PROBLEMS

Section 1-1: Types of Signals

1.1 Is each of these 1-D signals:

- Analog or digital?
- Continuous-time or discrete-time?

(a) Daily closes of the stock market
(b) Output from phonograph-record pickup
(c) Output from compact-disc pickup

1.2 Is each of these 2-D signals:

- Analog or digital?
- Continuous-space or discrete-space?

(a) Image in a telescope eyepiece
(b) Image displayed on digital TV
(c) Image stored in a digital camera

1.3 The following signals are 2-D in space and 1-D in time, so they are 3-D signals. Is each of these 3-D signals:

- Analog or digital?
- Continuous or discrete?

(a) The world as you see it
(b) A movie stored on film
(c) A movie stored on a DVD

Section 1-2: Signal Transformations

1.4 Given the waveform of $x_1(t)$ shown in Fig. P1.4(a), generate and plot the waveform of:

(a) $x_1(-2t)$
(b) $x_1[-2(t-1)]$

1.5 Given the waveform of $x_2(t)$ shown in Fig. P1.4(b), generate and plot the waveform of:

(a) $x_2[-(t+2)/2]$
(b) $x_2[-(t-2)/2]$

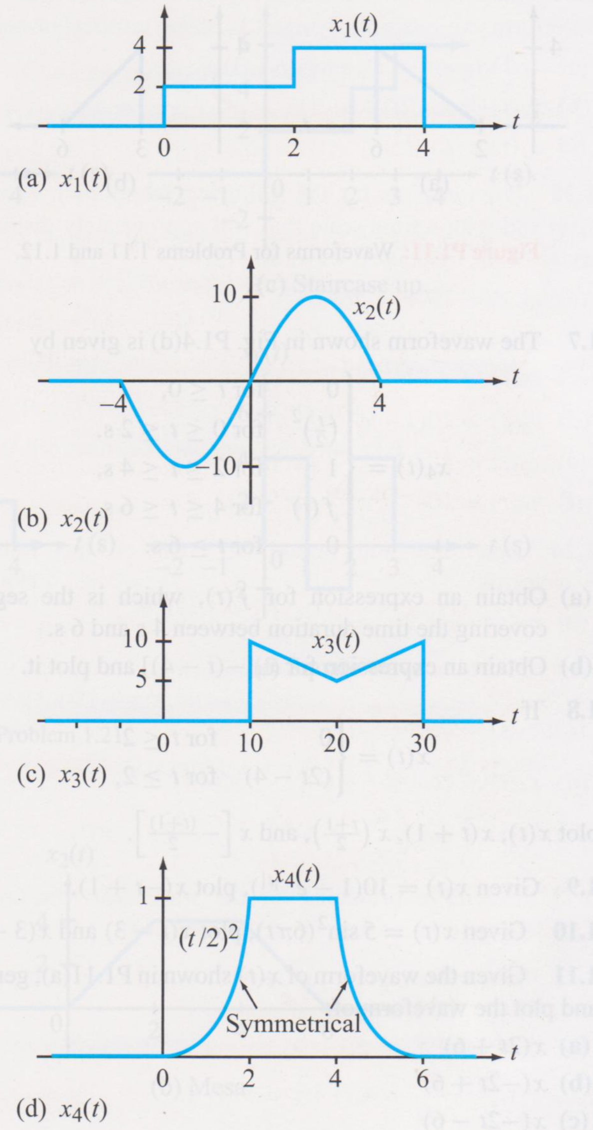


Figure P1.4: Waveforms for Problems 1.4 to 1.7.

1.6 Given the waveform of $x_3(t)$ shown in Fig. P1.4(c), generate and plot the waveform of:

*(a) $x_3[-(t+40)]$
(b) $x_3(-2t)$

*Answer(s) in Appendix E.

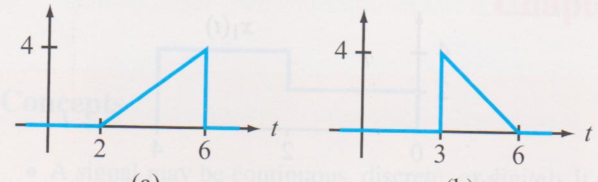


Figure P1.11: Waveforms for Problems 1.11 and 1.12.

1.7 The waveform shown in Fig. P1.4(d) is given by

$$x_4(t) = \begin{cases} 0 & \text{for } t \le 0, \\ \left(\frac{t}{2}\right)^2 & \text{for } 0 \le t \le 2 \text{ s}, \\ 1 & \text{for } 2 \le t \le 4 \text{ s}, \\ f(t) & \text{for } 4 \le t \le 6 \text{ s}, \\ 0 & \text{for } t \ge 6 \text{ s}. \end{cases}$$

(a) Obtain an expression for $f(t)$, which is the segment covering the time duration between 4 s and 6 s.

(b) Obtain an expression for $x_4[-(t-4)]$ and plot it.

1.8 If

$$x(t) = \begin{cases} 0 & \text{for } t \le 2 \\ (2t-4) & \text{for } t \ge 2, \end{cases}$$

plot $x(t)$, $x(t+1)$, $x\left(\frac{t+1}{2}\right)$, and $x\left[-\frac{(t+1)}{2}\right]$.

1.9 Given $x(t) = 10(1 - e^{-|t|})$, plot $x(-t+1)$.

1.10 Given $x(t) = 5\sin^2(6\pi t)$, plot $x(t-3)$ and $x(3-t)$.

1.11 Given the waveform of $x(t)$ shown in P1.11(a), generate and plot the waveform of:

(a) $x(2t+6)$

*__(b)__ $x(-2t+6)$

(c) $x(-2t-6)$

1.12 Given the waveform of $x(t)$ shown in P1.11(b), generate and plot the waveform of:

(a) $x(3t+6)$

(b) $x(-3t+6)$

(c) $x(-3t-6)$

1.13 If $x(t) = 0$ unless $a \le t \le b$, and $y(t) = x(ct+d)$ unless $e \le t \le f$, compute e and f in terms of a, b, c, and d. Assume $c > 0$ to make things easier for you.

1.14 If $x(t)$ is a musical note signal, what is $y(t) = x(4t)$? Consider sinusoidal $x(t)$.

1.15 Give an example of a non-constant signal that has the property $x(t) = x(at)$ for all $a > 0$.

Sections 1-3 and 1-4: Waveforms

1.16 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one.

(a) $x_1(t) = 3t^2 + 4t^4$

*__(b)__ $x_2(t) = 3t^3$

1.17 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one.

(a) $x_1(t) = 4[\sin(3t) + \cos(3t)]$

(b) $x_2(t) = \dfrac{\sin(4t)}{4t}$

1.18 For each of the following functions, indicate if it exhibits even symmetry, odd symmetry, or neither one.

(a) $x_1(t) = 1 - e^{-2t}$

(b) $x_2(t) = 1 - e^{-2t^2}$

1.19 Generate plots for each of the following step-function waveforms over the time span from −5 s to +5 s.

(a) $x_1(t) = -6u(t+3)$

(b) $x_2(t) = 10u(t-4)$

(c) $x_3(t) = 4u(t+2) - 4u(t-2)$

1.20 Generate plots for each of the following step-function waveforms over the time span from −5 s to +5 s.

(a) $x_1(t) = 8u(t-2) + 2u(t-4)$

*__(b)__ $x_2(t) = 8u(t-2) - 2u(t-4)$

(c) $x_3(t) = -2u(t+2) + 2u(t+4)$

1.21 Provide expressions in terms of step functions for the waveforms displayed in Fig. P1.21.

1.22 Generate plots for each of the following functions over the time span from −4 s to +4 s.

(a) $x_1(t) = 5r(t+2) - 5r(t)$

(b) $x_2(t) = 5r(t+2) - 5r(t) - 10u(t)$

*__(c)__ $x_3(t) = 10 - 5r(t+2) + 5r(t)$

(d) $x_4(t) = 10\,\text{rect}\left(\dfrac{t+1}{2}\right) - 10\,\text{rect}\left(\dfrac{t-3}{2}\right)$

(e) $x_5(t) = 5\,\text{rect}\left(\dfrac{t-1}{2}\right) - 5\,\text{rect}\left(\dfrac{t-3}{2}\right)$

1.23 Provide expressions for the waveforms displayed in Fig. P1.23 in terms of ramp and step functions.

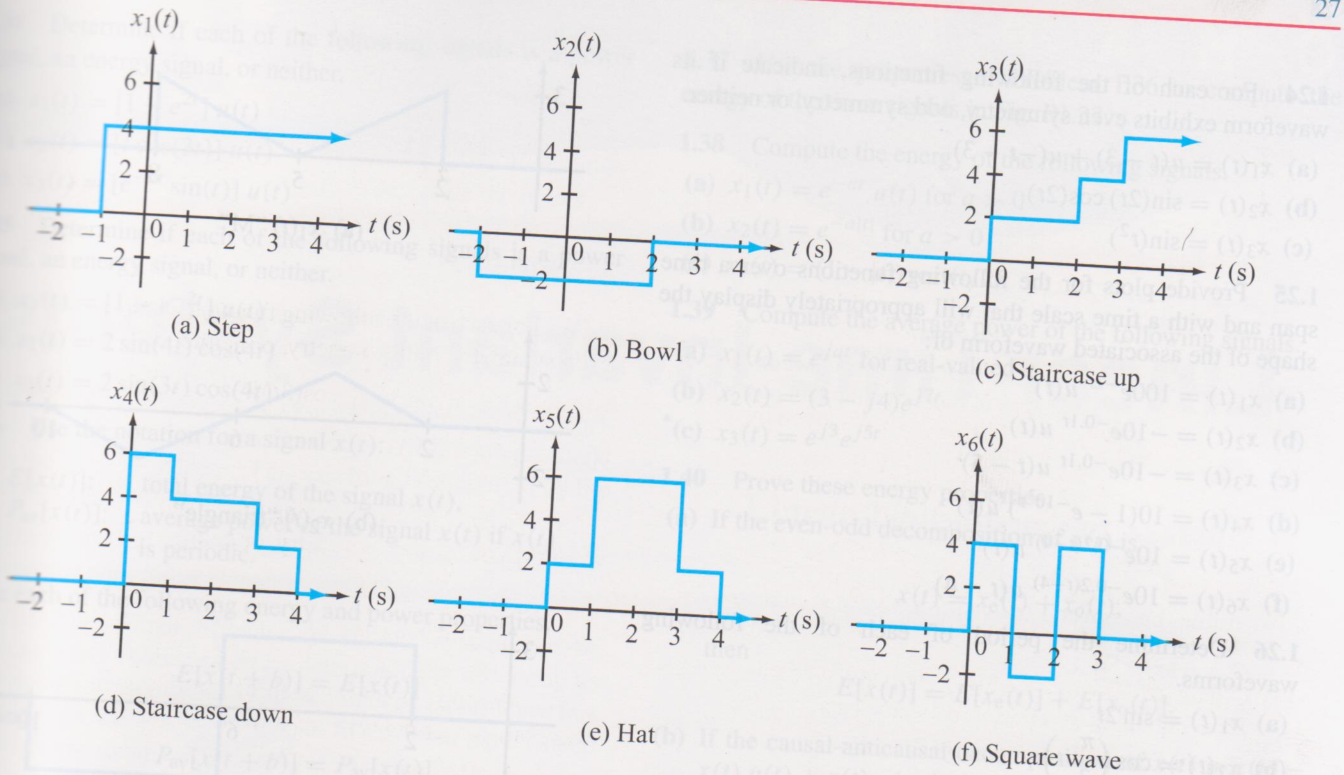


Figure P1.21: Waveforms for Problem 1.21.

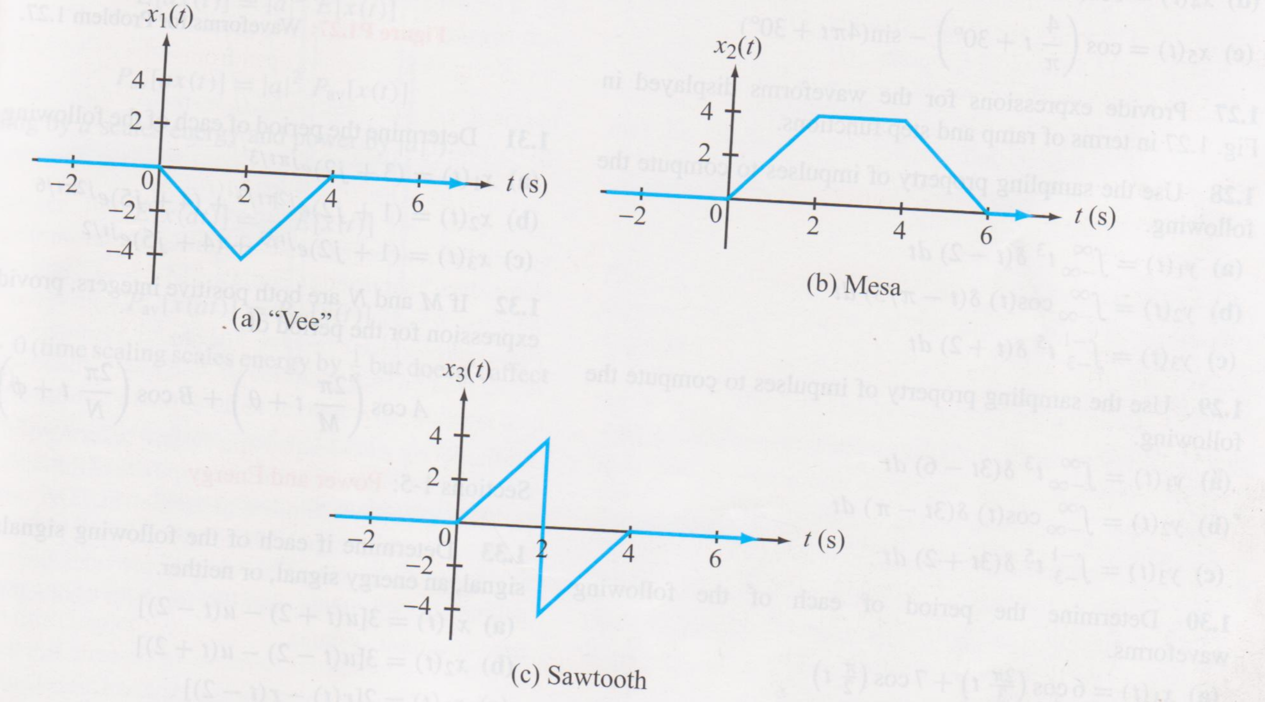


Figure P1.23: Waveforms for Problem 1.23.

1.24 For each of the following functions, indicate if its waveform exhibits even symmetry, odd symmetry, or neither.

(a) $x_1(t) = u(t-3) + u(-t-3)$

(b) $x_2(t) = \sin(2t)\cos(2t)$

(c) $x_3(t) = \sin(t^2)$

1.25 Provide plots for the following functions over a time span and with a time scale that will appropriately display the shape of the associated waveform of:

(a) $x_1(t) = 100e^{-2t}\ u(t)$

(b) $x_2(t) = -10e^{-0.1t}\ u(t)$

(c) $x_3(t) = -10e^{-0.1t}\ u(t-5)$

(d) $x_4(t) = 10(1 - e^{-10^3 t})\ u(t)$

(e) $x_5(t) = 10e^{-0.2(t-4)}\ u(t)$

(f) $x_6(t) = 10e^{-0.2(t-4)}\ u(t-4)$

1.26 Determine the period of each of the following waveforms.

(a) $x_1(t) = \sin 2t$

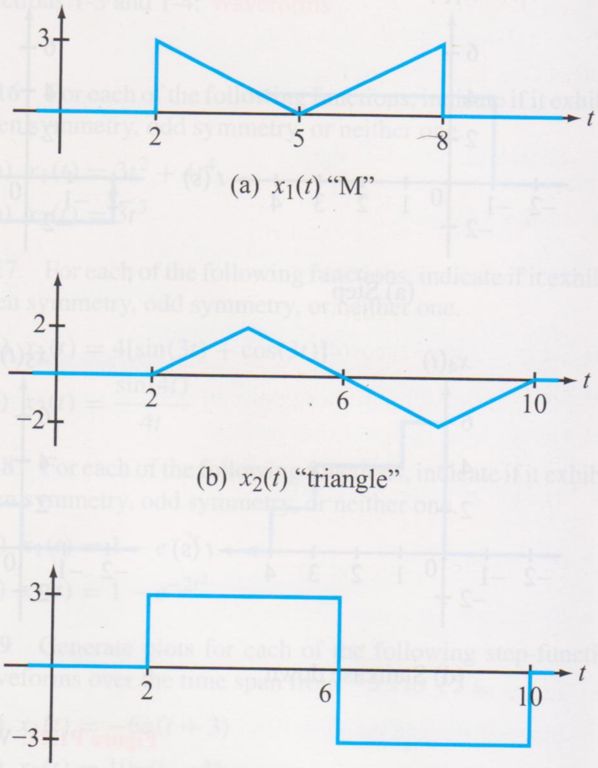
(b) $x_2(t) = \cos\left(\frac{\pi}{3}\,t\right)$

(c) $x_3(t) = \cos^2\left(\frac{\pi}{3}\,t\right)$

*(d) $x_4(t) = \cos(4\pi t + 60°) - \sin(4\pi t + 60°)$

(e) $x_5(t) = \cos\left(\frac{4}{\pi}\,t + 30°\right) - \sin(4\pi t + 30°)$

1.27 Provide expressions for the waveforms displayed in Fig. 1.27 in terms of ramp and step functions.



(a) $x_1(t)$ "M"

(b) $x_2(t)$ "triangle"

(c) $x_3(t)$ "Haar"

Figure P1.27: Waveforms for Problem 1.27.

1.28 Use the sampling property of impulses to compute the following.

(a) $y_1(t) = \int_{-\infty}^{\infty} t^3\ \delta(t-2)\ dt$

(b) $y_2(t) = \int_{-\infty}^{\infty} \cos(t)\ \delta(t - \pi/3)\ dt$

(c) $y_3(t) = \int_{-3}^{-1} t^5\ \delta(t+2)\ dt$

1.29 Use the sampling property of impulses to compute the following.

(a) $y_1(t) = \int_{-\infty}^{\infty} t^3\ \delta(3t-6)\ dt$

*(b) $y_2(t) = \int_{-\infty}^{\infty} \cos(t)\ \delta(3t-\pi)\ dt$

(c) $y_3(t) = \int_{-3}^{-1} t^5\ \delta(3t+2)\ dt$

1.30 Determine the period of each of the following waveforms.

(a) $x_1(t) = 6\cos\left(\frac{2\pi}{3}\,t\right) + 7\cos\left(\frac{\pi}{2}\,t\right)$

(b) $x_2(t) = 6\cos\left(\frac{2\pi}{3}\,t\right) + 7\cos(\pi\sqrt{2}\,t)$

(c) $x_3(t) = 6\cos\left(\frac{2\pi}{3}\,t\right) + 7\cos\left(\frac{2}{3}\,t\right)$

1.31 Determine the period of each of the following functions.

(a) $x_1(t) = (3 + j2)e^{j\pi t/3}$

(b) $x_2(t) = (1 + j2)e^{j2\pi t/3} + (4 + j5)e^{j2\pi t/6}$

(c) $x_3(t) = (1 + j2)e^{jt/3} + (4 + j5)e^{jt/2}$

1.32 If M and N are both positive integers, provide a general expression for the period of

$$A\cos\left(\frac{2\pi}{M}\,t + \theta\right) + B\cos\left(\frac{2\pi}{N}\,t + \phi\right).$$

Sections 1-5: Power and Energy

1.33 Determine if each of the following signals is a power signal, an energy signal, or neither.

(a) $x_1(t) = 3[u(t+2) - u(t-2)]$

(b) $x_2(t) = 3[u(t-2) - u(t+2)]$

(c) $x_3(t) = 2[r(t) - r(t-2)]$

(d) $x_4(t) = e^{-2t}\ u(t)$

1.34 Determine if each of the following signals is a power signal, an energy signal, or neither.

(a) $x_1(t) = [1 - e^{2t}]\ u(t)$

(b) $x_2(t) = [t\cos(3t)]\ u(t)$

(c) $x_3(t) = [e^{-2t}\sin(t)]\ u(t)$

1.35 Determine if each of the following signals is a power signal, an energy signal, or neither.

(a) $x_1(t) = [1 - e^{-2t}]\ u(t)$

(b) $x_2(t) = 2\sin(4t)\cos(4t)$

(c) $x_3(t) = 2\sin(3t)\cos(4t)$

1.36 Use the notation for a signal $x(t)$:

$E[x(t)]$: total energy of the signal $x(t)$,

$P_{\text{av}}[x(t)]$: average power of the signal $x(t)$ if $x(t)$ is periodic.

Prove each of the following energy and power properties.

(a)

$$E[x(t+b)] = E[x(t)]$$

and

$$P_{\text{av}}[x(t+b)] = P_{\text{av}}[x(t)]$$

(time shifts do not affect power or energy).

(b)

$$E[ax(t)] = |a|^2\ E[x(t)]$$

and

$$P_{\text{av}}[ax(t)] = |a|^2\ P_{\text{av}}[x(t)]$$

(scaling by a scales energy and power by $|a|^2$).

(c)

$$E[x(at)] = \frac{1}{a}\ E[x(t)]$$

and

$$P_{\text{av}}[x(at)] = P_{\text{av}}[x(t)]$$

if $a > 0$ (time scaling scales energy by $\frac{1}{a}$ but doesn't affect power).

1.37 Use the properties of Problem 1.36 to compute the energy of the three signals in Fig. P1.27.

1.38 Compute the energy of the following signals.

(a) $x_1(t) = e^{-at}\ u(t)$ for $a > 0$

(b) $x_2(t) = e^{-a|t|}$ for $a > 0$

(c) $x_3(t) = (1 - |t|)\ \text{rect}(t/2)$

1.39 Compute the average power of the following signals.

(a) $x_1(t) = e^{jat}$ for real-valued a

(b) $x_2(t) = (3 - j4)e^{j7t}$

*(c) $x_3(t) = e^{j3}e^{j5t}$

1.40 Prove these energy properties.

(a) If the even-odd decomposition of $x(t)$ is

$$x(t) = x_e(t) + x_o(t),$$

then

$$E[x(t)] = E[x_e(t)] + E[x_o(t)].$$

(b) If the causal-anticausal decomposition of $x(t)$ is $x(t) = x(t)\ u(t) + x(t)\ u(-t)$, then

$$E[x(t)] = E[x(t)\ u(t)] + E[x(t)\ u(-t)].$$