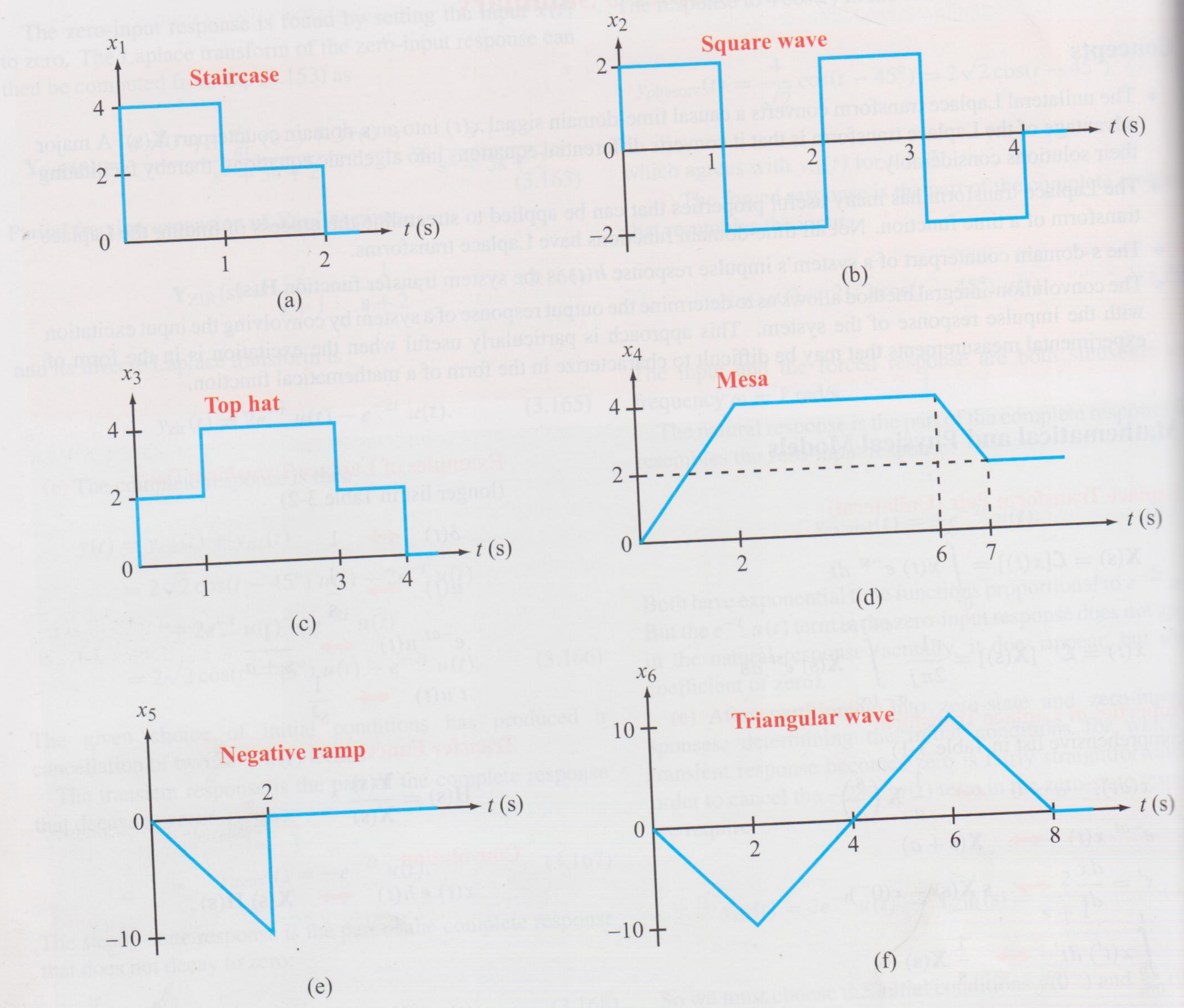

Figure P3.1: Waveforms for Problem 3.1.

PROBLEMS

Sections 3-1 to 3-3: Laplace Transform and Its Properties

3.1 Express each of the waveforms in Fig. P3.1 in terms of step functions and then determine its Laplace transfrom. [Recall that the ramp function is related to the step function by $r(t-T) = (t-T)\ u(t-T)$.] Assume that all waveforms are zero for $t < 0$.

(a) Staircase

(b) Square wave

(c) Top hat

(d) Mesa

(e) Negative ramp

(f) Triangular wave

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*Answer(s) in Appendix E.

3.2 Determine the Laplace transform of each of the *periodic* waveforms shown in Fig. P3.2. (Hint: See Exercise 3.2.)

(a) Sawtooth

(b) Interrupted ramps

(c) Impulses

(d) Periodic exponentials

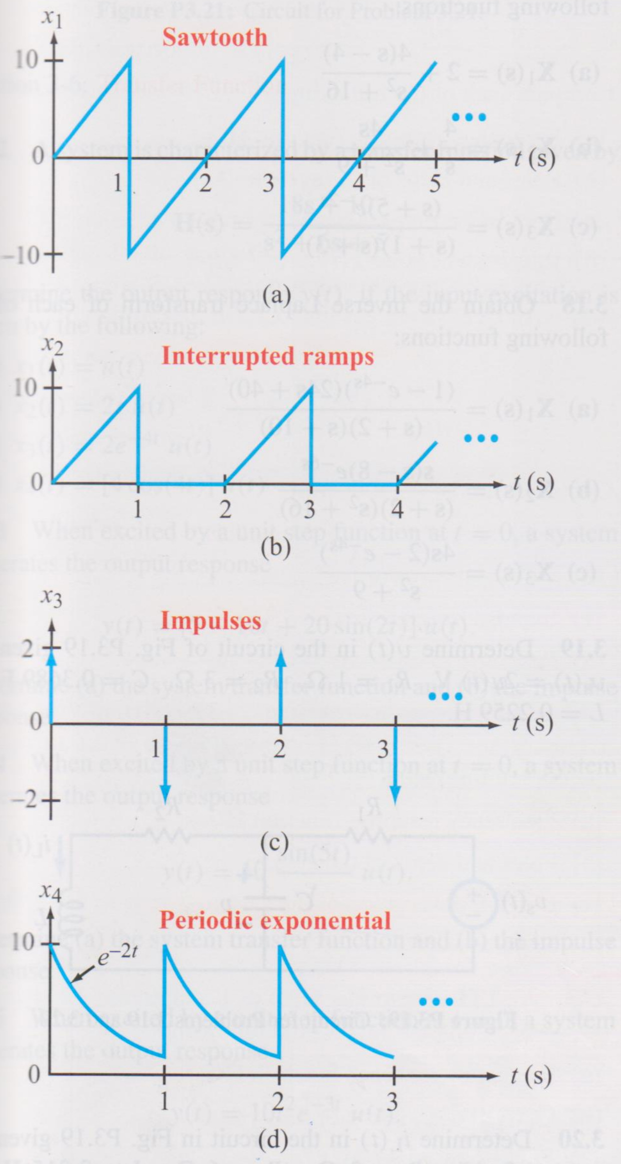

Figure P3.2: Periodic waveforms for Problem 3.2.

3.3 Determine the Laplace transform of each of the following functions by applying the properties given in Tables 3-1 and 3-2.

(a) $x_1(t) = 4te^{-2t}\, u(t)$

(b) $x_2(t) = 10\cos(12t + 60°)\, u(t)$

(c) $x_3(t) = 12e^{-3(t-4)}\, u(t-4)$

3.4 Determine the Laplace transform of each of the following functions by applying the properties given in Tables 3-1 and 3-2.

(a) $x_1(t) = 12te^{-3(t-4)}\, u(t-4)$

(b) $x_2(t) = 27t^2 \sin(6t - 60°)\, u(t)$

(c) $x_3(t) = 10t^3 e^{-2t}\, u(t)$

3.5 Determine the Laplace transform of each of the following functions by applying the properties given in Tables 3-1 and 3-2.

(a) $x_1(t) = 16e^{-2t}\cos 4t\; u(t)$

(b) $x_2(t) = 20te^{-2t}\sin 4t\; u(t)$

(c) $x_3(t) = 10e^{-3t}\, u(t-4)$

3.6 Determine the Laplace transform of each of the following functions by applying the properties given in Tables 3-1 and 3-2.

(a) $x_1(t) = 30(e^{-3t} + e^{3t})\, u(t)$

(b) $x_2(t) = 5(t-6)\, u(t-3)$

(c) $x_3(t) = 4e^{-2(t-3)}\, u(t-4)$

3.7 Determine the Laplace transform of the following functions:

(a) $x_1(t) = 25\cos(4\pi t + 30°)\, \delta(t)$

(b) $x_2(t) = 25\cos(4\pi t + 30°)\, \delta(t - 0.2)$

(c) $x_3(t) = 10\,\dfrac{\sin(3t)}{t}\, u(t)$

(d) $x_4(t) = \dfrac{d^2}{dt^2}\,[e^{-4t}\, u(t)]$

3.8 Determine the Laplace transform of the following functions:

(a) $x_1(t) = \dfrac{d}{dt}\,[4te^{-2t}\cos(4\pi t + 30°)\, u(t)]$

(b) $x_2(t) = e^{-3t}\cos(4t + 30°)\, u(t)$

(c) $x_3(t) = t^2[u(t) - u(t-4)]$

(d) $x_4(t) = 10\cos(6\pi t + 30°)\, \delta(t - 0.2)$

3.9 Determine $x(0^+)$ and $x(\infty)$ given that

$$\mathbf{X}(s) = \frac{4s^2 + 28s + 40}{s(s+3)(s+4)}.$$

3.10 Determine $x(0^+)$ and $x(\infty)$ given that

$$\mathbf{X}(s) = \frac{s^2+4}{2s^3+4s^2+10s}.$$

†3.11 Determine $x(0^+)$ and $x(\infty)$ given that

$$\mathbf{X}(s) = \frac{12e^{-2s}}{s(s+2)(s+3)}.$$

3.12 Determine $x(0^+)$ and $x(\infty)$ given that

$$\mathbf{X}(s) = \frac{19-e^{-s}}{s(s^2+5s+6)}$$

Section 3-4 and 3-5: Partial Fractions and Circuit Examples

3.13 Obtain the inverse Laplace transform of each of the following functions, by first applying the partial-fraction-expansion method.

(a) $\mathbf{X}_1(s) = \dfrac{6}{(s+2)(s+4)}$

(b) $\mathbf{X}_2(s) = \dfrac{4}{(s+1)(s+2)^2}$

(c) $\mathbf{X}_3(s) = \dfrac{3s^3+36s^2+131s+144}{s(s+4)(s^2+6s+9)}$

3.14 Obtain the inverse Laplace transform of each of the following functions:

(a) $\mathbf{X}_1(s) = \dfrac{s^2+17s+20}{s(s^2+6s+5)}$

(b) $\mathbf{X}_2(s) = \dfrac{2s^2+10s+16}{(s+2)(s^2+6s+10)}$

(c) $\mathbf{X}_3(s) = \dfrac{4}{(s+2)^3}$

3.15 Obtain the inverse Laplace transform of each of the following functions:

(a) $\mathbf{X}_1(s) = \dfrac{(s+2)^2}{s(s+1)^3}$

(b) $\mathbf{X}_2(s) = \dfrac{1}{(s^2+4s+5)^2}$

(c) $\mathbf{X}_3(s) = \dfrac{\sqrt{2}(s+1)}{s^2+6s+13}$

3.16 Obtain the inverse Laplace transform of each of the following functions:

(a) $\mathbf{X}_1(s) = \dfrac{2s^2+4s-16}{(s+6)(s+2)^2}$

(b) $\mathbf{X}_2(s) = \dfrac{2(s^3+12s^2+16)}{(s+1)(s+4)^3}$

(c) $\mathbf{X}_3(s) = \dfrac{-2(s^2+20)}{s(s^2+8s+20)}$

3.17 Obtain the inverse Laplace transform of each of the following functions:

(a) $\mathbf{X}_1(s) = 2 + \dfrac{4(s-4)}{s^2+16}$

(b) $\mathbf{X}_2(s) = \dfrac{4}{s} + \dfrac{4s}{s^2+9}$

(c) $\mathbf{X}_3(s) = \dfrac{(s+5)e^{-2s}}{(s+1)(s+3)}$

3.18 Obtain the inverse Laplace transform of each of the following functions:

(a) $\mathbf{X}_1(s) = \dfrac{(1-e^{-4s})(24s+40)}{(s+2)(s+10)}$

(b) $\mathbf{X}_2(s) = \dfrac{s(s-8)e^{-6s}}{(s+2)(s^2+16)}$

(c) $\mathbf{X}_3(s) = \dfrac{4s(2-e^{-4s})}{s^2+9}$

3.19 Determine $\upsilon(t)$ in the circuit of Fig. P3.19 given the $\upsilon_s(t) = 2u(t)$ V, $R_1 = 1\ \Omega$, $R_2 = 3\ \Omega$, $C = 0.3689$ F, and $L = 0.2259$ H.

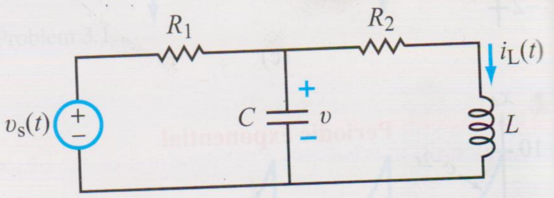

Figure P3.19: Circuit for Problems 3.19 and 3.20.

3.20 Determine $i_L(t)$ in the circuit in Fig. P3.19 given the $\upsilon_s(t) = 2u(t)$, $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $L = 2.215$ H, and $C = 0.0376$ F.

3.21 Determine $v_{out}(t)$ in the circuit in Fig. P3.21 given that $v_s(t) = 35u(t)$ V, $v_{C_1}(0^-) = 20$ V, $R_1 = 1\ \Omega$, $C_1 = 1$ F, $R_2 = 0.5\ \Omega$, and $C_2 = 2$ F.

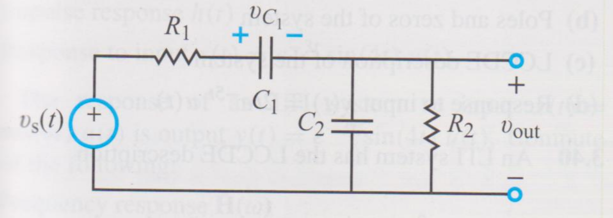

Figure P3.21: Circuit for Problem 3.21.

Section 3-6: Transfer Function

3.22 A system is characterized by a transfer function given by

$$\mathbf{H}(s) = \frac{18s + 10}{s^2 + 6s + 5}.$$

Determine the output response $y(t)$, if the input excitation is given by the following:

(a) $x_1(t) = u(t)$

(b) $x_2(t) = 2t\ u(t)$

(c) $x_3(t) = 2e^{-4t}\ u(t)$

(d) $x_4(t) = [4\cos(4t)]\ u(t)$

3.23 When excited by a unit step function at $t = 0$, a system generates the output response

$$y(t) = [5 - 10t + 20\sin(2t)]\ u(t).$$

Determine (a) the system transfer function and (b) the impulse response.

3.24 When excited by a unit step function at $t = 0$, a system generates the output response

$$y(t) = 10\ \frac{\sin(5t)}{t}\ u(t).$$

Determine (a) the system transfer function and (b) the impulse response.

3.25 When excited by a unit step function at $t = 0$, a system generates the output response

$$y(t) = 10t^2 e^{-3t}\ u(t).$$

Determine (a) the system transfer function and (b) the impulse response.

3.26 When excited by a unit step function at $t = 0$, a system generates the output response

$$y(t) = 9t^2 \sin(6t - 60°)\ u(t).$$

Determine (a) the system transfer function and (b) the impulse response.

3.27 For the circuit shown in Fig. P3.27, determine (a) $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{V}_i$ and (b) $h(t)$ given that $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 1\ \mu$F, and $C_2 = 2\ \mu$F.

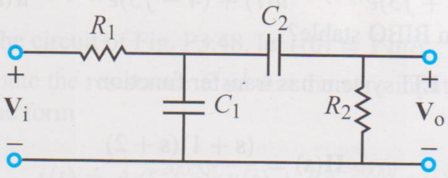

Figure P3.27: Circuit for Problem 3.27.

3.28 For the circuit shown in Fig. P3.28, determine (a) $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{V}_i$ and (b) $h(t)$ given that $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $L_1 = 1$ mH, and $L_2 = 2$ mH.

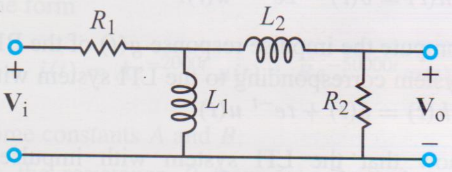

Figure P3.28: Circuit for Problem 3.28.

†**3.29** For the circuit shown in Fig. P3.29, determine (a) $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{V}_i$ and (b) $h(t)$ given that $R = 5\ \Omega$, $L = 0.1$ mH, and $C = 1\ \mu$F.

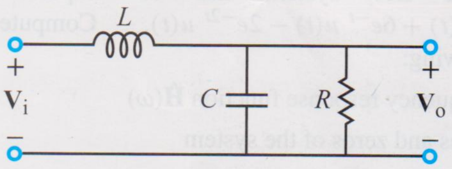

Figure P3.29: Circuit for Problem 3.29.

Section 3-7: LTI System Stability

3.30 An LTI system is described by the LCCDE

$$\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = \frac{dx}{dt} + 5x.$$

Is the system BIBO stable?

3.31 The response of an LTI system to input $x(t) = \delta(t) - 4e^{-3t}\ u(t)$ is output $y(t) = e^{-2t}\ u(t)$. Is the system BIBO stable?

3.32 An LTI system has impulse response $h(t) = (4 + j5)e^{(2+j3)t}\ u(t) + (4 - j5)e^{(2-j3)t}\ u(t)$. Is the system BIBO stable?

3.33 An LTI system has transfer function

$$\mathbf{H}(\mathbf{s}) = \frac{(\mathbf{s}+1)(\mathbf{s}+2)}{\mathbf{s}(\mathbf{s}+3)}.$$

Is it BIBO stable?

Section 3-8: Invertible Systems

†**3.34** Compute the impulse response $g(t)$ of the BIBO stable inverse system corresponding to the LTI system with impulse response $h(t) = \delta(t) - 2e^{-3t}\ u(t)$.

3.35 Compute the impulse response $g(t)$ of the BIBO stable inverse system corresponding to the LTI system with impulse response $h(t) = \delta(t) + te^{-t}\ u(t)$.

3.36 Show that the LTI system with impulse response $h(t) = \delta(t) - 4e^{-3t}\ u(t)$ does not have a BIBO stable inverse system.

3.37 Show that the LTI system with impulse response $h(t) = e^{-t}\ u(t)$ does not have a BIBO stable inverse system.

Section 3-10: Interrelating Descriptions

3.38 An LTI system has an impulse response $h(t) = \delta(t) + 6e^{-t}\ u(t) - 2e^{-2t}\ u(t)$. Compute each of the following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) Poles and zeros of the system

(c) LCCDE description of the system

(d) Response to input $x(t) = e^{-3t}\ u(t) - e^{-4t}\ u(t)$

3.39 An LTI system has an impulse resp $h(t) = \delta(t) + 4e^{-3t}\cos(2t)\ u(t)$. Compute each of following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) Poles and zeros of the system

(c) LCCDE description of the system

(d) Response to input $x(t) = 2te^{-5t}\ u(t)$

3.40 An LTI system has the LCCDE description

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{dx}{dt} + 2x.$$

Compute each of the following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) Poles and zeros of the system

(c) Impulse response $h(t)$

(d) Response to input $x(t) = e^{-2t}\ u(t)$

3.41 An LTI system has the LCCDE description

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = \frac{dx}{dt} + 2x.$$

Compute each of the following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) Poles and zeros of the system

(c) Impulse response $h(t)$

(d) Response to input $x(t) = e^{-2t}\ u(t)$

3.42 The response of an LTI system to $x(t) = \delta(t) - 2e^{-3t}\ u(t)$ is output $y(t) = e^{-2t}\ u(t)$ Co each of the following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) Poles and zeros of the system

(c) LCCDE description of the system

(d) Impulse response $h(t)$

3.43 An LTI system has $\mathbf{H}(0) = 1$ and zeros: $\{-1\}$; p $\{-3, -5\}$. Compute each of the following:

(a) Frequency response function $\hat{\mathbf{H}}(\omega)$

(b) LCCDE description of the system

(c) Impulse response $h(t)$

(d) Response to input $x(t) = e^{-t}\ u(t)$

An LTI system has $\mathbf{H}(0) = 15$ and zeros: $\{-3 \pm j4\}$; ... $\{-1 \pm j2\}$. Compute each of the following:

- Frequency response function $\hat{\mathbf{H}}(\omega)$
- LCCDE description of the system
- Impulse response $h(t)$
- Response to input $x(t) = e^{-3t} \sin(4t)\, u(t)$

The response of an LTI system to input $x(t) =$ ... $\cos(4t)\, u(t)$ is output $y(t) = e^{-3t} \sin(4t)\, u(t)$. Compute ... of the following:

- Frequency response $\hat{\mathbf{H}}(\omega)$
- Poles and zeros of the system
- LCCDE description of the system
- Impulse response $h(t)$

...ion 3-11: Partitions of Responses

Compute the following system responses for the circuit ... in Fig. P3.46 given that $x(t) = 25\cos(3t)\, u(t)$, $R =$ ... kΩ, $C = 1\ \mu$F, and the capacitor was initially charged to ...

- Zero-input response.
- Zero-state response.
- Transient response.
- Steady-state response.
- Natural response.
- Forced response.

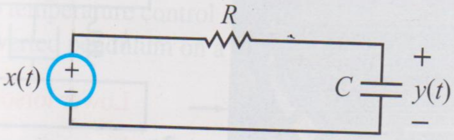

Figure P3.46: Circuit for Problem 3.46.

[†]**3.47** If the capacitor in the circuit of Fig. P3.46 is initially charged to $y(0)$ volts, instead of 2 V, for what value of $y(0)$ is the transient response identically equal to zero (i.e., no transient)?

3.48 For the circuit in Fig. P3.48, compute the steady-state unit-step response for $i(t)$ in terms of R.

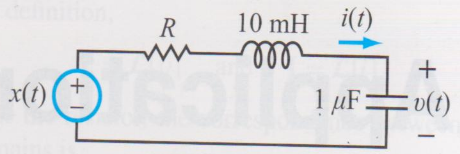

Figure P3.48: Circuit for Problems 3.48 to 3.50.

3.49 In the circuit of Fig. P3.48, let $i(0) = 1$ mA.

(a) Compute the resistance R so that the zero-input response has the form

$$i(t) = Ae^{-20000t}\, u(t) + Be^{-5000t}\, u(t)$$

for some constants A and B.

(b) Using the resistance R from (a), compute the initial capacitor voltage $\upsilon(0)$ so that the zero-input response is $i(t) = Be^{-5000t}\, u(t)$ for some constant B.

3.50 In the circuit of Fig. P3.48, $i(0) = 0$.

(a) Compute the resistance R so that the zero-input response has the form

$$i(t) = Ae^{-2000t}\, u(t) + Be^{-50000t}\, u(t)$$

for some constants A and B.

(b) Using the resistance R from (a), show that no initial capacitor voltage $\upsilon(0)$ will make the zero-input response be $i(t) = Be^{-50000t}\, u(t)$ for some constant B.