Problem 1

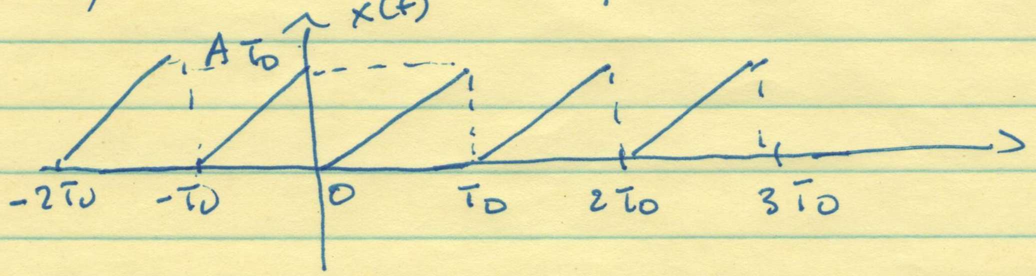
$$p(t) = \begin{cases} At & 0 \le t < T \\ 0 & \text{elsewhere} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} p(t - nT_0)$$

$T = T_0$

$$p(t) = \begin{cases} At & 0 \le t \le T_0 \\ 0 & \text{elsewhere} \end{cases}$$

a)



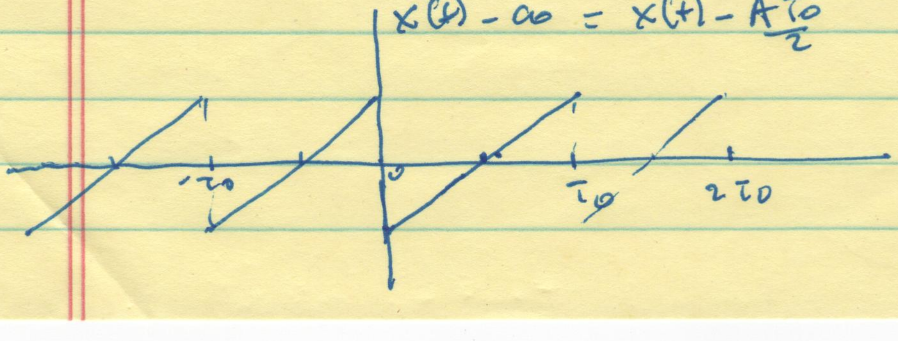
b) Expand $x(t)$ in trigonometric Fourier series

1) DC value a_0

$$a_0 = \frac{1}{T_0}\int_0^{T_0} At\,dt = \frac{1}{T_0} A\frac{t^2}{2}\Big|_0^{T_0} = \frac{1}{T_0} A \frac{T_0^2}{2}$$

$$= \frac{AT_0}{2}$$

2) Plot $x(t) - a_0$ to find out if there is any possible symmetry



$$x(t) - a_0 = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

is odd so there is no an therefore we need to only calculate bn

$$\omega_0 = \frac{2\pi}{T_0}$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} At \sin\left(\frac{2\pi n}{T_0} t\right) dt$$

let $u = \frac{2\pi}{T_0} t \implies du = \frac{2\pi}{T_0} dt$

$$t = \frac{T_0}{2\pi} u \qquad dt = \frac{T_0}{2\pi} du$$

$$b_n = \frac{2}{T_0} \int_0^{2\pi} \frac{T_0}{2\pi} Au \sin nu \frac{T_0}{2\pi} du$$

$$b_n = \frac{AT_0}{(2\pi)^2} \int_0^{2\pi} u \sin nu \, du$$

$$\cancel{\left((a+bu)\sin nu\right)}$$

$$\int x \sin nx \, dx$$

$$u' = \sin nx \, d \implies u = -\frac{1}{n} \cos nx$$

$$v = x \implies v' = dx$$

$$\int u'v = -\cancel{x}\frac{1}{n} \cos nx \Big|_0^{2\pi} + \frac{1}{n} \int_0^{2\pi} \cos nx \, dx$$

$$= -\frac{2\pi}{n}.$$

$$-\frac{x}{n}\cos nx\Big|_0^{2\pi}+\frac{1}{n}\sin nx\Big|_0^{2\pi}$$

$$=-\frac{2\pi}{n}+$$

$$b_n=-\frac{A T_0}{n\cancel{2}\pi}$$

$$x(t)=\frac{A T_0}{2}-\sum_{n=1}^{\infty}\frac{A T_0}{\cancel{2}\pi n}\sin\frac{2\pi n t}{T_0}$$

c)
$$P=\frac{1}{T_0}\int_0^{T_0}A^2t^2\,dt=\frac{A^2}{T_0}\frac{t^3}{3}\Big|_0^{T_0}$$
$$=\frac{A^2T_0^2}{3}$$

Pb 2

$$H(j\omega)=\frac{1}{1+j\omega RC}\qquad x(t)=\cos 120\pi t$$
$$\omega_0=120\pi$$

$$Y(j\omega)=X(j\omega)H(j\omega)=\frac{[\pi\delta(\omega-120\pi)+\pi\delta(\omega+120\pi)]}{1+j\omega RC}$$

$$Y(j\omega)=\frac{\pi\delta(\omega-120\pi)}{1+j120\pi RC}+\frac{\pi\delta(\omega+120\pi)}{1-j120\pi RC}$$

c)

$$\cos 120\pi t \rightarrow \boxed{} \rightarrow y(t) = |H(j120\pi)| \cos(120\pi t - \theta(\omega))$$

$$|H(j120\pi)| = \frac{1}{\sqrt{1 + (120\pi RC)^2}}$$

$$\theta(\omega) = \tan^{-1}(120\pi RC)$$

$$y(t) = \frac{1}{\sqrt{1 + (120\pi RC)^2}} \cos\left(120\pi t - \tan^{-1}(120\pi RC)\right)$$

d) find RC such that Amplitude of

$$y(t) = \frac{1}{\sqrt{2}}$$

$$\sqrt{1 + (120\pi RC)^2} = \sqrt{2}$$

$$(120\pi RC)^2 = 1 \qquad RC = \frac{1}{120\pi}$$

Problem 3

$$\frac{d^2 y(t)}{dt^2} + \frac{3\,dy}{dt} + 2y(t) = f(t)$$

$$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] + 3\mathcal{L}\left[\frac{dy}{dt}\right] + 2\mathcal{L}[y(t)] = \mathcal{L}[f(t)$$

$$f(t) = (t^2 + 3t)\, u(t)$$

$$\mathcal{L}[f(t)] = F(s) = \frac{2}{s^3} + \frac{3}{s^2}$$

$$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] = s^2 Y(s) - s y(0^-) - y'(0^-)$$

$$\mathcal{L}\left[\frac{dy}{dt}\right] = s Y(s) - y(0^-)$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$s^2 Y(s) - s y(0^-) - y'(0^-) + 3\left[s Y(s) - y(0^-)\right]$$

$$+ 2 Y(s) = \frac{2}{s^3} + \frac{3}{s^2}$$

$$(s^2 + 3s + 2) Y(s) = s y(0^-) + y'(0^-) + 3 y(0^-)$$

$$+ \frac{2}{s^3} + \frac{3}{s^2}$$

$$Y(s) = \frac{(s+3) y(0^-) + y'(0^-)}{s^2 + 3s + 2}$$

$$+ \frac{2 + 3s}{s^3 (s^2 + 3s + 2)}$$

$$Y(s) = \frac{2(s+3) - 8}{s^2 + 3s + 2} + \frac{3s + 2}{s^3 (s^2 + 3s + 2)}$$

$(s+1)(s+2)$

$$Y(s) = \frac{2s-2}{(s+1)(s+2)} + \frac{3s+2}{s^3(s+1)(s+2)}$$

$$\frac{2s-2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$6 - \frac{1}{2}$

$$= -\frac{4}{s+1} + \frac{6}{s+2} \qquad \frac{12-1}{2} = \frac{11}{2}$$

$$\frac{3s+2}{s^3(s+1)(s+2)} = \frac{1}{s^3} + \frac{1}{s+1} - \frac{1/2}{s+2} - \frac{1/2}{s}$$

$$Y(s) = \frac{1}{2}\frac{2}{s^3} - \frac{1/2}{s} - \frac{3}{s+1} + \frac{11/2}{(s+2)}$$

$$\boxed{y(t) = \frac{1}{2}t^2 u(t) - \frac{1}{2}u(t) - 3e^{-t}u(t) + \frac{11}{2}e^{-2t}u(t)}$$

$$y'(t) = t\,u(t) + 3e^{-t}u(t) - 11e^{-2t}u(t)$$

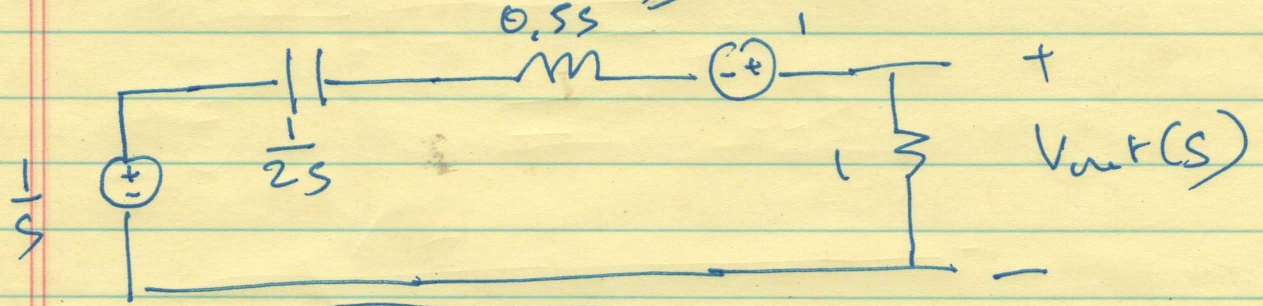
$$y''(t) = u(t) - 3e^{-t}u(t) + 22e^{-2t}u(t)$$

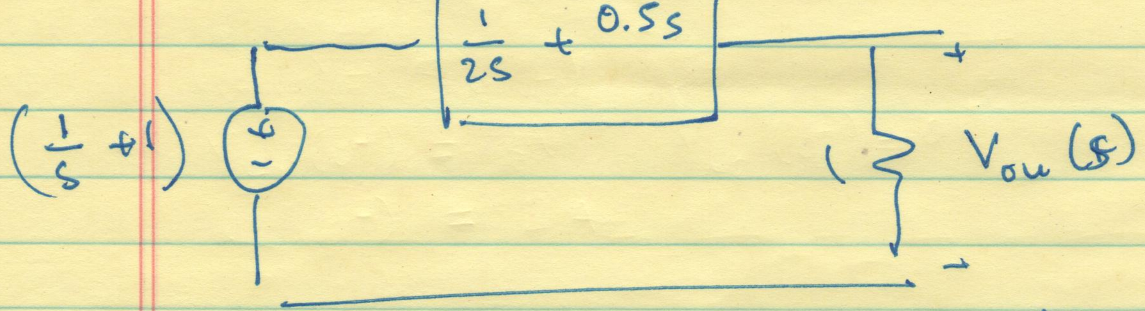
$$3y'(t) = 3t\,u(t) + 9e^{-t}u(t) - 33e^{-2t}u(t)$$

$$2y(t) = t^2u(t) - u(t) - 6e^{-t}u(t) + 11e^{-2t}u(t)$$

$$= t^2u(t) + 3t\,u(t)$$

$$\frac{\frac{1}{s} \times \frac{1}{s}}{\frac{2}{s}} = \frac{1}{2s}$$





$$V_{out}(s) = \frac{\left(\frac{1}{s}+1\right) \times 1}{\frac{1}{2s} + 0.5s + 1} = \frac{\frac{s+1}{s}}{\frac{1}{s}\left[\frac{1}{2} + 0.5s^2 + s\right]}$$

$$= \frac{s+1}{0.5s^2 + s + \frac{1}{2}}$$

$$= \frac{2(s+1)}{s^2+2s+1} = \frac{2(s+1)}{(s+1)^2}$$

$$= \frac{2}{(s+1)}$$

$$v_{out}(t) = 2e^{-t}u(t)$$