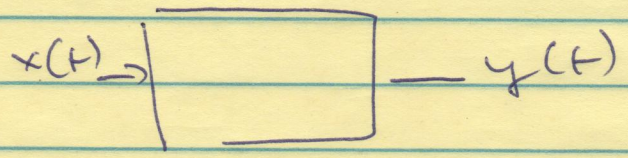
Solution Homework 3

Pb 2.1



a) $y(t) = 3x(t) + 1$ not linear because

$S(x_1 + x_2) = 3x_1(t) + 3x_2(t) + 1$

and $S(x_1) + S(x_2) = 3x_1(t) + 1 + 3x_2(t) + 1$

$= 3x_1(t) + 3x_2(t) + 2$

b) $y(t) = 3\sin(t)\, x(t)$ is linear

c) $\frac{dy}{dt} + y(t) = x(t)$ is linear

d) $\frac{dy}{dt} + 2y(t) = 3\frac{dx}{dt}$ is linear

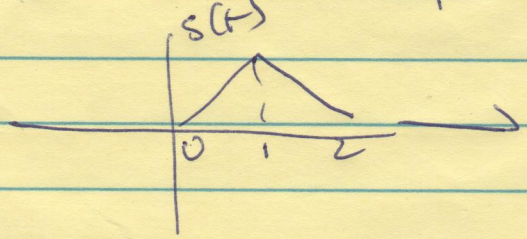
e) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ is linear

f) $y(t) = \int_{0}^{t} x(\tau)\, d\tau$ is linear

g) $y(t) = \int_{t-1}^{t+1} x(\tau)\, d\tau$ is linear

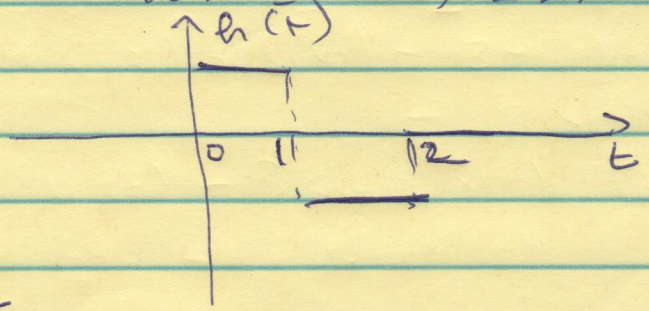
Pb 2.3

find the impulse response of an LTI system whose step response is



$h(t) = \frac{ds}{dt}$ the impulse response is the derivative of the step respons

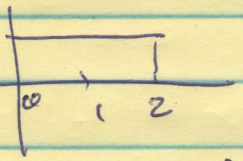
$$s(t) = r(t) - 2r(t-1) + r(t-2)$$
$$h(t) = u(t) - 2u(t-1) + u(t-2)$$



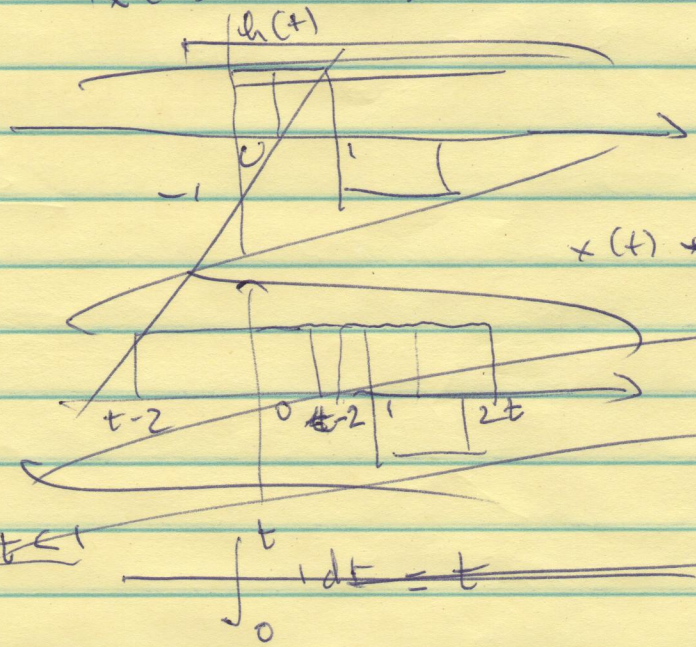
Pb 2.5

a) $h(t)$ is the same as in probl. 2.3

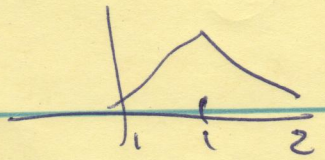
a $x(t)$



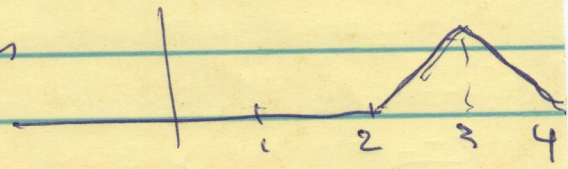
$$x(t) = u(t) - u(t-2)$$



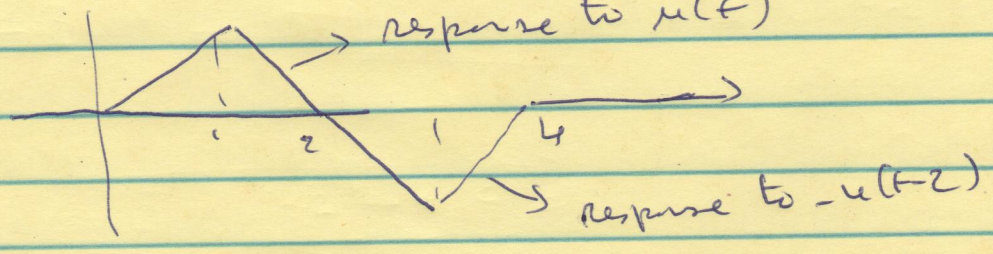
the response to $u(t)$ is



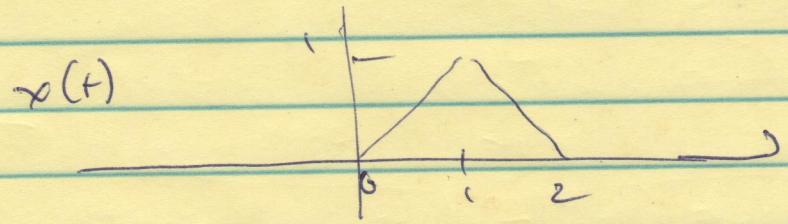
the response to $u(t-2)$ is shifted by 2

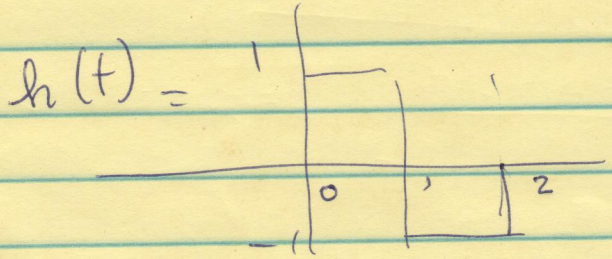


so the response to $u(t) - u(t-2)$ is



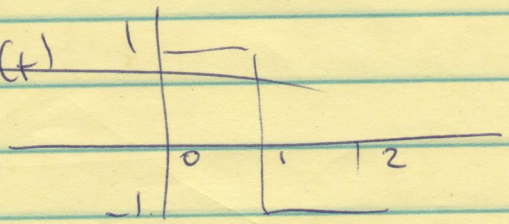
b)





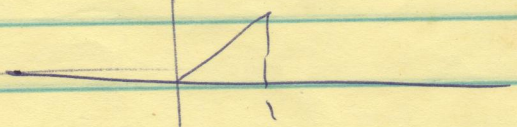
perform the graphical convolution

c) $x(t) = h(t)$



$y(t) = x(t) * h(t)$

$x(t)$

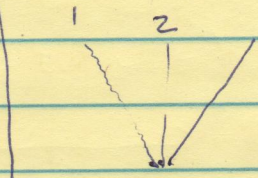


d)

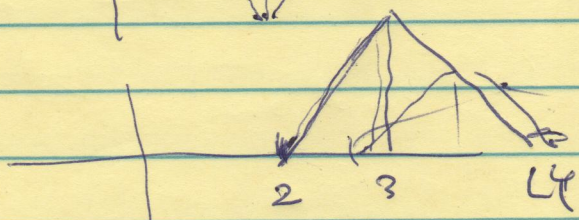
c) $x(t) = u(t) - 2u(t-1) + u(t-2)$

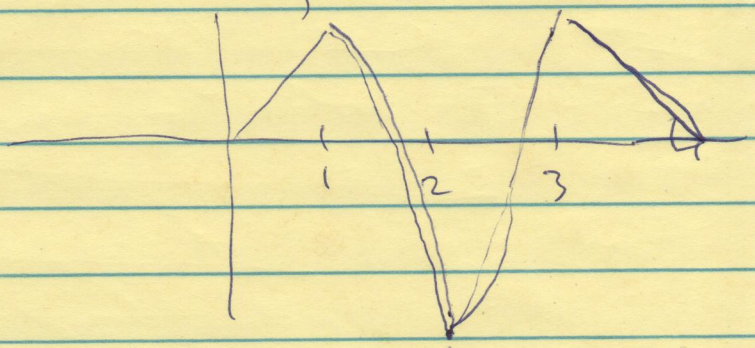
response to $u(t)$ →

response to $-2u(t-1)$



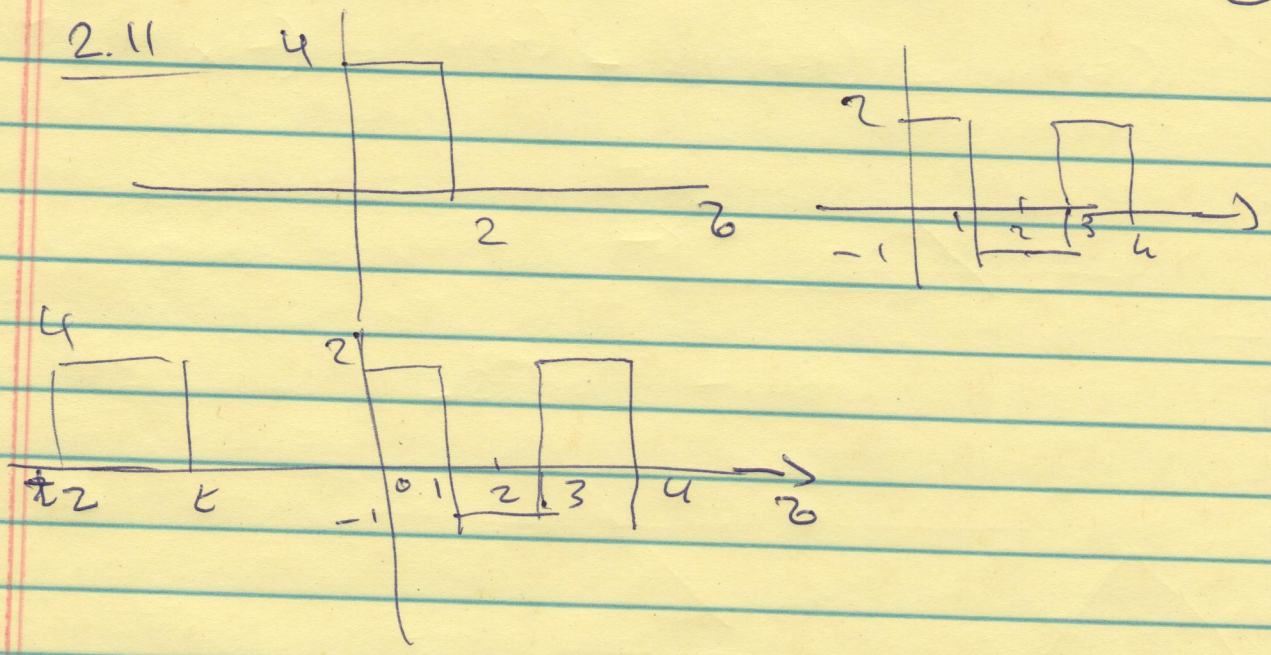
response to $u(t-2)$





d) Graphical convolution

2.11



Graphical convolution

See the solution of practice test

2.12

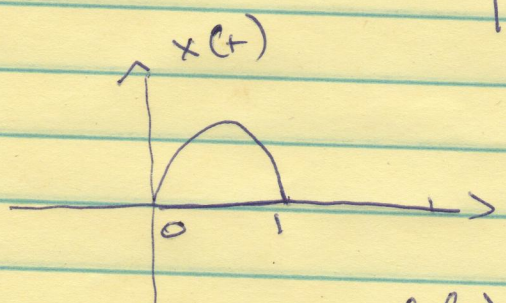
Same, do not do the analytical method

2.14

$$x(t) = \begin{cases} 0 & t < 0 \\ \sin \pi t & 0 \le t \le 1s \\ 0 & \text{elsewhere} \end{cases}$$

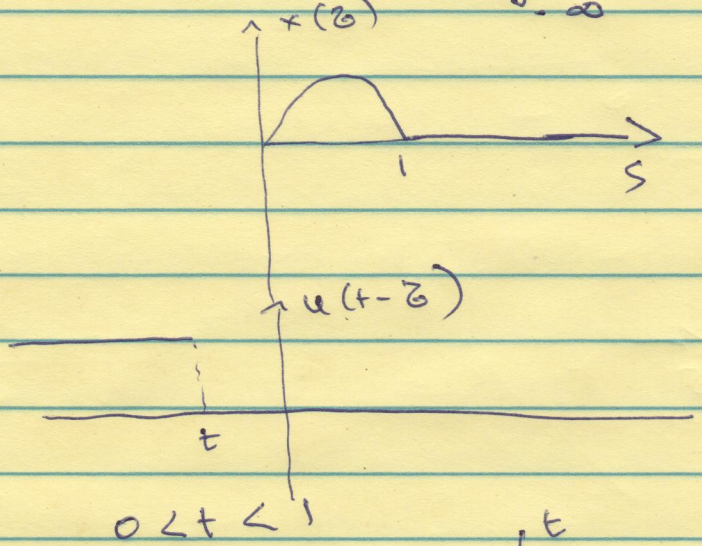
$\omega_0 = \pi = \frac{2\pi}{T_0}$

$T_0 = 2s$



$$h(t) = u(t)$$

$$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$



$0 < t < 1$

$$y(t) = \int_0^t \sin \pi\tau \, d\tau$$

$$= -\frac{1}{\pi}\left[\cos \pi\tau \Big|_0^t\right]$$

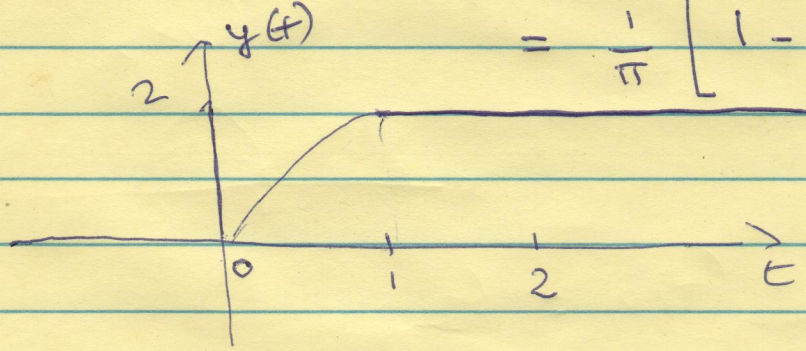
$$= -\frac{1}{\pi}\left[\cos \pi t - 1\right]$$

$$= \frac{1}{\pi}\left[1 - \cos \pi t\right]$$

$t \geq 1$

$$y(t) = \int_0^1 \sin \pi\tau \, d\tau$$

$$= \frac{1}{\pi}\left[1 - \cos \pi\right] = 2$$



2.15

a) $u(t) * [\delta(t) - 3\delta(t-1) + 2\delta(t-2)]$

$= u(t) * \delta(t) - 3u(t) * \delta(t-1) + 2u(t) * \delta(t-2)$

for any function $f(t)$

$f(t) * \delta(t-T_0) = f(t-T_0)$

so

$= u(t) - 3u(t-1) + 2u(t-2)$

b) $u(t) * [2u(t) - 2u(t-3)]$

$= 2u(t) * u(t) - 2u(t) * u(t-3)$

$u(t) * u(t) = r(t)$

$= 2r(t) - 2u(t) * u(t-3)$

c) $u(t) * [(t-1)u(t-1)]$

$u(t) * r(t-1)$