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Problems

Plane Wall

3.1 Consider the plane wall of Figure 3.1, separating hot and cold fluids at temperatures $T_{\infty,1}$ and $T_{\infty,2}$, respectively. Using surface energy balances as boundary conditions at $x = 0$ and $x = L$ (see Equation 2.32), obtain the temperature distribution within the wall and the heat flux in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1 , h_2 , k , and L .

3.2 The rear window of an automobile is defogged by passing warm air over its inner surface.

(a) If the warm air is at $T_{\infty,i} = 40^\circ\text{C}$ and the corresponding convection coefficient is $h_i = 30 \text{ W/m}^2 \cdot \text{K}$, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{\infty,o} = -10^\circ\text{C}$ and the associated convection coefficient is $h_o = 65 \text{ W/m}^2 \cdot \text{K}$?

(b) In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2, 65,$ and $100 \text{ W/m}^2 \cdot \text{K}$, compute and plot the inner and outer surface temperatures as a function of $T_{\infty,o}$ for $-30 \leq T_{\infty,o} \leq 0^\circ\text{C}$.

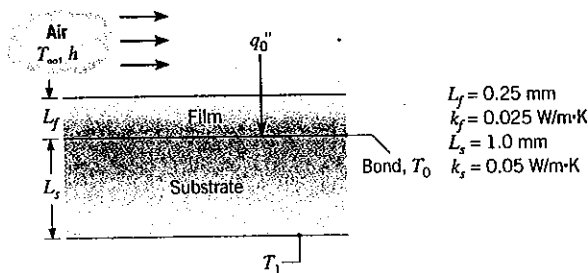
3.3 The rear window of an automobile is defogged by attaching a thin, transparent, film-type heating element to its inner surface. By electrically heating this element, a uniform heat flux may be established at the inner surface.

(a) For 4-mm-thick window glass, determine the electrical power required per unit window area to maintain an inner surface temperature of 15°C when the interior air temperature and convection coefficient are $T_{\infty,i} = 25^\circ\text{C}$ and $h_i = 10 \text{ W/m}^2 \cdot \text{K}$, while the exterior (ambient) air temperature and convection coefficient are $T_{\infty,o} = -10^\circ\text{C}$ and $h_o = 65 \text{ W/m}^2 \cdot \text{K}$.

(b) In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2, 20, 65,$ and $100 \text{ W/m}^2 \cdot \text{K}$, determine and plot the electrical power

requirement as a function of $T_{\infty,o}$ for $-30 \leq T_{\infty,o} \leq 0^\circ\text{C}$. From your results, what can you conclude about the need for heater operation at low values of h_o ? How is this conclusion affected by the value of $T_{\infty,o}$? If $h \propto V^n$, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

3.4 In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch. To cure the bond at a temperature T_0 , a radiant source is used to provide a heat flux q_0'' (W/m^2), all of which is absorbed at the bonded surface. The back of the substrate is maintained at T_1 while the free surface of the film is exposed to air at T_∞ and a convection heat transfer coefficient h .



(a) Show the thermal circuit representing the steady-state heat transfer situation. Be sure to label all elements, nodes, and heat rates. Leave in symbolic form.

(b) Assume the following conditions: $T_\infty = 20^\circ\text{C}$, $h = 50 \text{ W/m}^2 \cdot \text{K}$, and $T_1 = 30^\circ\text{C}$. Calculate the heat flux q_0'' that is required to maintain the bonded surface at $T_0 = 60^\circ\text{C}$.

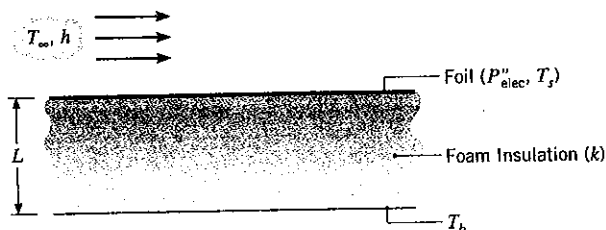
(c) Compute and plot the required heat flux as a function of the film thickness for $0 \leq L_f \leq 1 \text{ mm}$.

(d) If the film is not transparent and all of the radiant heat flux is absorbed at its upper surface, determine

the heat flux required to achieve bonding. Plot your results as a function of L_f for $0 \leq L_f \leq 1$ mm.

3.5/ The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity $k_i = 0.046 \text{ W/m} \cdot \text{K}$ and thickness $L_i = 50$ mm and steel panels, each of thermal conductivity $k_p = 60 \text{ W/m} \cdot \text{K}$ and thickness $L_p = 3$ mm. If the wall separates refrigerated air at $T_{\infty,i} = 4^\circ\text{C}$ from ambient air at $T_{\infty,o} = 25^\circ\text{C}$, what is the heat gain per unit surface area? Coefficients associated with natural convection at the inner and outer surfaces may be approximated as $h_i = h_o = 5 \text{ W/m}^2 \cdot \text{K}$.

3.6 A technique for measuring convection heat transfer coefficients involves bonding one surface of a thin metallic foil to an insulating material and exposing the other surface to the fluid flow conditions of interest.



By passing an electric current through the foil, heat is dissipated uniformly within the foil and the corresponding flux, P''_{elec} , may be inferred from related voltage and current measurements. If the insulation thickness L and thermal conductivity k are known and the fluid, foil, and insulation temperatures (T_∞ , T_s , T_b) are measured, the convection coefficient may be determined. Consider conditions for which $T_\infty = T_b = 25^\circ\text{C}$, $P''_{\text{elec}} = 2000 \text{ W/m}^2$, $L = 10$ mm, and $k = 0.040 \text{ W/m} \cdot \text{K}$.

- (a) With water flow over the surface, the foil temperature measurement yields $T_s = 27^\circ\text{C}$. Determine the convection coefficient. What error would be incurred by assuming all of the dissipated power to be transferred to the water by convection?
- (b) If, instead, air flows over the surface and the temperature measurement yields $T_s = 125^\circ\text{C}$, what is the convection coefficient? The foil has an emissivity of 0.15 and is exposed to large surroundings at 25°C . What error would be incurred by assuming all of the dissipated power to be transferred to the air by convection?
- (c) Typically, heat flux gages are operated at a fixed temperature (T_s), in which case the power dissipation provides a direct measure of the convection coefficient. For $T_s = 27^\circ\text{C}$, plot P''_{elec} as a function of h_o for $10 \leq h_o \leq 1000 \text{ W/m}^2 \cdot \text{K}$. What effect does h_o have on the error associated with neglecting conduction through the insulation?

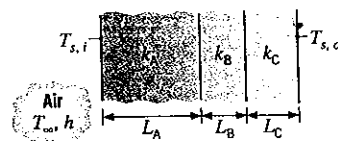
3.7 The *wind chill*, which is experienced on a cold, windy day, is related to increased heat transfer from exposed human skin to the surrounding atmosphere. Consider a layer of fatty tissue that is 3 mm thick and whose interior surface is maintained at a temperature of 36°C . On a calm day the convection heat transfer coefficient at the outer surface is $25 \text{ W/m}^2 \cdot \text{K}$, but with 30 km/h winds it reaches $65 \text{ W/m}^2 \cdot \text{K}$. In both cases the ambient air temperature is -15°C .

- (a) What is the ratio of the heat loss per unit area from the skin for the calm day to that for the windy day?
- (b) What will be the skin outer surface temperature for the calm day? For the windy day?
- (c) What temperature would the air have to assume on the calm day to produce the same heat loss occurring with the air temperature at -15°C on the windy day?

3.8 A thermopane window consists of two pieces of glass 7 mm thick that enclose an air space 7 mm thick. The window separates room air at 20°C from outside ambient air at -10°C . The convection coefficient associated with the inner (room-side) surface is $10 \text{ W/m}^2 \cdot \text{K}$.

- (a) If the convection coefficient associated with the outer (ambient) air is $h_o = 80 \text{ W/m}^2 \cdot \text{K}$, what is the heat loss through a window that is 0.8 m long by 0.5 m wide? Neglect radiation, and assume the air enclosed between the panes to be stagnant.
- (b) Compute and plot the effect of h_o on the heat loss for $10 \leq h_o \leq 100 \text{ W/m}^2 \cdot \text{K}$. Repeat this calculation for a triple-pane construction in which a third pane and a second air space of equivalent thickness are added.

3.9 The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 20 \text{ W/m} \cdot \text{K}$ and $k_C = 50 \text{ W/m} \cdot \text{K}$, and known thickness, $L_A = 0.30$ m and $L_C = 0.15$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.15$ m, but unknown thermal conductivity k_B .



Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20^\circ\text{C}$, an inner surface temperature of $T_{s,i} = 600^\circ\text{C}$, and an oven air temperature of $T_\infty = 800^\circ\text{C}$. The inside convection coefficient h is known to be $25 \text{ W/m}^2 \cdot \text{K}$. What is the value of k_B ?

3.10 A testing lab is contracted to measure the thermal conductivity of various liquids as a function of the liquid temperature. Typically, the lab would measure the thermal conductivity and its temperature dependence

stored at the prescribed temperature of 55°C. The cylindrical storage tank (with flat ends) has a capacity of 100 gallons, and foamed urethane is used to insulate the side and end walls from ambient air at an annual average temperature of 20°C. The resistance to heat transfer is dominated by conduction in the insulation and by free convection in the air, for which $h \approx 2 \text{ W/m}^2 \cdot \text{K}$. If electric resistance heating is used to compensate for the losses and the cost of electric power is $\$0.08/\text{kW} \cdot \text{h}$, specify tank and insulation dimensions for which the annual cost associated with the heat losses is less than $\$50$.

3.37 A thin electrical heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temperature of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of $10 \text{ W/m} \cdot \text{K}$. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01 \text{ m} \cdot \text{K/W}$. The outer surface of the heater is exposed to a fluid with $T_\infty = -10^\circ\text{C}$ and a convection coefficient of $h = 100 \text{ W/m}^2 \cdot \text{K}$. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25^\circ\text{C}$.

3.38 In the previous problem, the electrical power required to maintain the heater at $T_o = 25^\circ\text{C}$ depends on the thermal conductivity of the wall material k , the thermal contact resistance $R'_{t,c}$, and the convection coefficient h . Compute and plot the separate effect of changes in k ($1 \leq k \leq 200 \text{ W/m} \cdot \text{K}$), $R'_{t,c}$ ($0 \leq R'_{t,c} \leq 0.1 \text{ m} \cdot \text{K/W}$), and h ($10 \leq h \leq 1000 \text{ W/m}^2 \cdot \text{K}$) on the total heater power requirement, as well as the rate of heat transfer to the inner surface of the tube and to the fluid.

3.39 A stainless steel (AISI 304) tube used to transport a chilled pharmaceutical has an inner diameter of 36 mm and a wall thickness of 2 mm. The pharmaceutical and ambient air are at temperatures of 6°C and 23°C, respectively, while the corresponding inner and outer convection coefficients are $400 \text{ W/m}^2 \cdot \text{K}$ and $6 \text{ W/m}^2 \cdot \text{K}$, respectively.

(a) What is the heat gain per unit tube length?

(b) What is the heat gain per unit length if a 10-mm-thick layer of calcium silicate insulation ($k_{\text{ins}} = 0.050 \text{ W/m} \cdot \text{K}$) is applied to the tube?

3.40 Superheated steam at 575°C is routed from a boiler to the turbine of an electric power plant through steel tubes ($k = 35 \text{ W/m} \cdot \text{K}$) of 300 mm inner diameter and 30 mm wall thickness. To reduce heat loss to the surroundings and to maintain a *safe-to-touch* outer surface temperature, a layer of calcium silicate insulation ($k = 0.10 \text{ W/m} \cdot \text{K}$) is applied to the tubes, while degradation of the insulation is reduced by

wrapping it in a thin sheet of aluminum having an emissivity of $\epsilon = 0.20$. The air and wall temperatures of the power plant are 27°C.

(a) Assuming that the inner surface temperature of a steel tube corresponds to that of the steam and the convection coefficient outside the aluminum sheet is $6 \text{ W/m}^2 \cdot \text{K}$, what is the minimum insulation thickness needed to insure that the temperature of the aluminum does not exceed 50°C? What is the corresponding heat loss per meter of tube length?

(b) Explore the effect of the insulation thickness on the temperature of the aluminum and the heat loss per unit tube length.

3.41 A thin electrical heater is inserted between a long circular rod and a concentric tube with inner and outer radii of 20 and 40 mm. The rod (A) has a thermal conductivity of $k_A = 0.15 \text{ W/m} \cdot \text{K}$, while the tube (B) has a thermal conductivity of $k_B = 1.5 \text{ W/m} \cdot \text{K}$ and its outer surface is subjected to convection with a fluid of temperature $T_\infty = -15^\circ\text{C}$ and heat transfer coefficient $50 \text{ W/m}^2 \cdot \text{K}$. The thermal contact resistance between the cylinder surfaces and the heater is negligible.

(a) Determine the electrical power per unit length of the cylinders (W/m) that is required to maintain the outer surface of cylinder B at 5°C.

(b) What is the temperature at the center of cylinder A?

3.42 A wire of diameter $D = 2 \text{ mm}$ and uniform temperature T has an electrical resistance of $0.01 \Omega/\text{m}$ and a current flow of 20 A.

(a) What is the rate at which heat is dissipated per unit length of wire? What is the heat dissipation per unit volume within the wire?

(b) If the wire is not insulated and is in ambient air and large surrounding for which $T_\infty = T_{\text{sur}} = 20^\circ\text{C}$, what is the temperature T of the wire? The wire has an emissivity of 0.3, and the coefficient associated with heat transfer by natural convection may be approximated by an expression of the form, $h = C[(T - T_\infty)/D]^{1/4}$, where $C = 1.25 \text{ W/m}^{7/4} \cdot \text{K}^{5/4}$.

(c) If the wire is coated with plastic insulation of 2-mm thickness and a thermal conductivity of $0.25 \text{ W/m} \cdot \text{K}$, what are the inner and outer surface temperatures of the insulation? The insulation has an emissivity of 0.9, and the convection coefficient is given by the expression of part (b). Explore the effect of the insulation thickness on the surface temperatures.

3.43 A 2-mm-diameter electrical wire is insulated by a 2-mm-thick rubberized sheath ($k = 0.13 \text{ W/m} \cdot \text{K}$), and the wire/sheath interface is characterized by a thermal

(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation ($k = 0.5 \text{ W/m} \cdot \text{K}$) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

3.48 A 0.20-m-diameter, thin-walled steel pipe is used to transport saturated steam at a pressure of 20 bars in a room for which the air temperature is 25°C and the convection heat transfer coefficient at the outer surface of the pipe is $20 \text{ W/m}^2 \cdot \text{K}$.

(a) What is the heat loss per unit length from the bare pipe (no insulation)? Estimate the heat loss per unit length if a 50-mm-thick layer of insulation (magnesia, 85%) is added. The steel and magnesia may each be assumed to have an emissivity of 0.8, and the steam-side convection resistance may be neglected.

(b) The costs associated with generating the steam and installing the insulation are known to be $\$4/10^9 \text{ J}$ and $\$100/\text{m}$ of pipe length, respectively. If the steam line is to operate 7500 h/yr, how many years are needed to pay back the initial investment in insulation?

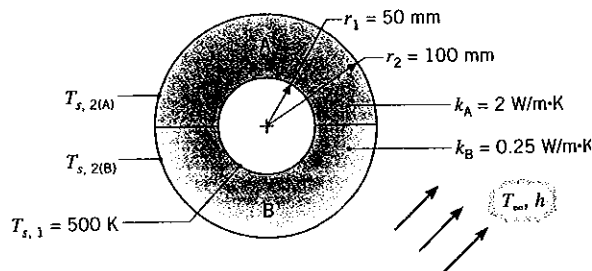
3.49 Steam at a temperature of 250°C flows through a steel pipe (AISI 1010) of 60-mm inside diameter and 75-mm outside diameter. The convection coefficient between the steam and the inner surface of the pipe is $500 \text{ W/m}^2 \cdot \text{K}$, while that between the outer surface of the pipe and the surroundings is $25 \text{ W/m}^2 \cdot \text{K}$. The pipe emissivity is 0.8, and the temperature of the air and the surroundings is 20°C . What is the heat loss per unit length of pipe?

3.50 We wish to determine the effect of adding a layer of magnesia insulation to the steam pipe of the foregoing problem. The convection coefficient at the outer surface of the insulation may be assumed to remain at $25 \text{ W/m}^2 \cdot \text{K}$, and the emissivity is $\epsilon = 0.8$. Determine and plot the heat loss per unit length of pipe and the outer surface temperature as a function of insulation thickness. If the cost of generating the steam is $\$4/10^9 \text{ J}$ and the steam line operates 7000 h/yr, recommend an insulation thickness and determine the corresponding annual savings in energy costs. Plot the temperature distribution in the insulation for the recommended thickness.

3.51 An uninsulated, thin-walled pipe of 100-mm diameter is used to transport water to equipment that operates outdoors and uses the water as a coolant. During particularly harsh winter conditions, the pipe wall achieves a temperature of -15°C and a cylindrical layer of ice forms on the inner surface of the wall. If the mean water temperature is 3°C and a convection coefficient of

$2000 \text{ W/m}^2 \cdot \text{K}$ is maintained at the inner surface of the ice, which is at 0°C , what is the thickness of the ice layer?

3.52 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which $T_\infty = 300 \text{ K}$ and $h = 25 \text{ W/m}^2 \cdot \text{K}$.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

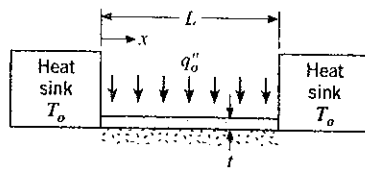
(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

3.53 A bakelite coating is to be used with a 10-mm-diameter conducting rod, whose surface is maintained at 200°C by passage of an electrical current. The rod is in a fluid at 25°C , and the convection coefficient is $140 \text{ W/m}^2 \cdot \text{K}$. What is the critical radius associated with the coating? What is the heat transfer rate per unit length for the bare rod and for the rod with a coating of bakelite that corresponds to the critical radius? How much bakelite should be added to reduce the heat transfer associated with the bare rod by 25%?

Spherical Wall

3.54 A storage tank consists of a cylindrical section that has a length and inner diameter of $L = 2 \text{ m}$ and $D_i = 1 \text{ m}$, respectively, and two hemispherical end sections. The tank is constructed from 20-mm-thick glass (Pyrex) and is exposed to ambient air for which the temperature is 300 K and the convection coefficient is $10 \text{ W/m}^2 \cdot \text{K}$. The tank is used to store heated oil, which maintains the inner surface at a temperature of 400 K . Determine the electrical power that must be supplied to a heater submerged in the oil if the prescribed conditions are to be maintained. Radiation

plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q_o'' .



- Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.
- Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

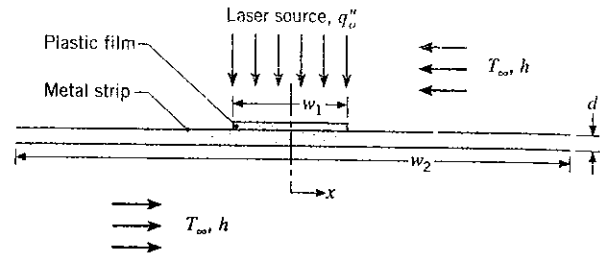
3.102 Consider the flat plate of Problem 3.101, but with the heat sinks at different temperatures, $T(0) = T_o$ and $T(L) = T_L$, and with the bottom surface no longer insulated. Convection heat transfer is now allowed to occur between this surface and a fluid at T_∞ , with a convection coefficient h .

- Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.
- Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

(c) For $q_o'' = 20,000 \text{ W/m}^2$, $T_o = 100^\circ\text{C}$, $T_L = 35^\circ\text{C}$, $T_\infty = 25^\circ\text{C}$, $k = 25 \text{ W/m}\cdot\text{K}$, $h = 50 \text{ W/m}^2\cdot\text{K}$, $L = 100 \text{ mm}$, $t = 5 \text{ mm}$, and a plate width of $W = 30 \text{ mm}$, plot the temperature distribution and determine the sink heat rates, $q_x(0)$ and $q_x(L)$. On the same graph, plot three additional temperature distributions corresponding to changes in the following parameters, with the remaining parameters unchanged: (i) $q_o'' = 30,000 \text{ W/m}^2$, (ii) $h = 200 \text{ W/m}^2\cdot\text{K}$, and (iii) the value of q_o'' for which $q_x(0) = 0$ when $h = 200 \text{ W/m}^2\cdot\text{K}$.

3.103 A bonding operation utilizes a laser to provide a constant heat flux, q_o'' , across the top surface of a thin adhesive-backed, plastic film to be affixed to a metal strip as shown in the sketch. The metal strip has a thickness $d = 1.25 \text{ mm}$ and its width is large relative to that of the film. The thermophysical properties of the strip are $\rho = 7850 \text{ kg/m}^3$, $c_p = 435 \text{ J/kg}\cdot\text{K}$, and $k = 60 \text{ W/m}\cdot\text{K}$. The thermal resistance of the plastic film of width $w_1 = 40 \text{ mm}$ is negligible. The upper and lower surfaces of the strip (including the plastic film) experience convection with air at 25°C and a convection coefficient of $10 \text{ W/m}^2\cdot\text{K}$. The strip and film are very

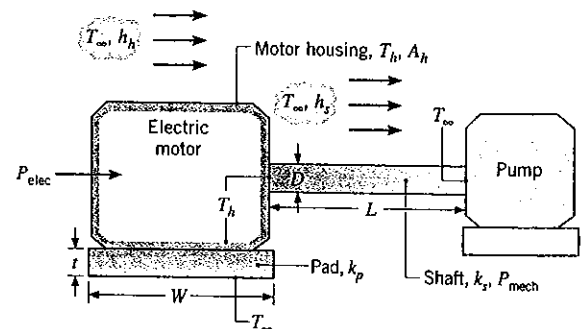
long in the direction normal to the page. Assume the edges of the metal strip are at the air temperature (T_∞).



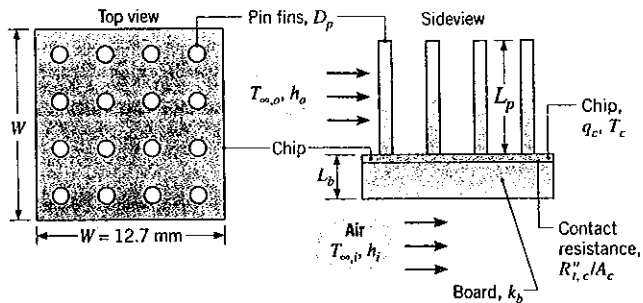
- Derive an expression for the temperature distribution in the portion of the steel strip with the plastic film ($-w_1/2 \leq x \leq +w_1/2$).
- If the heat flux provided by the laser is $10,000 \text{ W/m}^2$, determine the temperature of the plastic film at the center ($x = 0$) and its edges ($x = \pm w_1/2$).
- Plot the temperature distribution for the entire strip and point out its special features.

3.104 A thin metallic wire of thermal conductivity k , diameter D , and length $2L$ is annealed by passing an electrical current through the wire to induce a uniform volumetric heat generation \dot{q} . The ambient air around the wire is at a temperature T_∞ , while the ends of the wire at $x = \pm L$ are also maintained at T_∞ . Heat transfer from the wire to the air is characterized by the convection coefficient h . Obtain an expression for the steady-state temperature distribution $T(x)$ along the wire.

3.105 A motor draws electric power P_{elec} from a supply line and delivers mechanical power P_{mech} to a pump through a rotating copper shaft of thermal conductivity k_s , length L , and diameter D . The motor is mounted on a square pad of width W , thickness t , and thermal conductivity k_p . The surface of the housing exposed to ambient air at T_∞ is of area A_h , and the corresponding convection coefficient is h_h . Opposite ends of the shaft are at temperatures of T_h and T_∞ , and heat transfer from the shaft to the ambient air is characterized by the convection coefficient h_s . The base of the pad is at T_∞ .



3.134 As more and more components are placed on a single integrated circuit (chip), the amount of heat that is dissipated continues to increase. However, this increase is limited by the maximum allowable chip operating temperature, which is approximately 75°C. To maximize heat dissipation, it is proposed that a 4 × 4 array of copper pin fins be metallurgically joined to the outer surface of a square chip that is 12.7 mm on a side.

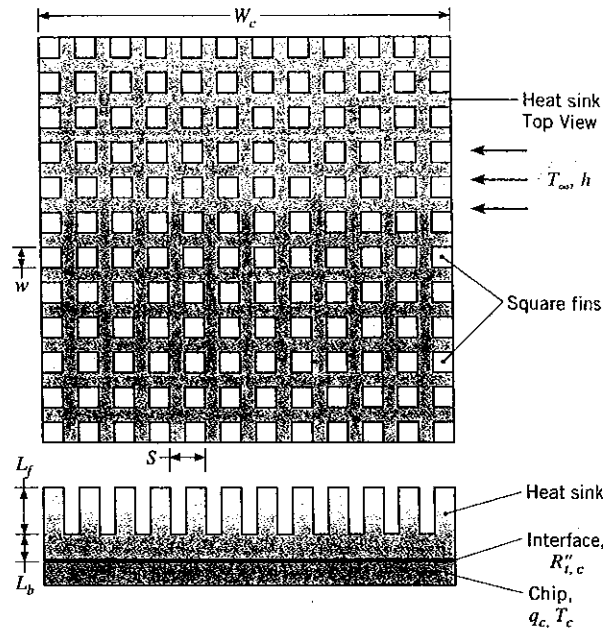


- (a) Sketch the equivalent thermal circuit for the pin-chip-board assembly, assuming one-dimensional, steady-state conditions and negligible contact resistance between the pins and the chip. In variable form, label appropriate resistances, temperatures, and heat rates.
- (b) For the conditions prescribed in Problem 3.27, what is the maximum rate at which heat can be dissipated in the chip when the pins are in place? That is, what is the value of q_c for $T_c = 75^\circ\text{C}$? The pin diameter and length are $D_p = 1.5$ mm and $L_p = 15$ mm.

3.135 In Problem 3.134, the prescribed value of $h_o = 1000$ $\text{W/m}^2 \cdot \text{K}$ is large and characteristic of liquid cooling. In practice it would be far more preferable to use air cooling, for which a reasonable upper limit to the convection coefficient would be $h_o = 250$ $\text{W/m}^2 \cdot \text{K}$. Assess the effect of changes in the pin fin geometry on the chip heat rate if the remaining conditions of Problem 3.134, including a maximum allowable chip temperature of 75°C, remain in effect. Parametric variations that may be considered include the total number of pins, N , in the square array, the pin diameter D_p , and the pin length L_p . However, the product $N^{1/2}D_p$ should not exceed 9 mm to ensure adequate air flow passage through the array. Recommend a design that enhances chip cooling.

3.136 As a means of enhancing heat transfer from high-performance logic chips, it is common to attach a *heat sink* to the chip surface in order to increase the surface area available for convection heat transfer. Because of the ease with which it may be manufactured (by taking orthogonal sawcuts in a block of material), an attractive

option is to use a heat sink consisting of an array of square fins of width w on a side. The spacing between adjoining fins would be determined by the width of the sawblade, with the sum of this spacing and the fin width designated as the fin pitch S . The method by which the heat sink is joined to the chip would determine the interfacial contact resistance, $R''_{t,c}$.



Consider a square chip of width $W_c = 16$ mm and conditions for which cooling is provided by a dielectric liquid with $T_\infty = 25^\circ\text{C}$ and $h = 1500$ $\text{W/m}^2 \cdot \text{K}$. The heat sink is fabricated from copper ($k = 400$ $\text{W/m} \cdot \text{K}$), and its characteristic dimensions are $w = 0.25$ mm, $S = 0.50$ mm, $L_f = 6$ mm, and $L_b = 3$ mm. The prescribed values of w and S represent minima imposed by manufacturing constraints and the need to maintain adequate flow in the passages between fins.

- (a) If a metallurgical joint provides a contact resistance of $R''_{t,c} = 5 \times 10^{-6}$ $\text{m}^2 \cdot \text{K/W}$ and the maximum allowable chip temperature is 85°C, what is the maximum allowable chip power dissipation q_c ? Assume all of the heat to be transferred through the heat sink.
- (b) It may be possible to increase the heat dissipation by increasing w , subject to the constraint that $(S - w) \geq 0.25$ mm, and/or increasing L_f (subject to manufacturing constraints that $L_f \leq 10$ mm). Assess the effect of such changes.

3.137 Because of the large number of devices in today's PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin