for pressures of 1, 5, and 10 atm. Assuming a transition Reynolds number of 5×10^5, determine the distance from the leading edge of a flat plate at which transition will occur for air at 350 K at pressures of 1, 5, and 10 atm with $u_\infty = 2$ m/s.

Similarity and Dimensionless Parameters

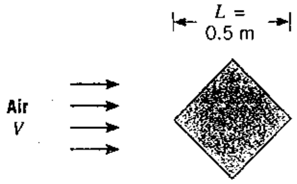
6.18 An object of irregular shape has a characteristic length of $L = 1$ m and is maintained at a uniform surface temperature of $T_s = 400$ K. When placed in atmospheric air at a temperature of $T_\infty = 300$ K and moving with a velocity of $V = 100$ m/s, the average heat flux from the surface to the air is 20,000 W/m². If a second object of the same shape, but with a characteristic length of $L = 5$ m, is maintained at a surface temperature of $T_s = 400$ K and is placed in atmospheric air at $T_\infty = 300$ K, what will the value of the average convection coefficient be if the air velocity is $V = 20$ m/s?

6.19 Experiments have shown that, for airflow at $T_\infty = 35°\text{C}$ and $V_1 = 100$ m/s, the rate of heat transfer from a turbine blade of characteristic length $L_1 = 0.15$ m and surface temperature $T_{s,1} = 300°\text{C}$ is $q_1 = 1500$ W. What would be the heat transfer rate from a second turbine blade of characteristic length $L_2 = 0.3$ m operating at $T_{s,2} = 400°\text{C}$ in airflow of $T_\infty = 35°\text{C}$ and $V_2 = 50$ m/s?

The surface area of the blade may be assumed to be directly proportional to its characteristic length.

6.20 Experimental measurements of the convection heat transfer coefficient for a square bar in cross flow yielded the following values:

$\bar{h}_1 = 50\ \mathrm{W/m^2 \cdot K}$ when $V_1 = 20\ \mathrm{m/s}$
$\bar{h}_2 = 40\ \mathrm{W/m^2 \cdot K}$ when $V_2 = 15\ \mathrm{m/s}$



Assume that the functional form of the Nusselt number is $\overline{Nu} = C\,Re^m\,Pr^n$, where C, m, and n are constants.

(a) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 15$ m/s?

(b) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 30$ m/s?

(c) Would your results be the same if the side of the bar, rather than its diagonal, were used as the characteristic length?

6.21 Experimental results for heat transfer over a flat plate with an extremely rough surface were found to be correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.9}\,Pr^{1/3}$$

where Nu_x is the local value of the Nusselt number at a position x measured from the leading edge of the plate. Obtain an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ to the local coefficient h_x.

6.22 Consider conditions for which a fluid with a free stream velocity of $V = 1$ m/s flows over a surface with a characteristic length of $L = 1$ m, providing an average convection heat transfer coefficient of $\bar{h} = 100\ \mathrm{W/m^2 \cdot K}$. Calculate the dimensionless parameters $\overline{Nu}_L$, Re_L, Pr, and $\bar{j}_H$ for the following fluids: air, engine oil, mercury, and water. Assume the fluids to be at 300 K.

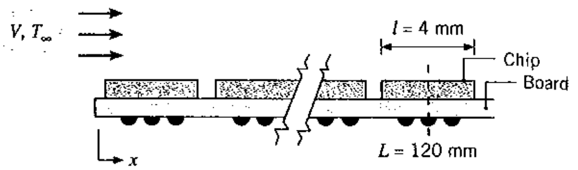
6.23 For flow over a flat plate of length L, the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge of the plate. What is the ratio of the average Nusselt number for the entire plate ($\overline{Nu}_L$) to the local Nusselt number at $x = L$ (Nu_L)?

6.24 For laminar boundary layer flow over a flat plate with air at 20°C and 1 atm, the thermal boundary layer thickness δ_t is approximately 13% larger than the velocity boundary layer thickness δ. Determine the ratio δ/δ_t if the fluid is ethylene glycol under the same flow conditions.

6.25 Sketch the variation of the velocity and thermal boundary layer thicknesses with distance from the leading edge of a flat plate for the laminar flow of air, water, engine oil, and mercury. For each case assume a mean fluid temperature of 300 K.

6.26 Forced air at $T_\infty = 25$°C and $V = 10$ m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.85}\,Pr^{1/3}$$



Estimate the surface temperature of the chip if it is dissipating 30 mW.

6.27 Consider the electronic elements that are cooled by forced convection in Problem 6.26. The cooling system is designed and tested at sea level ($P \approx 1$ atm), but the circuit board is sold to a customer in Mexico City, with an elevation of 2250 m and atmospheric pressure of 76.5 kPa.

(a) Estimate the surface temperature of the chip located 120 mm from the leading edge of the board when the board is operated in Mexico City. The dependence of various thermophysical properties upon pressure is noted in Problem 6.17.

(b) It is highly desirable for the chip operating temperature to be independent of the location of the customer. What air velocity is required for operation in Mexico City if the chip temperature is to be the same as at sea level?

6.28 Consider the chip on the circuit board of Problem 6.26. To ensure reliable operation over extended periods, the chip temperature should not exceed 85°C. Assuming the availability of forced air at $T_\infty = 25$°C and applicability of the prescribed heat transfer correlation, compute and plot the maximum allowable chip power dissipation P_c as a function of air velocity for $1 \le V \le 25$ m/s. If the chip surface has an emissivity of 0.80 and the board is mounted in a large enclosure whose walls are at 25°C, what is the effect of radiation on the P_c–V plot?

6.29 A major contributor to product defects in electronic modules relates to stresses induced during thermal