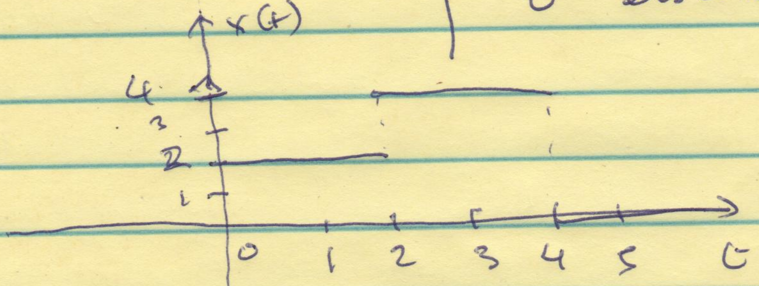
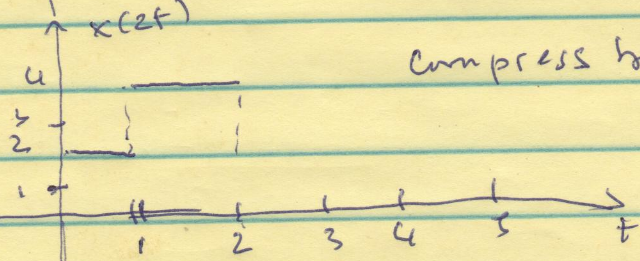
Solution Homework 2

Problem 1

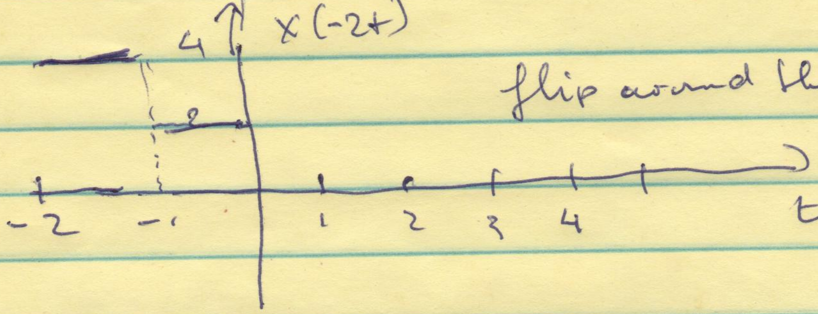
plot $x(t) = \begin{cases} 2 & 0 \le t < 2 \\ 4 & 2 \le t \le 4 \\ 0 & \text{elsewhere} \end{cases}$



$x(2t)$



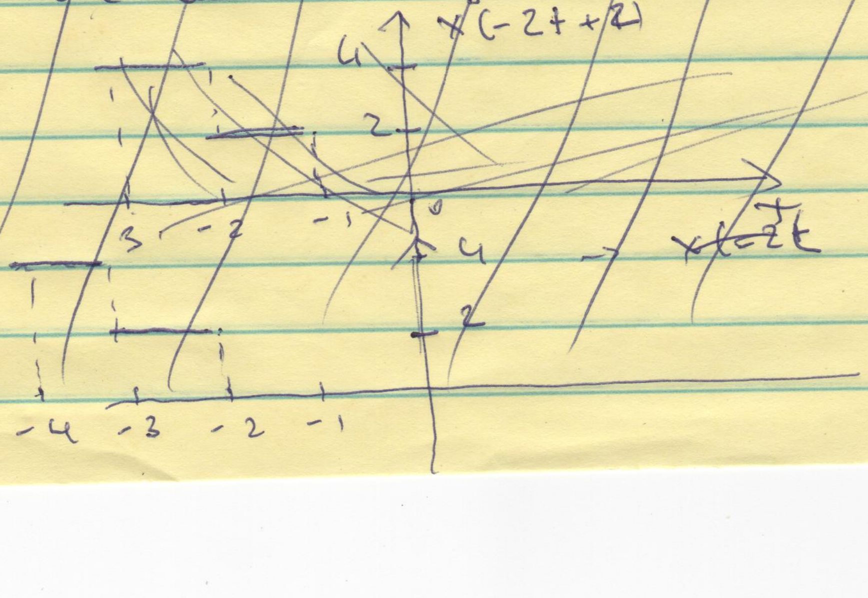
compress by 2



flip around the vertical axis

~~Pro $x(-2(t+1)) = x(-2t+2)$~~

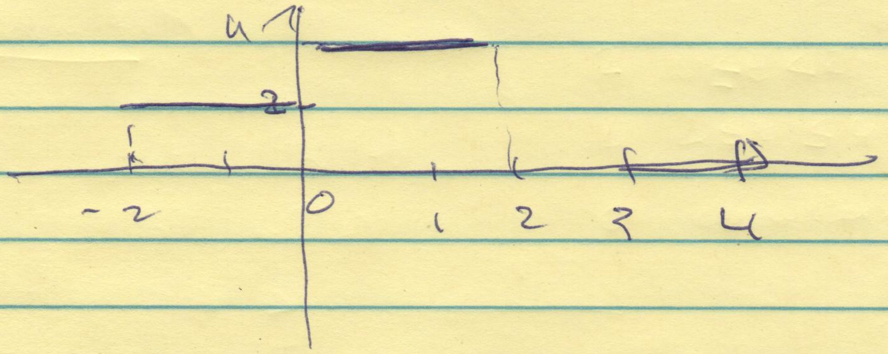
~~we need to shift $x(-2t)$ by 2 to the left~~



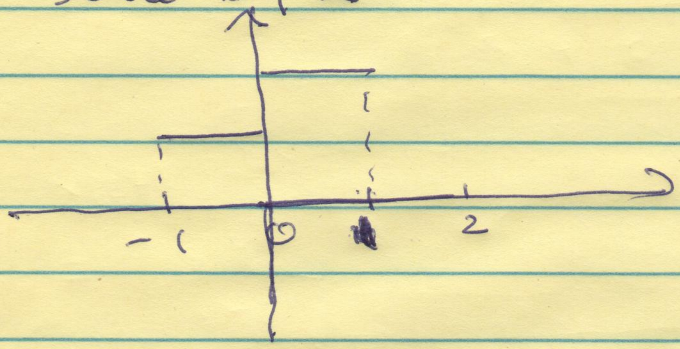
~~$x(-2t$ $x(-2(t-1))$~~

$x(-2(t-1)) = x(-2t+2)$

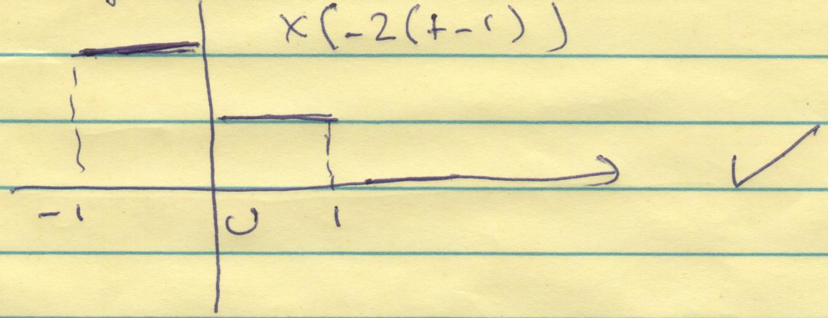
shift to the left by 2



Scale by 2



then flip around the vertical axis

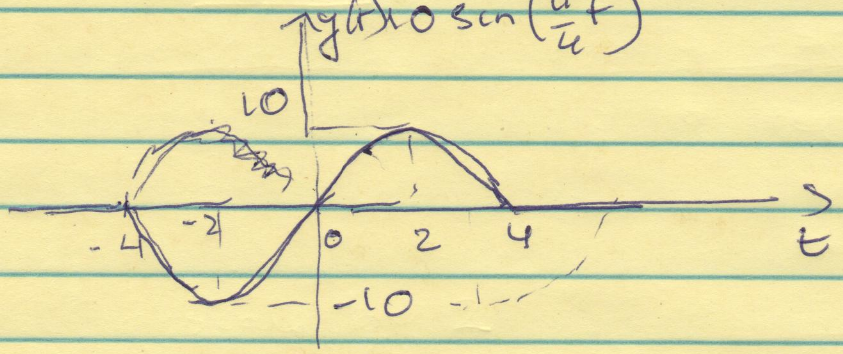


Problem 2

plot $y(t) = \begin{cases} 10\left(\frac{\pi}{4}t\right) & \text{for } -4 < t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

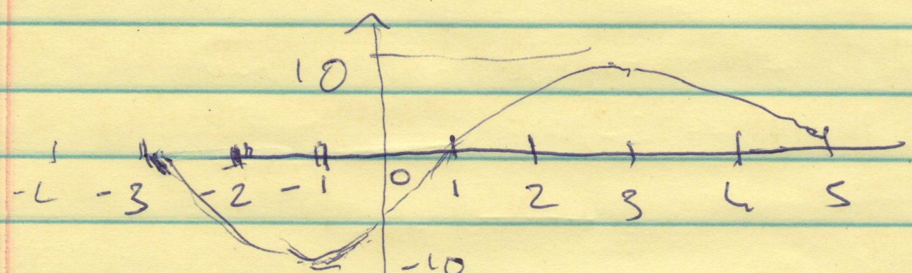
$\omega = \frac{\pi}{4} = \frac{2\pi}{T}$

$\frac{T}{4} = \frac{2\pi}{\pi} = 2 \Rightarrow T = 8s$

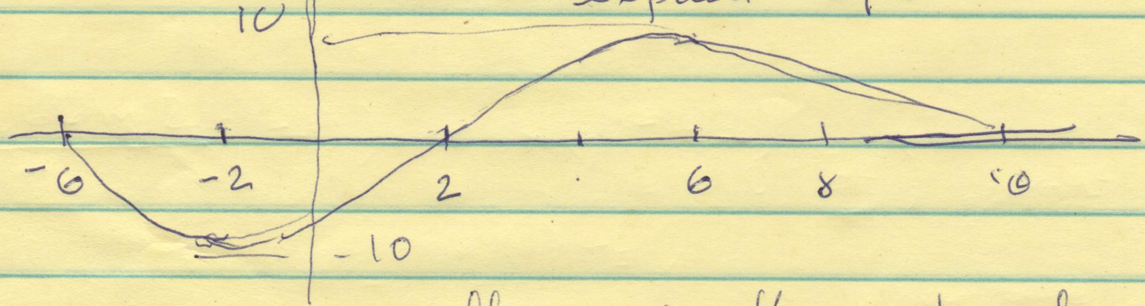


$y\left(-\frac{1}{2}(t+2)\right) = y\left(-\frac{1}{2}t - 1\right)$

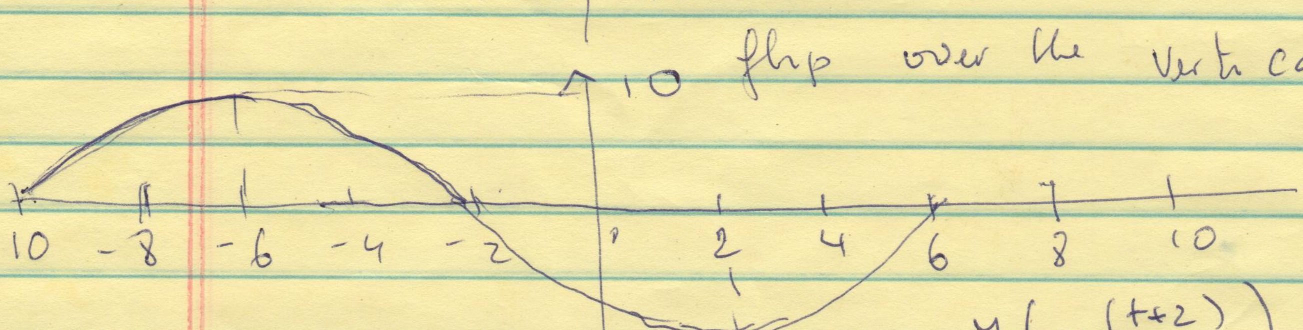
shift to the right by one



expand by 2



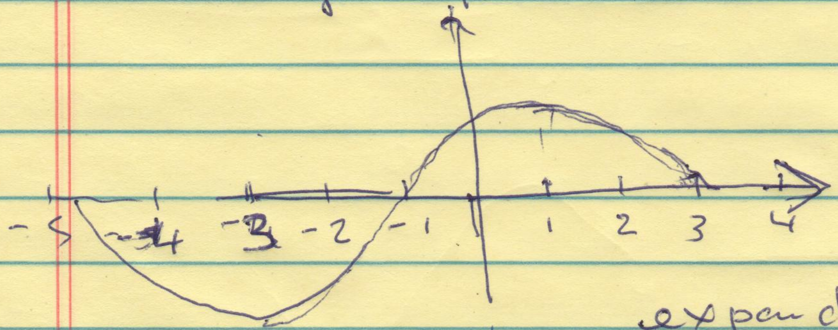
flip over the vertical



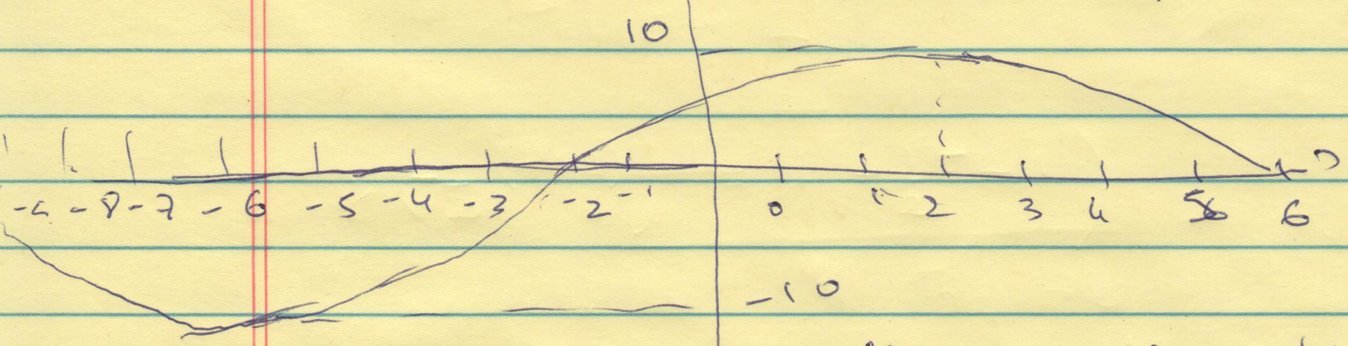
$y\left(-\frac{(t+2)}{2}\right)$

$$y\left(-\frac{(t-2)}{2}\right) = y\left(-\frac{1}{2}t + 1\right)$$

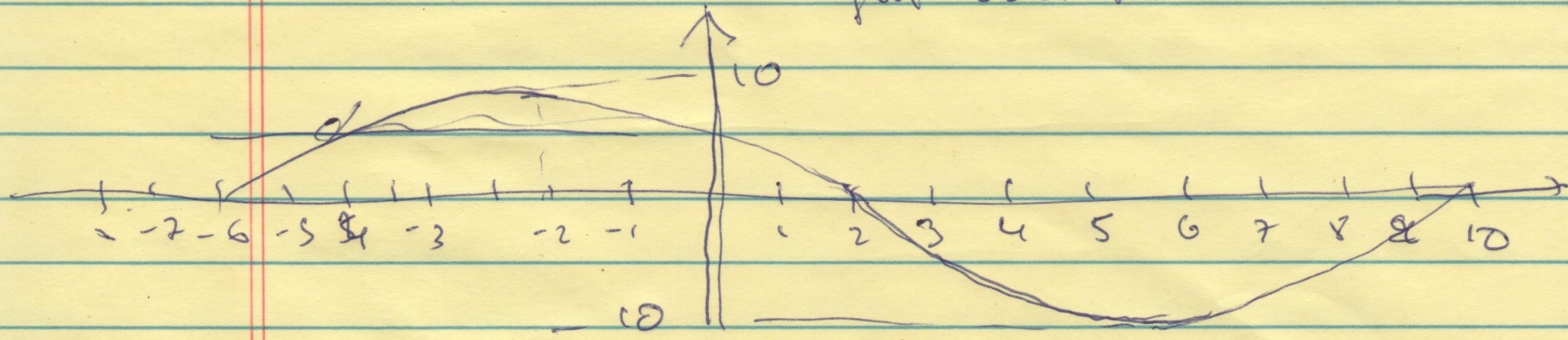
Shift by one to the left



expand by 2



flip over the vertical axis

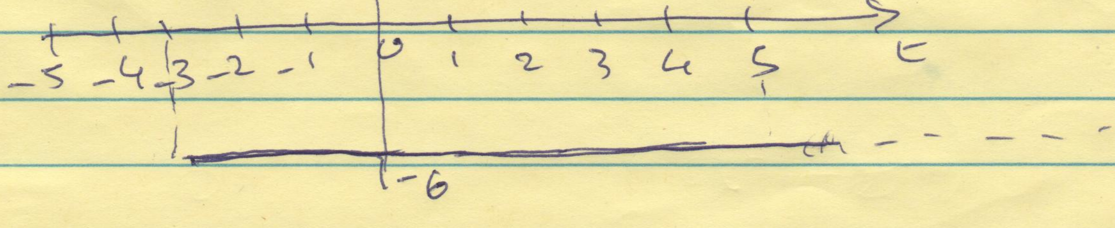


$$y\left(-\frac{(t-2)}{2}\right)$$

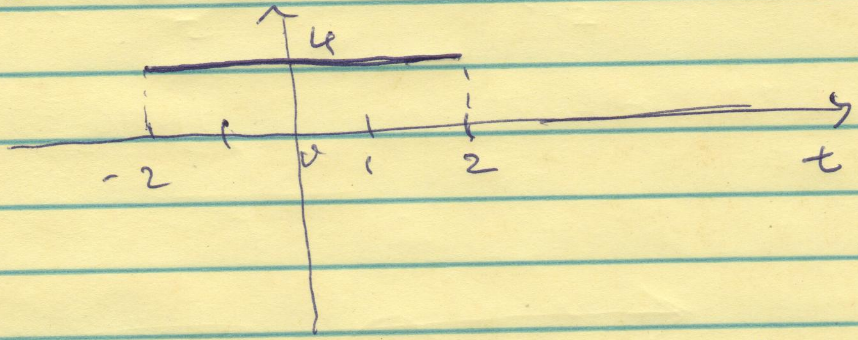
Problem 2

Generate ~~and pl~~ plots $-5 \le t < 5$

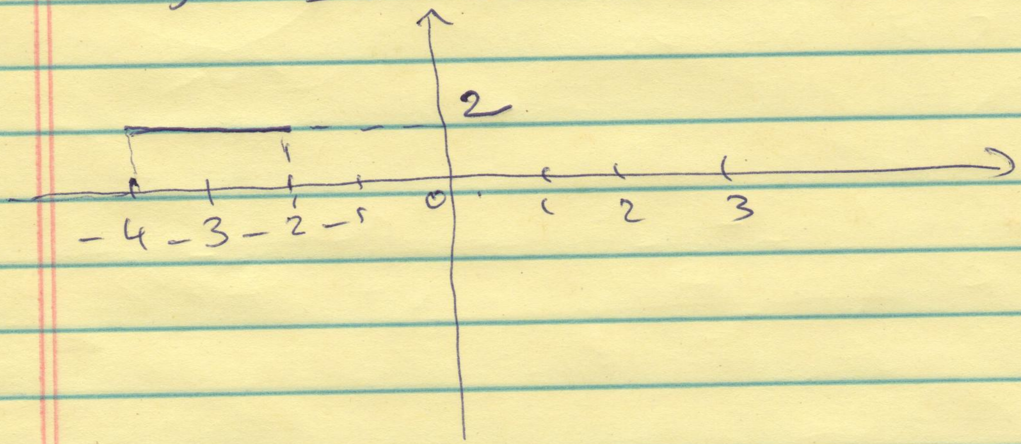
$x_1(t) = -6u(t+3)$



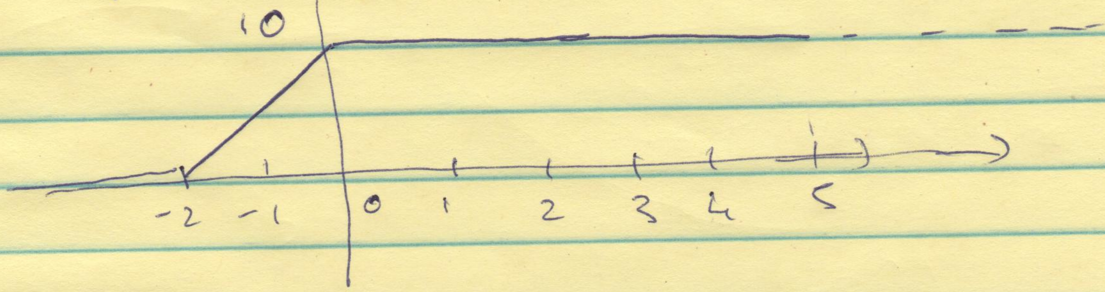
$$x_2(t) = 4u(t+2) - 4u(t-2)$$



$$x_3(t) = -2u(t+2) + 2u(t+4)$$



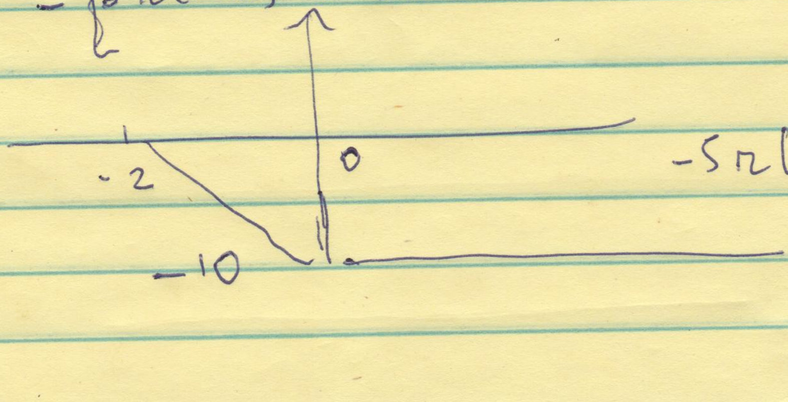
$$x_4(t) = 5(r+2) - 5r(t)$$



$$x_5(t) = 10 - \cancel{8r}\ 5r(t+2) + 5r(t)$$

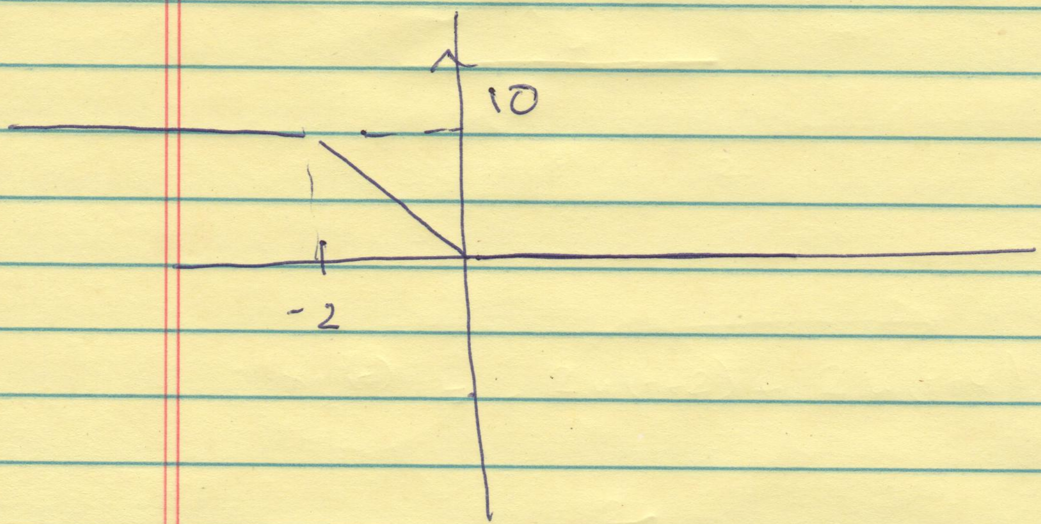
$$= 10 - [5r(t+2) - 5r(t)]$$

$$-[5r(t+2) + 5r(t)$$



$-5r(t+2) + r(t)$

$10 - 5r(t+2) + r(t)$



Pb 4

a) $x_1(t) = 3t^2 + 4t^4$ is even. (polynomial with even exponent)

b) $x_2(t) = 3t^3$ is odd monomial with odd exponent

c) $x_3(t) = 4[\sin 3t + \cos 3t]$

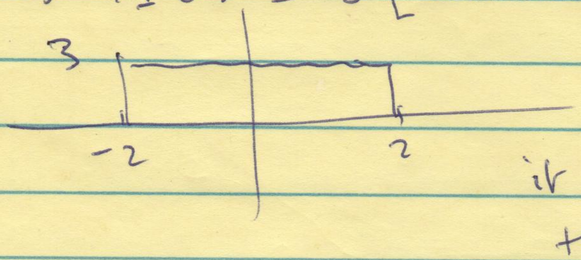
neither even nor odd

d) $x_4(t) = \dfrac{\sin 4t}{4t}$ is even (ratio of two odd signals)

Problem 5

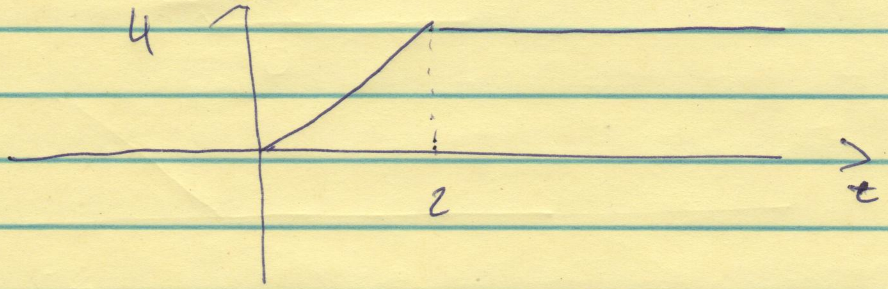
Determine if each of the following signals is a power signal, an energy signal or neither

a) $x_1(t) = 3[u(t+2) - u(t-2)]$



energy signal since it vanishes at $\pm\infty$ and the maximum is finite $= 3$

$x_2(t) = 2\left[r(t) - r(t-2)\right]$



it is not ~~an~~ energy signal because it does not vanish at $+\infty$. Is it a power signal

$$P_{av} = \lim_{T\to+\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_2^2(t)\,dt$$

$$= \lim_{T\to+\infty} \frac{1}{T}\left[\int_{0}^{2} 4t^2\,dt + \int_{2}^{T/2} (4)^2\,dt\right]$$

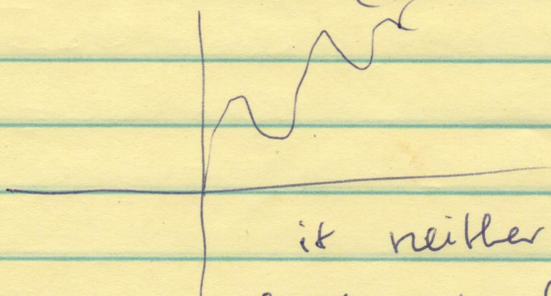
we start at 0 because $r(t)$ does not exist $t<0$

This quantity is finite and it is not 0

so $x_2(t)$ is a power signal

c) $x_3(t) = e^{-2t}u(t)$ is an energy signal because it vanishes at $\pm\infty$

d) $x_4(t) = t\cos(3t)u(t)$



it neither power nor energy signal because first it does not vanish at $+\infty$ and

and the quantity $\lim_{T\to+\infty} \frac{1}{T}\int_{-T/2}^{T/2} t^2\cos^2(3t)\,dt \to +\infty$

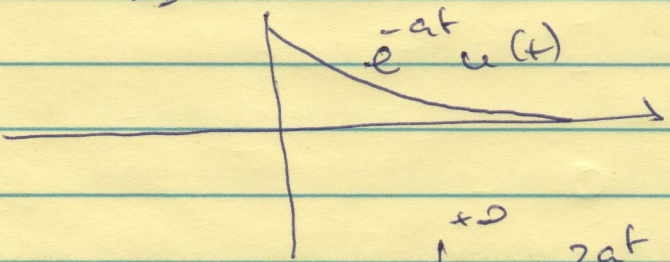
$x_5(t) = 2\sin(4t)\cos(4t) = \sin 8t$

is a power signal because it is a periodic signal

Problem 6

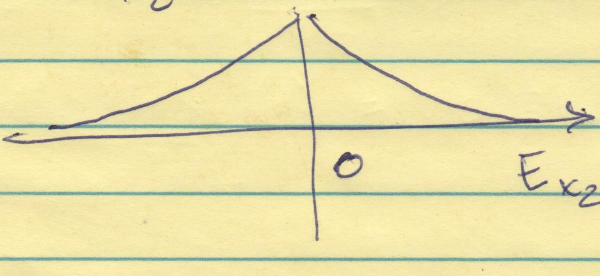
Compute the energy of the following signals

$$x_1(t) = e^{-at}\,u(t) \qquad a > 0$$



$$E_{x_1(t)} = \int_{-\infty}^{+\infty} e^{-2at}\,u(t)\,dt = \int_{0}^{+\infty} e^{-2at}\,dt = \frac{1}{2a}$$

$$x_2(t) = e^{-a|t|}$$



$$E_{x_2(t)} = 2\,E_{x_1(t)} = \frac{1}{2a} \times 2 = \frac{1}{a}$$

Problem 7

Compute the average power of the following signals

a) $x_1(t) = 2\cos 5t$

it is periodic signal

$$\omega = 5 = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{5}$$

$$P_{av} = \frac{1}{T}\int_0^T 2^2 \cos^2 5t \, dt$$

$$= \frac{1}{T}\int_0^T 4 \cos^2 5t \, dt$$

but $\cos^2 5t = \frac{1}{2}(1 + \cos 10t)$

$$P_{av} = \frac{1}{T}\int_0^T \frac{4}{2}(1 + \cos 10t)\, dt$$

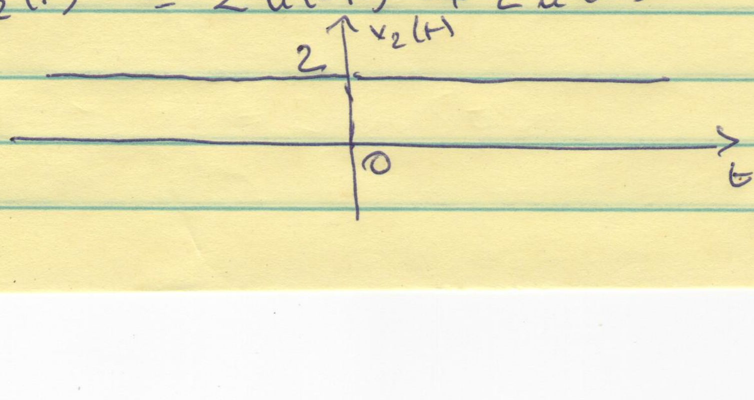
$$= \underbrace{\frac{1}{T}\int_0^T 2\, dt}_{2} + \underbrace{\frac{1}{T}\int_0^T \cos(10t)\, dt}_{0}$$

because it is an integral over a period

$P_{av} = 2$

in general the average power of a signal such as $A\cos(\omega t + \theta)$ or $A\sin(\omega + \theta)$ is $\frac{A^2}{2}$

b) $x_2(t) = 2u(-t) + 2u(t)$



$$P_{av} = \lim_{T \to +\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$\lim_{T \to +\infty} \frac{1}{T} \int_{-T/2}^{T/2} 4\, dt$$

$$= \lim_{T \to +\infty} \frac{1}{T} \left[4 \left(\frac{T}{2} + \frac{T}{2} \right) \right] = \frac{1}{T} 4T$$

$$= 4$$