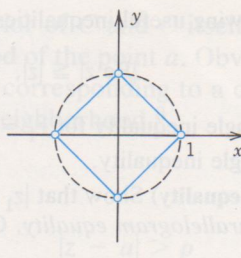Fig. 291. $\sqrt[3]{1}$ Fig. 292. $\sqrt[4]{1}$ Fig. 293. $\sqrt[5]{1}$
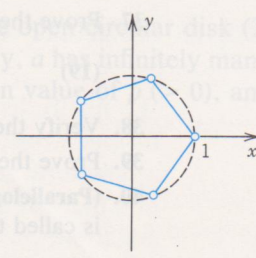
The student should do the problems related to the polar representation with particular care, since we shall need this representation quite often in our work.

Problem Set 12.2

1. **(Multiplication by i)** Show that multiplication of a complex number by i corresponds to a **counterclockwise rotation** of the corresponding vector through the angle $\pi/2$.

Find

2. $|-0.2i|$ 3. $|1.5 + 2i|$ 4. $|z|^4, \; |z^4|$ 5. $|\cos\theta + i\sin\theta|$

6. $\left|\dfrac{\bar{z}}{z}\right|$ 7. $\left|\dfrac{5 + 7i}{7 - 5i}\right|$ 8. $\left|\dfrac{z + 1}{z - 1}\right|$ 9. $\left|\dfrac{(1 + i)^6}{i^3(1 + 4i)^2}\right|$

Represent in polar form:

10. $2i, \; -2i$ 11. $1 + i$ 12. -3 13. $6 + 8i$

14. $\dfrac{1 + i}{1 - i}$ 15. $\dfrac{i\sqrt{2}}{4 + 4i}$ 16. $\dfrac{3\sqrt{2} + 2i}{-\sqrt{2} - 2i/3}$ 17. $\dfrac{2 + 3i}{5 + 4i}$

Determine the principal value of the arguments of

18. $-6 - 6i$ 19. $-10 - i$ 20. $-\pi$ 21. $2 + 2i$

Represent in the form $x + iy$:

22. $4(\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi)$ 23. $2\sqrt{2}(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi)$

24. $10(\cos 0.4 + i\sin 0.4)$ 25. $\cos(-1.8) + i\sin(-1.8)$

Find all values of the following roots and plot them in the complex plane.

26. $\sqrt{i}$ 27. $\sqrt{-8i}$ 28. $\sqrt{-7 - 24i}$ 29. $\sqrt[8]{1}$

30. $\sqrt[4]{-7 + 24i}$ 31. $\sqrt[4]{-1}$ 32. $\sqrt[5]{-1}$ 33. $\sqrt[3]{1 + i}$

Solve the equations:

34. $z^2 + z + 1 - i = 0$ 35. $z^2 - (5 + i)z + 8 + i = 0$

36. $z^4 - 3(1 + 2i)z^2 - 8 + 6i = 0$