(d) Derive an expression for the steady-state temperature $T(x, \infty) = T_f$, leaving your result in terms of plate parameters (M, c_p), thermal conditions (T_i, T_∞, h), the surface temperature $T(L, t)$, and the heating time t_o.

Lumped Capacitance Method

5.5 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20\ \text{W/m}^2 \cdot \text{K}$. Assuming the properties of the steel to be $k = 40\ \text{W/m} \cdot \text{K}$, $\rho = 7800\ \text{kg/m}^3$, and $c = 600\ \text{J/kg} \cdot \text{K}$, estimate the time required for the cooling process.

5.6 Consider the steel balls of Problem 5.5, except now the air temperature increases with time as $T_\infty(t) = 325\ \text{K} + at$ where $a = 0.1875$ K/s.

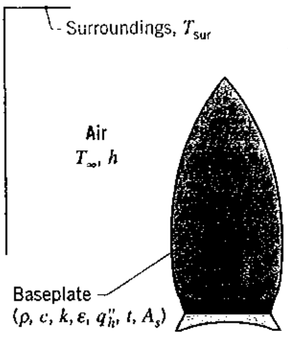
(a) Sketch the ball temperature versus time for $0 \leq t \leq 1$ h. Also show the ambient temperature, T_∞, in your graph. Explain special features of the ball temperature behavior.

(b) Find an expression for the ball temperature as a function of time, $T(t)$, and plot the ball temperature for $0 \leq t \leq 1$ h. Was your sketch correct?

5.7 The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

5.8 A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m · K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_\infty = 100$°C and $h = 3300\ \text{W/m}^2 \cdot \text{K}$. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance ($\rho c V$) is small compared to that of the steel sphere.

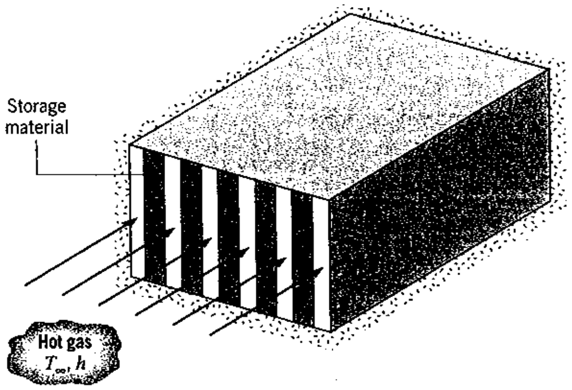
5.9 The base plate of an iron has a thickness of $L = 7$ mm and is made from an aluminum alloy ($\rho = 2800\ \text{kg/m}^3$, $c = 900\ \text{J/kg} \cdot \text{K}$, $k = 180\ \text{W/m} \cdot \text{K}$, $\varepsilon = 0.80$). An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The areas of both the inner and outer surfaces are $A_s = 0.040\ \text{m}^2$.



If an approximately uniform heat flux of $q''_h = 1.25 \times 10^4\ \text{W/m}^2$ is applied to the inner surface of the base plate and the convection coefficient at the outer surface is $h = 10\ \text{W/m}^2 \cdot \text{K}$, estimate the time required for the plate to reach a temperature of 135°C. *Hint:* Numerical integration is suggested in order to solve the problem.

5.10 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of $100\ \text{W/m}^2 \cdot \text{K}$. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

5.11 A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.

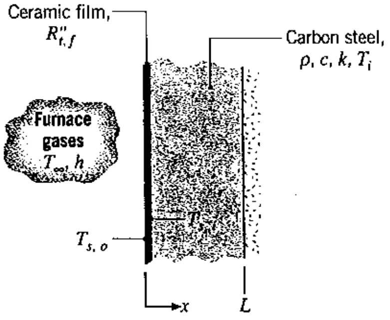


Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_\infty = 600$°C and $h = 100\ \text{W/m}^2 \cdot \text{K}$ throughout the channel. How long will it take to achieve 75% of the maximum possible

Consider a situation for which a chemical of density $\rho = 1200\ \text{kg/m}^3$ and specific heat $c = 2200\ \text{J/kg}\cdot\text{K}$ occupies a volume of $V = 2.25\ \text{m}^3$ in an insulated vessel. The chemical is to be heated from room temperature, $T_i = 300\ \text{K}$, to a process temperature of $T = 450\ \text{K}$ by passing saturated steam at $T_h = 500\ \text{K}$ through a coiled, thin-walled, 20-mm-diameter tube in the vessel. Steam condensation within the tube maintains an interior convection coefficient of $h_i = 10{,}000\ \text{W/m}^2\cdot\text{K}$, while the highly agitated liquid in the stirred vessel maintains an outside convection coefficient of $h_o = 2000\ \text{W/m}^2\cdot\text{K}$.

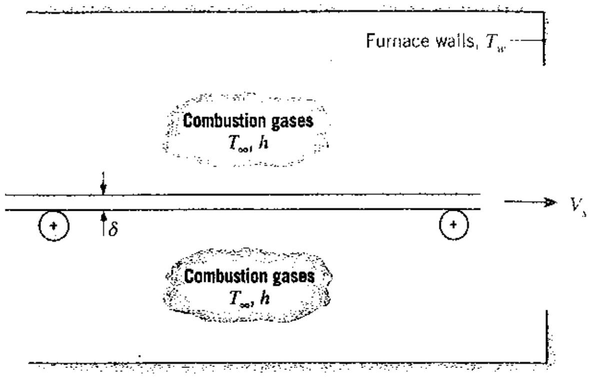
If the chemical is to be heated from 300 to 450 K in 60 minutes, what is the required length L of the submerged tubing?

5.16 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60\ \text{W/m}\cdot\text{K}$, $\rho = 7850\ \text{kg/m}^3$, $c = 430\ \text{J/kg}\cdot\text{K}$) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01\ \text{m}^2\cdot\text{K/W}$. The opposite surface is well insulated from the surroundings.



At furnace start-up the wall is at an initial temperature of $T_i = 300\ \text{K}$, and combustion gases at $T_\infty = 1300\ \text{K}$ enter the furnace, providing a convection coefficient of $h = 25\ \text{W/m}^2\cdot\text{K}$ at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200\ \text{K}$? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

5.17 A steel strip of thickness $\delta = 12$ mm is annealed by passing it through a large furnace whose walls are maintained at a temperature T_w corresponding to that of combustion gases flowing through the furnace ($T_w = T_\infty$). The strip, whose density, specific heat, thermal conductivity, and emissivity are $\rho = 7900\ \text{kg/m}^3$, $c_p = 640\ \text{J/kg}\cdot\text{K}$, $k = 30\ \text{W/m}\cdot\text{K}$, and $\varepsilon = 0.7$, respectively, is to be heated from 300°C to 600°C.



(a) For a uniform convection coefficient of $h = 100\ \text{W/m}^2\cdot\text{K}$ and $T_w = T_\infty = 700°\text{C}$, determine the time required to heat the strip. If the strip is moving at 0.5 m/s, how long must the furnace be?

(b) The annealing process may be accelerated (the strip speed increased) by increasing the environmental temperatures. For the furnace length obtained in part (a), determine the strip speed for $T_w = T_\infty = 850°\text{C}$ and $T_w = T_\infty = 1000°\text{C}$. For each set of environmental temperatures (700, 850, and 1000°C), plot the strip temperature as a function of time over the range $25°\text{C} \le T \le 600°\text{C}$. Over this range, also plot the radiation heat transfer coefficient, h_r, as a function of time.

5.18 A long wire of diameter $D = 1$ mm is submerged in an oil bath of temperature $T_\infty = 25°\text{C}$. The wire has an electrical resistance per unit length of $R'_e = 0.01\ \Omega/\text{m}$. If a current of $I = 100$ A flows through the wire and the convection coefficient is $h = 500\ \text{W/m}^2\cdot\text{K}$, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? The properties of the wire are $\rho = 8000\ \text{kg/m}^3$, $c = 500\ \text{J/kg}\cdot\text{K}$, and $k = 20\ \text{W/m}\cdot\text{K}$.

5.19 Consider the system of Problem 5.1 where the temperature of the plate is spacewise isothermal during the transient process.

(a) Obtain an expression for the temperature of the plate as a function of time $T(t)$ in terms of q''_o, T_∞, h, L, and the plate properties ρ and c.

(b) Determine the thermal time constant and the steady-state temperature for a 12-mm-thick plate of pure copper when $T_\infty = 27°\text{C}$, $h = 50\ \text{W/m}^2\cdot\text{K}$, and $q''_o = 5000\ \text{W/m}^2$. Estimate the time required to reach steady-state conditions.

(c) For the conditions of part (b), as well as for $h = 100$ and $200\ \text{W/m}^2\cdot\text{K}$, compute and plot the corresponding temperature histories of the plate for $0 \le t \le 2500$ s.

5.20 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

(a) Consider such a system of mass M, specific heat c, and surface area A_s, which is initially in equilibrium with the environment at T_∞. Suddenly, the electronic device is energized such that a constant heat generation $\dot{E}_g$ (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where $\theta \equiv T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \rightarrow \infty$; $\theta_i = T_i - T(\infty)$; T_i = initial temperature of device; R = thermal resistance $1/\bar{h}A_s$; and C = thermal capacitance Mc.

(b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

5.21 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at $T_{sur} = 1000°C$. The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

5.22 A metal sphere of diameter D, which is at a uniform temperature T_i, is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature T_∞ and the surrounding walls at a temperature T_{sur}.

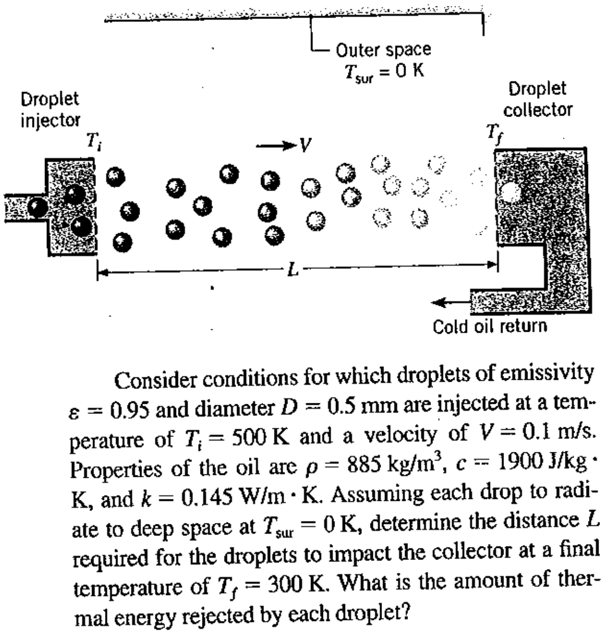
(a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature T.

(b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature T.

(c) How would you go about determining the time required for the sphere to cool to the temperature T if both convection and radiation are of the same order of magnitude?

(d) Consider an anodized aluminum sphere ($\varepsilon = 0.75$) 50 mm in diameter, which is at an initial temperature of $T_i = 800$ K. Both the air and surroundings are at 300 K, and the convection coefficient is 10 W/m$^2 \cdot$ K. For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere ($\varepsilon = 0.1$).

5.23 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance L, over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.



Consider conditions for which droplets of emissivity $\varepsilon = 0.95$ and diameter $D = 0.5$ mm are injected at a temperature of $T_i = 500$ K and a velocity of $V = 0.1$ m/s. Properties of the oil are $\rho = 885$ kg/m^3, $c = 1900$ J/kg $\cdot$ K, and $k = 0.145$ W/m $\cdot$ K. Assuming each drop to radiate to deep space at $T_{sur} = 0$ K, determine the distance L required for the droplets to impact the collector at a final temperature of $T_f = 300$ K. What is the amount of thermal energy rejected by each droplet?

5.24 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with $\rho = 8600$ kg/m^3, $c = 290$ J/kg $\cdot$ K, and $k = 63$ W/m $\cdot$ K.