- What is the physical significance of each term appearing in the *heat equation*?
- Cite some examples of *thermal energy generation*. If the rate at which thermal energy is generated per unit volume, $\dot{q}$, varies with location in a medium of volume V, how can the rate of energy generation for the entire medium, $\dot{E}_g$, be determined from knowledge of $\dot{q}(x, y, z)$?
- For a chemically reacting medium, what kind of reaction provides a *source* of thermal energy ($\dot{q} > 0$)? What kind of reaction provides a *sink* for thermal energy ($\dot{q} < 0$)?
- To solve the *heat equation* for the temperature distribution in a medium, *boundary conditions* must be prescribed at the surfaces of the medium. What physical conditions are commonly suitable for this purpose?

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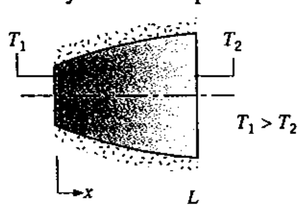
Problems

Fourier's Law

2.1 Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown below.

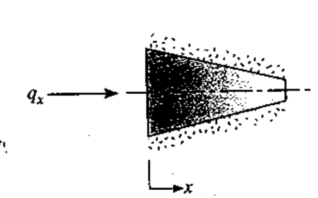


Assuming constant properties and no internal heat generation, sketch the temperature distribution on T–x coordinates. Briefly explain the shape of your curve.

2.2 A hot water pipe with outside radius r_1 has a temperature T_1. A thick insulation applied to reduce the heat loss has an outer radius r_2 and temperature T_2. On T–r coordinates, sketch the temperature distribution in the insulation for one-dimensional, steady-state heat transfer with constant properties. Give a brief explanation, justifying the shape of your curve.

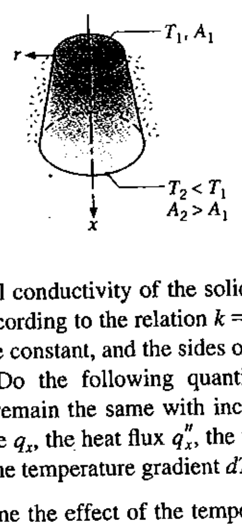
2.3 A spherical shell with inner radius r_1 and outer radius r_2 has surface temperatures T_1 and T_2, respectively, where $T_1 > T_2$. Sketch the temperature distribution on T–r coordinates assuming steady-state, one-dimensional conduction with constant properties. Briefly justify the shape of your curve.

2.4 Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.



Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in kelvins, and x in meters.

2.5 A solid, truncated cone serves as a support for a system that maintains the top (truncated) face of the cone at a temperature T_1, while the base of the cone is at a temperature $T_2 < T_1$.



The thermal conductivity of the solid depends on temperature according to the relation $k = k_o - aT$, where a is a positive constant, and the sides of the cone are well insulated. Do the following quantities increase, decrease, or remain the same with increasing x: the heat transfer rate q_x, the heat flux q_x'', the thermal conductivity k, and the temperature gradient dT/dx?

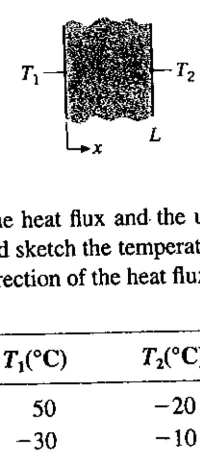
2.6 To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as

$$k = k_o + aT$$

where k_o is a positive constant and a is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

2.7 A young engineer is asked to design a thermal protection barrier for a sensitive electronic device that might be exposed to irradiation from a high-powered infrared laser. Having learned as a student that a low thermal conductivity material provides good insulating characteristics, the engineer specifies use of a nanostructured aerogel, characterized by a thermal conductivity of $k_a = 0.005$ W/m·K, for the protective barrier. The engineer's boss questions the wisdom of selecting the aerogel *because* it has a low thermal conductivity. Consider the sudden laser irradiation of (a) pure aluminum, (b) glass, and (c) aerogel. The laser provides irradiation of $G = 10 \times 10^6$ W/m^2. The absorptivities of the materials are $\alpha = 0.2$, 0.9, and 0.8 for the aluminum, glass, and aerogel, respectively, and the initial temperature of the barrier is $T_i = 300$ K. Explain why the boss is concerned. *Hint:* All materials experience thermal expansion (or contraction), and local stresses that develop within a material are, to a first approximation, proportional to the local temperature gradient.

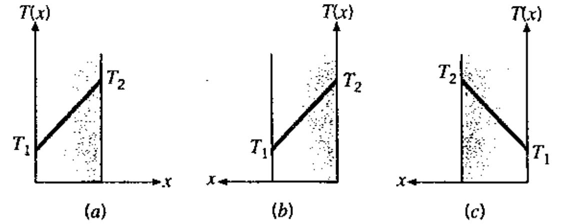
2.8 Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.



Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

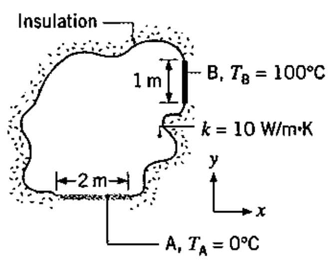
Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	−20	
2	−30	−10	
3	70		160
4		40	−80
5		30	200

2.9 Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux q_x'' and the temperature gradient dT/dx for the coordinate systems shown.



2.10 A cylinder of radius r_o, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

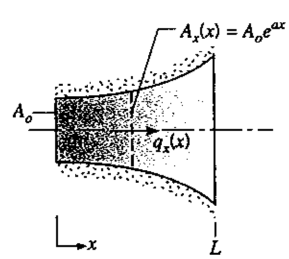


2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m · K) that are 1 m long and have a cross-sectional area of 0.005 m^2. Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft–pipeline joint ($x = 0$) and at the shaft–ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through $L = 10$-mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The thermal conductivities of the aerogel and polycarbonate are $k_{ag} = 0.014$ W/m · K and $k_{pc} = 0.21$ W/m · K, respectively. Evaluate the thermal conductivity of the soda lime glass at 300 K. If the aircraft has 130 windows and the cost to heat the cabin air is \$1/kW · h, compare the costs associated with the heat loss through the windows for an 8-hour intercontinental flight.

2.15 Gold is commonly used in semiconductor packaging to form interconnections (also known as interconnects) that carry electrical signals between different devices in the package. In addition to being a good electrical conductor, gold interconnects are also effective at protecting the heat-generating devices to which they are attached by conducting thermal energy away from the devices to surrounding, cooler regions. Consider a thin film of gold that has a cross section of 60 nm × 250 nm.

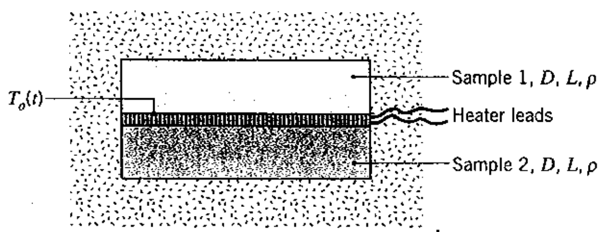
(a) For an applied temperature difference of 20°C, determine the energy conducted along a 1-μm-long, thin-film interconnect. Evaluate properties at 300 K.

(b) Plot the lengthwise (in the 1-μm direction) and spanwise (in the thinnest direction) thermal conductivities of the gold film as a function of the film thickness, L, for $30 \le L \le 140$ nm.

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn't the thickness of the insulating material that counts, it's the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood, or just 6 in. of the manufacturer's insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don't know the R-value is defined as L/k, where L (in.) is the thickness of the insulation and k (Btu · in./hr · ft^2 · °F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two

For a particular test run, the electrical heater dissipates 15.0 W for a period of Δt_o = 120 s and the temperature at the interface is $T_o(30\text{ s})$ = 24.57°C after 30 s of heating. A long time after the heater is deenergized, $t \gg \Delta t_o$, the samples reach the uniform temperature of $T_o(\infty)$ = 33.50°C. The density of the sample materials, determined by measurement of volume and mass, is ρ = 3965 kg/m^3.



Determine the specific heat and thermal conductivity of the test material. By looking at values of the thermophysical properties in Table A.1 or A.2, identify the test sample material.

The Heat Equation

2.20 At a given instant of time the temperature distribution within an infinite homogeneous body is given by the function

$$T(x, y, z) = x^2 - 2y^2 + z^2 - xy + 2yz$$

Assuming constant properties and no internal heat generation, determine the regions where the temperature changes with time.

2.21 A pan is used to boil water by placing it on a stove, from which heat is transferred at a fixed rate q_o. There are two stages to the process. In Stage 1, the water is taken from its initial (room) temperature T_i to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient h may be assumed, while the bulk temperature of the water increases with time, $T_\infty = T_\infty(t)$. In Stage 2, the water has come to a boil, and its temperature remains at a fixed value, $T_\infty = T_b$, as heating continues. Consider a pan bottom of thickness L and diameter D, with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water, respectively.

(a) Write the form of the heat equation and the boundary/initial conditions that determine the variation of temperature with position and time, $T(x,t)$, in the pan bottom during Stage 1. Express your result in terms of the parameters q_o, D, L, h, and T_∞, as well as appropriate properties of the pan material.

(b) During Stage 2, the surface of the pan in contact with the water is at a fixed temperature, $T(L, t) = T_L > T_b$. Write the form of the heat equation and boundary conditions that determine the temperature distribution, $T(x)$, in the pan bottom. Express your result in terms of the parameters q_o, D, L, and T_L, as well as appropriate properties of the pan material.

2.22 Uniform internal heat generation at $\dot{q} = 5 \times 10^7$ W/m^3 is occurring in a cylindrical nuclear reactor fuel rod of 50-mm diameter, and under steady-state conditions the temperature distribution is of the form $T(r) = a + br^2$, where T is in degrees Celsius and r is in meters, while a = 800°C and $b = -4.167 \times 10^5$°C/m^2. The fuel rod properties are k = 30 W/m · K, ρ = 1100 kg/m^3, and c_p = 800 J/kg · K.

(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at r = 25 mm (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q}_2 = 10^8$ W/m^3, what is the initial time rate of temperature change at $r = 0$ and r = 25 mm?

2.23 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m · K and thickness 50 mm is observed to be $T(°\text{C}) = a + bx^2$, where a = 200°C, b = −2000°C/m^2, and x is in meters.

(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

2.24 The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where T is in degrees Celsius and x is in meters, a = 200°C, b = −200°C/m, and c = 30°C/m^2. The wall has a thermal conductivity of 1 W/m · K.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at 100°C, what is the convection coefficient?

2.25 A plane wall of thickness $2L$ = 40 mm and thermal conductivity k = 5 W/m · K experiences uniform volumetric heat generation at a rate $\dot{q}$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature T_∞ = 20°C. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where a = 82.0°C, b = −210°C/m, $c = -2 \times 10^4$°C/m^2, and x is in meters. The origin of the x-coordinate is at the midplane of the wall.

(a) Sketch the temperature distribution and identify significant physical features.