PROBLEMS

Section 5-1: Phasor-Domain Technique

A system is characterized by the differential equation

$$c_1 \frac{dy}{dt} + c_2 y = 10\cos(400t - 30^\circ).$$

Determine $y(t)$, given that $c_1 = 10^{-2}$ and $c_2 = 3$.

Determine $y(t)$, given that $c_1 = 10^{-2}$ and $c_2 = 0.3$.

A system is characterized by the differential equation

$$c_1 \frac{d^2 y}{dt^2} + c_2 \frac{dy}{dt} + c_3 y = A\cos(\omega t + \phi).$$

Determine $y(t)$ for the following:

$c_1 = 10^{-6}$, $c_2 = 3 \times 10^{-3}$, $c_3 = 3$, $A = 12$, $\omega = 10^3$ rad/s, and $\phi = 60^\circ$.

$c_1 = 5 \times 10^{-4}$, $c_2 = 10^{-2}$, $c_3 = 1$, $A = 16$, $\omega = 200$ rad/s, and $\phi = -30^\circ$.

$c_1 = 5 \times 10^{-6}$, $c_2 = 1$, $c_3 = 10^6$, $A = 4$, $\omega = 10^6$ rad/s, and $\phi = -60^\circ$.

Repeat part (a) of Problem 5.2 after replacing the cosine with a sine.

A system is characterized by

$$c_1 \frac{d^2 y}{dt^2} + c_2 \frac{dy}{dt} + c_3 y = A_1\cos(\omega t + \phi_1) + A_2\sin(\omega t + \phi_2).$$

Determine $y(t)$, given that $c_1 = 10^{-6}$, $c_2 = 3 \times 10^{-3}$, $c_3 = 3$, $A_1 = 10$, $A_2 = 20$, $\omega = 10^3$ rad/s, $\phi_1 = 30^\circ$, and $\phi_2 = 30^\circ$. (Hint: Apply the superposition property of LTI systems.)

A system is characterized by

$$4 \times 10^{-3} \frac{dy}{dt} + 3y = 5\cos(1000t) - 10\cos(2000t).$$

Determine $y(t)$. (*Hint:* Apply the superposition property of LTI systems.)

*Answer(s) in Appendix E.

†MATLAB® is a registered trademark of The MathWorks, Inc.

Sections 5-3 and 5-4: Fourier Series

Follow these instruction for each of the waveforms in Problems 5.6 through 5.15.

(a) Determine if the waveform has dc, even, or odd symmetry.

(b) Obtain its cosine/sine Fourier series representation.

(c) Convert the representation to amplitude/phase format and plot the line spectra for the first five non-zero terms.

(d) Convert the representation to complex exponential format and plot the line spectra for the first five non-zero terms.

(e) Use MATLAB®† or MathScript RT Module to plot the waveform using a truncated Fourier series representation with $n_{max} = 100$.

5.6 Waveform in Fig. P5.6 with $A = 10$.

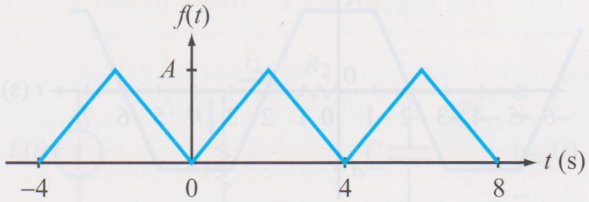

Figure P5.6: Waveform for Problem 5.6.

5.7 Waveform in Fig. P5.7 with $A = 4$.

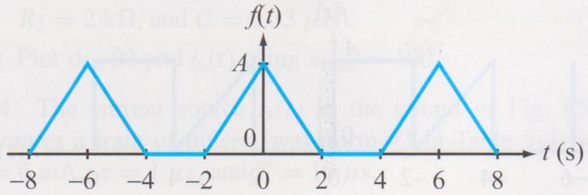

Figure P5.7: Waveform for Problem 5.7.

5.8 Waveform in Fig. P5.8 with $A = 6$.

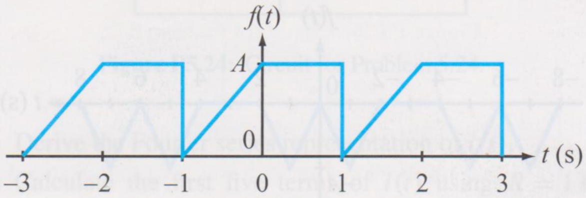

Figure P5.8: Waveform for Problem 5.8.

*5.9 Waveform in Fig. P5.9 with $A = 10$.

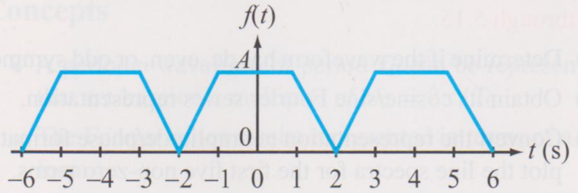

Figure P5.9: Waveform for Problem 5.9.

5.10 Waveform in Fig. P5.10 with $A = 20$.

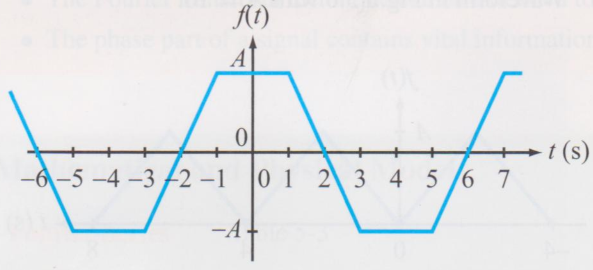

Figure P5.10: Waveform for Problem 5.10.

5.11 Waveform in Fig. P5.11 with $A = 100$.

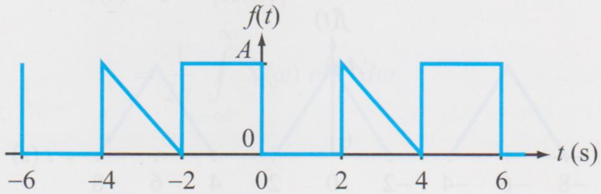

Figure P5.11: Waveform for Problem 5.11.

5.12 Waveform in Fig. P5.12 with $A = 4$.

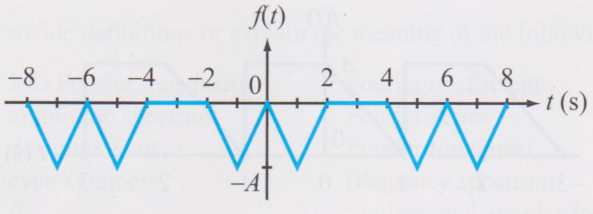

Figure P5.12: Waveform for Problem 5.12.

5.13 Waveform in Fig. P5.13 with $A = 10$.

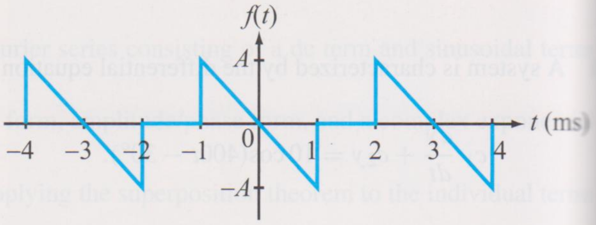

Figure P5.13: Waveform for Problem 5.13.

5.14 Waveform in Fig. P5.14 with $A = 10$.

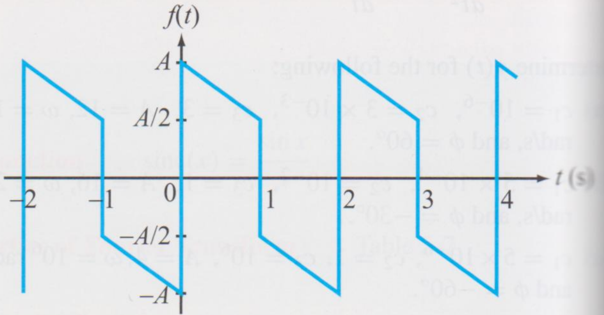

Figure P5.14: Waveform for Problem 5.14.

5.15 Waveform in Fig. P5.15 with $A = 20$.

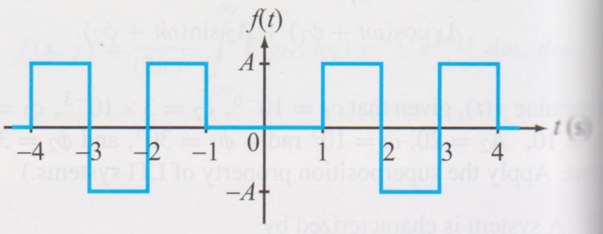

Figure P5.15: Waveform for Problem 5.15.

5.16 Obtain the cosine/sine Fourier-series representation for $f(t) = \cos^2(4\pi t)$, and use MATLAB® or MathScript RT Module software to plot it with $n_{\max} = 100$.

5.17 Repeat Problem 5.16 for $f(t) = \sin^2(4\pi t)$.

5.18 Repeat Problem 5.16 for $f(t) = |\sin(4\pi t)|$.

5.19 Which of the six waveforms shown in Figs. P5.6 through P5.11 will exhibit the Gibbs oscillation phenomenon when represented by a Fourier series? Why?

5.20 Consider the sawtooth waveform shown in Fig. 5-3(a). Evaluate the Gibbs phenomenon in the neighborhood of $t = 4$ s by plotting the Fourier-series representation with $n_{\max} = 100$ over the range between 4.01 s and 4.3 s.

5.21 The Fourier series of the periodic waveform shown in Fig. P5.21(a) is given by

$$f_1(t) = 10 - \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi t}{2}\right).$$

Determine the Fourier series of waveform $f_2(t)$ in Fig. P5.21(b).

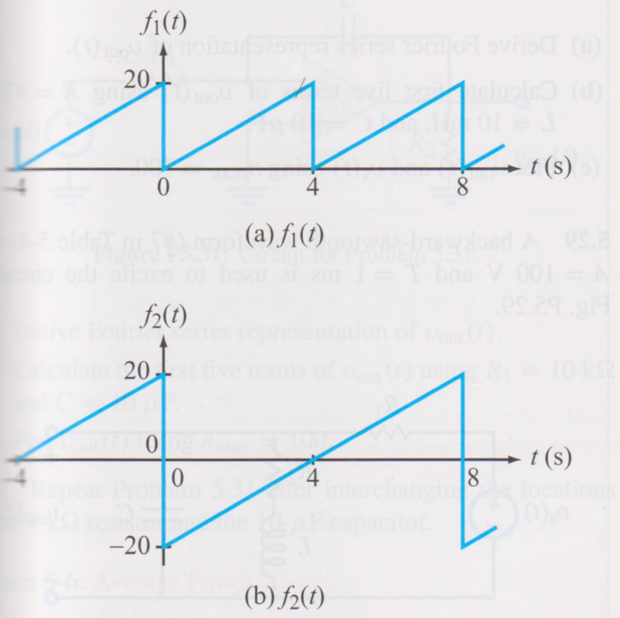

Figure P5.21: Waveforms of Problem 5.21.

Section 5-5: Circuit Applications

5.22 The voltage source $v_s(t)$ in the circuit of Fig. P5.22 generates a square wave (waveform #1 in Table 5-4) with $A = 10$ V and $T = 1$ ms.

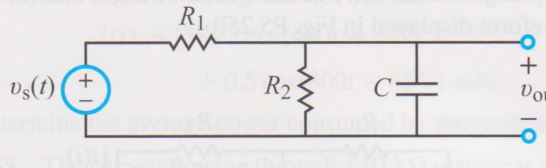

Figure P5.22: Circuit for Problem 5.22.

(a) Derive the Fourier series representation of $v_{\text{out}}(t)$.

(b) Calculate the first five terms of $v_{\text{out}}(t)$ using $R_1 = R_2 = 2$ kΩ, $C = 1$ μF.

(c) Plot $v_{\text{out}}(t)$ using $n_{\max} = 100$.

5.23 The current source $i_s(t)$ in the circuit of Fig. P5.23 generates a sawtooth wave (waveform in Fig. 5-3(a)) with a peak amplitude of 20 mA and a period $T = 5$ ms.

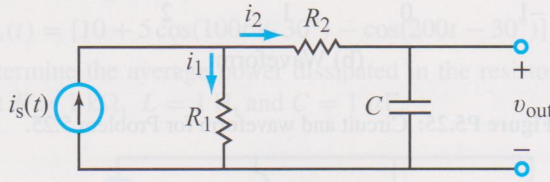

Figure P5.23: Circuit for Problem 5.23.

(a) Derive the Fourier series representation of $v_{\text{out}}(t)$.

(b) Calculate the first five terms of $v_{\text{out}}(t)$ using $R_1 = 500$ Ω, $R_2 = 2$ kΩ, and $C = 0.33$ μF.

(c) Plot $v_{\text{out}}(t)$ and $i_s(t)$ using $n_{\max} = 100$.

*__5.24__ The current source $i_s(t)$ in the circuit of Fig. P5.24 generates a train of pulses (waveform #3 in Table 5-4) with $A = 6$ mA, $\tau = 1$ μs, and $T = 10$ μs.

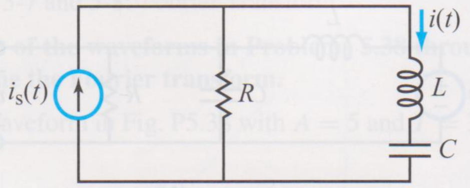

Figure P5.24: Circuit for Problem 5.24.

(a) Derive the Fourier series representation of $i(t)$.

(b) Calculate the first five terms of $i(t)$ using $R = 1$ kΩ, $L = 1$ mH, and $C = 1$ μF.

(c) Plot $i(t)$ and $i_s(t)$ using $n_{\max} = 100$.

5.25 Voltage source $\upsilon_s(t)$ in the circuit of Fig. P5.25(a) has the waveform displayed in Fig. P5.25(b).

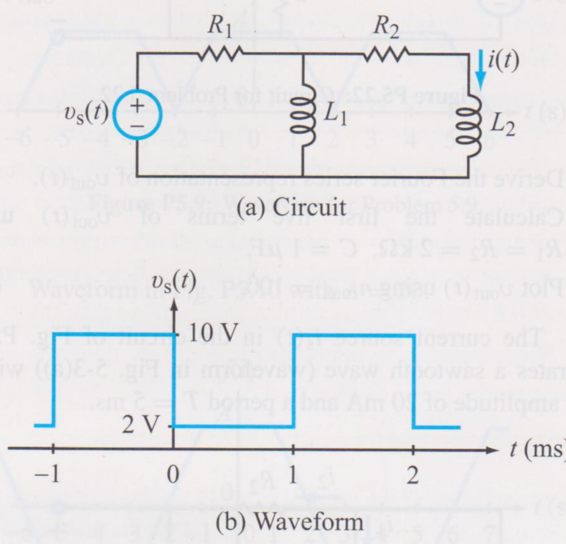

Figure P5.25: Circuit and waveform for Problem 5.25.

(a) Derive the Fourier series representation of $i(t)$.

(b) Calculate the first five terms of $i(t)$ using $R_1 = R_2 = 10\ \Omega$ and $L_1 = L_2 = 10$ mH.

(c) Plot $i(t)$ and $\upsilon_s(t)$ using $n_{max} = 100$.

5.26 Determine the output voltage $\upsilon_{out}(t)$ in the circuit of Fig. P5.26, given that the input voltage $\upsilon_{in}(t)$ is a full-wave rectified sinusoid (waveform #8 in Table 5-4) with $A = 120$ V and $T = 1\ \mu$s.

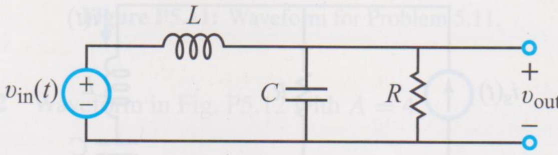

Figure P5.26: Circuit for Problem 5.26.

(a) Derive the Fourier series representation of $\upsilon_{out}(t)$.

(b) Calculate the first five terms of $\upsilon_{out}(t)$ using $R = 1$ kΩ, $L = 1$ mH, and $C = 1$ nF.

(c) Plot $\upsilon_{out}(t)$ and $\upsilon_{in}(t)$ using $n_{max} = 100$.

5.27

(a) Repeat Example 5-6, after replacing the capacitor with an inductor $L = 0.1$ H and reducing the value of R to 1 Ω.

(b) Calculate the first five terms of $\upsilon_{out}(t)$.

(c) Plot $\upsilon_{out}(t)$ and $\upsilon_s(t)$ using $n_{max} = 100$.

5.28 Determine $\upsilon_{out}(t)$ in the circuit of Fig. P5.28, given the input excitation is characterized by a triangular waveform (#4 in Table 5-4) with $A = 24$ V and $T = 20$ ms.

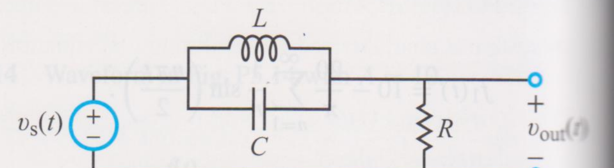

Figure P5.28: Circuit for Problem 5.28.

(a) Derive Fourier series representation of $\upsilon_{out}(t)$.

(b) Calculate first five terms of $\upsilon_{out}(t)$ using $R = 470\ \Omega$, $L = 10$ mH, and $C = 10\ \mu$F.

(c) Plot $\upsilon_{out}(t)$ and $\upsilon_s(t)$ using $n_{max} = 100$.

5.29 A backward-sawtooth waveform (#7 in Table 5-4) with $A = 100$ V and $T = 1$ ms is used to excite the circuit in Fig. P5.29.

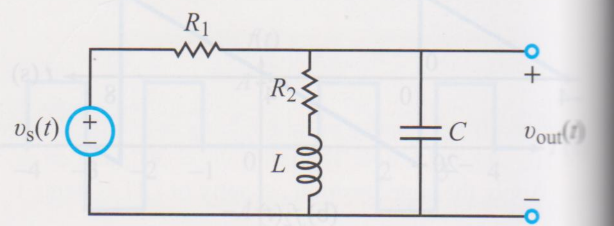

Figure P5.29: Circuit for Problem 5.29.

(a) Derive Fourier series representation of $\upsilon_{out}(t)$.

(b) Calculate the first five terms of $\upsilon_{out}(t)$ using $R_1 = 1$ kΩ, $R_2 = 100\ \Omega$, $L = 1$ mH, and $C = 1\ \mu$F.

(c) Plot $\upsilon_{out}(t)$ and $\upsilon_s(t)$ using $n_{max} = 100$.

The circuit in Fig. P5.30 is excited by the source waveform shown in Fig. P5.25(b).

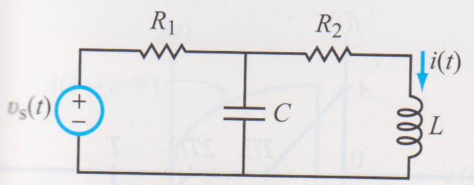

Figure P5.30: Circuit for Problem 5.30.

Derive Fourier series representation of $i(t)$.

Calculate the first five terms of $v_{out}(t)$ using $R_1 = R_2 = 100\ \Omega$, $L = 1$ mH, and $C = 1\ \mu$F.

Plot $i(t)$ and $v_s(t)$ using $n_{max} = 100$.

The RC op-amp integrator circuit of Fig. P5.31 excited by a square wave (waveform #1 in Table 5-4) with $A = 4$ V and $T = 2$ s.

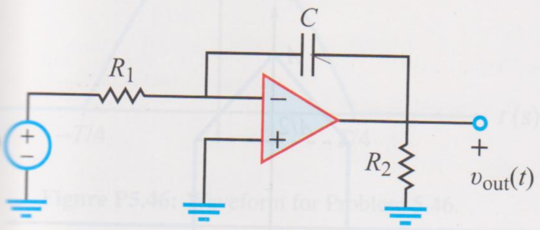

Figure P5.31: Circuit for Problem 5.31.

Derive Fourier series representation of $v_{out}(t)$.

Calculate the first five terms of $v_{out}(t)$ using $R_1 = 10$ kΩ and $C = 10\ \mu$F.

Plot $v_{out}(t)$ using $n_{max} = 100$.

Repeat Problem 5.31 after interchanging the locations of the 1-kΩ resistor and the 10-μF capacitor.

Section 5-6: Average Power

The voltage across the terminals of a certain circuit and the current entering into its (+) voltage terminal are given by

$$v(t) = [4 + 12\cos(377t + 60°) - 6\cos(754t - 30°)]\text{ V},$$

$$i(t) = [5 + 10\cos(377t + 45°) + 2\cos(754t + 15°)]\text{ mA}.$$

Determine the average power consumed by the circuit, and the power fraction.

5.34 The current flowing through a 2-kΩ resistor is given by

$$i(t) = [5 + 2\cos(400t + 30°) + 0.5\cos(800t - 45°)]\text{ mA}.$$

Determine the average power consumed by the resistor.

5.35 The current flowing through a 10-kΩ resistor is given by a triangular waveform (#4 in Table 5-4) with $A = 4$ mA and $T = 0.2$ s.

(a) Determine the exact value of the average power consumed by the resistor.

(b) Using a truncated Fourier-series representation of the waveform with only the first four terms, obtain an approximate value for the average power consumed by the resistor.

(c) What is the percentage of error in the value given in (b)?

*__5.36__ The current source in the parallel RLC circuit of Fig. P5.36 is given by

$$i_s(t) = [10 + 5\cos(100t + 30°) - \cos(200t - 30°)]\text{ mA}.$$

Determine the average power dissipated in the resistor given that $R = 1$ kΩ, $L = 1$ H, and $C = 1\ \mu$F.

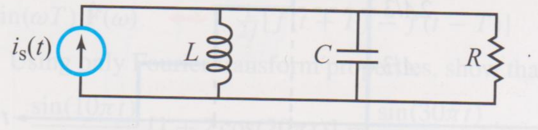

Figure P5.36: Circuit for Problem 5.36.

5.37 A series RC circuit is connected to a voltage source whose waveform is given by waveform #5 in Table 5-4, with $A = 12$ V and $T = 1$ ms. Using a truncated Fourier-series representation composed of only the first three non-zero terms, determine the average power dissipated in the resistor, given that $R = 2$ kΩ and $C = 1\ \mu$F.

Sections 5-7 and 5-8: Fourier Transform

For each of the waveforms in Problems 5.38 through 5.47, determine the Fourier transform.

5.38 Waveform in Fig. P5.38 with $A = 5$ and $T = 3$ s.

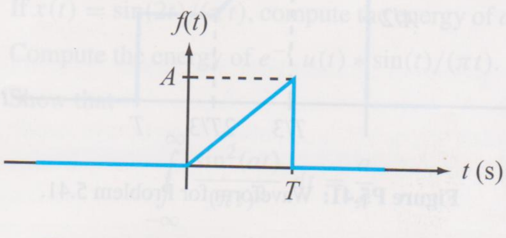

Figure P5.38: Waveform for Problem 5.38.

5.39 Waveform in Fig. P5.39 with $A = 10$ and $T = 6$ s.

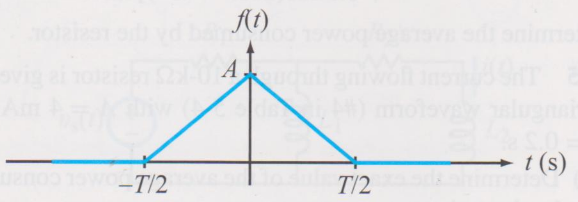

Figure P5.39: Waveform for Problem 5.39.

5.40 Waveform in Fig. P5.40 with $A = 12$ and $T = 3$ s.

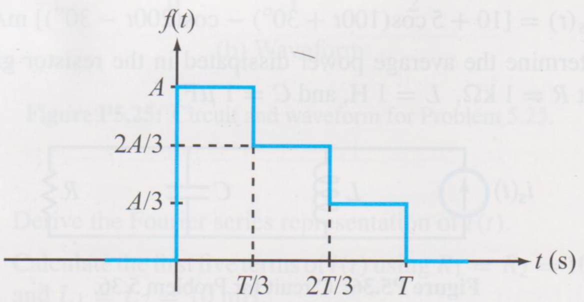

Figure P5.40: Waveform for Problem 4.50.

5.41 Waveform in Fig. P5.41 with $A = 2$ and $T = 12$ s.

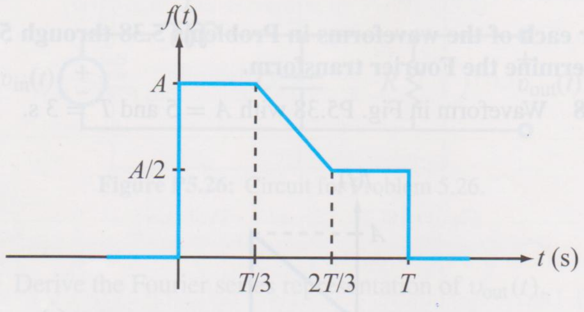

Figure P5.41: Waveform for Problem 5.41.

5.42 Waveform in Fig. P5.42 with $A = 1$ and $T = 3$ s.

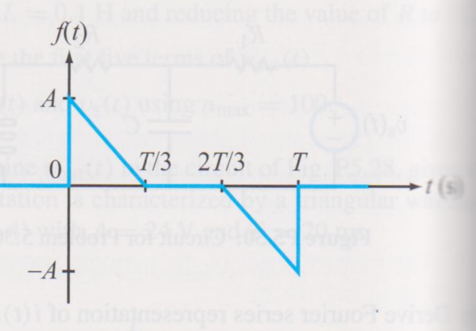

Figure P5.42: Waveform for Problem 5.42.

5.43 Waveform in Fig. P5.43 with $A = 1$ and $T = 2$ s.

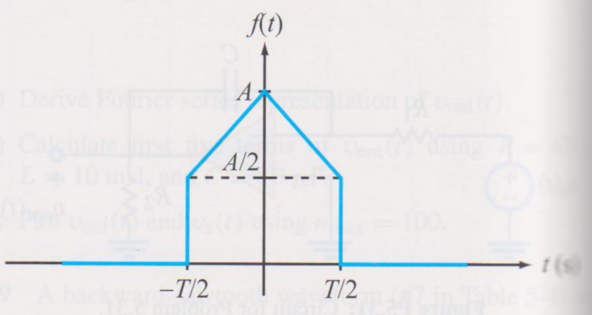

Figure P5.43: Waveform for Problem 5.43.

5.44 Waveform in Fig. P5.44 with $A = 3$ and $T = 1$ s.

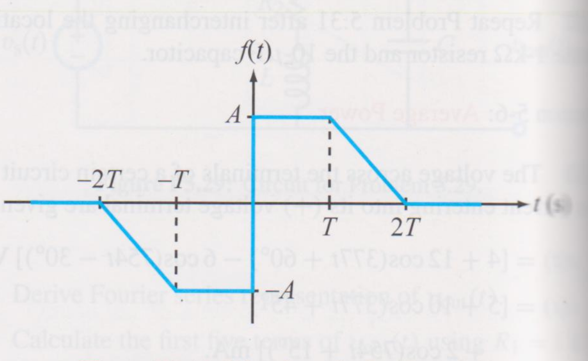

Figure P5.44: Waveform for Problem 5.44.

5.45 Waveform in Fig. P5.45 with $A = 5$, $T = 1$ s, and $\alpha = 10\ \text{s}^{-1}$.

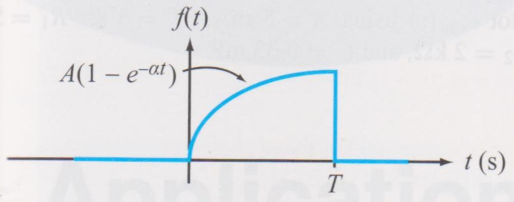

Figure P5.45: Waveform for Problem 5.45.

5.46 Waveform in Fig. P5.46 with $A = 10$ and $T = 2$ s.

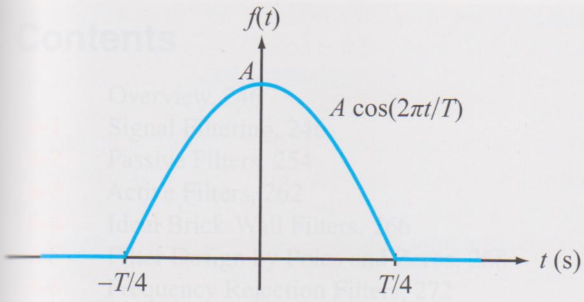

Figure P5.46: Waveform for Problem 5.46.

5.47 Find the Fourier transform of the following signals with $A = 2$, $\omega_0 = 5$ rad/s, $\alpha = 0.5\ \text{s}^{-1}$, and $\phi_0 = \pi/5$.

(a) $f(t) = A\cos(\omega_0 t - \phi_0), \quad -\infty \le t \le \infty$

(b) $g(t) = e^{-\alpha t}\cos(\omega_0 t)\, u(t)$

5.48 Find the Fourier transform of the following signals with $A = 3$, $B = 2$, $\omega_1 = 4$ rad/s, and $\omega_2 = 2$ rad/s.

(a) $f(t) = [A + B\sin(\omega_1 t)]\sin(\omega_2 t)$

(b) $g(t) = A|t|, \quad |t| < (2\pi/\omega_1)$

5.49 Find the Fourier transform of the following signals with $\alpha = 0.5\ \text{s}^{-1}$, $\omega_1 = 4$ rad/s, and $\omega_2 = 2$ rad/s.

(a) $f(t) = e^{-\alpha t}\sin(\omega_1 t)\cos(\omega_2 t)\, u(t)$

(b) $g(t) = te^{-\alpha t}, \quad 0 \le t \le 10\alpha$

5.50 Using the definition of Fourier transform, prove that

$$\mathcal{F}[t\ f(t)] = j\,\frac{d}{d\omega}\,\mathcal{F}(\omega).$$

5.51 Let the Fourier transform of $f(t)$ be

$$\hat{\mathbf{F}}(\omega) = \frac{A}{(B + j\omega)}.$$

Determine the transforms of the following signals (using $A = 5$ and $B = 2$).

(a) $f(3t - 2)$

(b) $t\ f(t)$

(c) $d\ f(t)/dt$

5.52 Let the Fourier transform of $f(t)$ be

$$\hat{\mathbf{F}}(\omega) = \frac{1}{(A + j\omega)}\, e^{-j\omega} + B.$$

Determine the Fourier transforms of the following signals (set $A = 2$ and $B = 1$).

(a) $f\left(\frac{5}{8}\,t\right)$

(b) $f(t)\cos(At)$

(c) $d^3 f/dt^3$

5.53 Prove the following two Fourier transform pairs.

(a) $\cos(\omega T)\ \hat{\mathbf{F}}(\omega) \leftrightarrow \frac{1}{2}[f(t - T) + f(t + T)]$

(b) $\sin(\omega T)\ \hat{\mathbf{F}}(\omega) \leftrightarrow \frac{1}{2j}[f(t + T) - f(t - T)]$

5.54 Using only Fourier transform properties, show that

$$\frac{\sin(10\pi t)}{\pi t}\,[1 + 2\cos(20\pi t)] = \frac{\sin(30\pi t)}{\pi t}.$$

5.55 Show that the spectrum of

$$\frac{\sin(20\pi t)}{\pi t}\ \frac{\sin(10\pi t)}{\pi t}$$

is zero for $|\omega| > 30\pi$.

*__5.56__ A square wave $x(t)$ has the Fourier series given by

$$x(t) = \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots.$$

Compute $y(t) = x(t) * 3e^{-|t|} * [\sin(4t)/(\pi t)]$.

Section 5-9: Parseval's Theorem for Fourier Integral

5.57 If $x(t) = \sin(2t)/(\pi t)$, compute the energy of d^2x/dt^2.

5.58 Compute the energy of $e^{-t}\, u(t) * \sin(t)/(\pi t)$.

5.59 Show that

$$\int_{-\infty}^{\infty} \frac{\sin^2(at)}{(\pi t)^2}\, dt = \frac{a}{\pi}$$

if $a > 0$.

Section 5-12: Circuit Analysis with Fourier Transform

5.60 The circuit in Fig. P5.23 is excited by the source waveform shown in Fig. P5.47.

(a) Derive the expression for $\upsilon_{\text{out}}(t)$ using Fourier analysis.

(b) Plot $\upsilon_{\text{out}}(t)$ using $A = 5$ V, $T = 3$ ms, $R_1 = 500\ \Omega$, $R_2 = 2\ \text{k}\Omega$, and $C = 0.33\ \mu\text{F}$.

(c) Repeat part (b) with $C = 0.33$ mF and comment on the results.

5.61 The circuit in Fig. P5.23 is excited by the source waveform shown in Fig. P5.39.

(a) Derive the expression for $\upsilon_{\text{out}}(t)$ using Fourier analysis.

(b) Plot $\upsilon_{\text{out}}(t)$ using $A = 5$ mA, $T = 3$ s, $R_1 = 500\ \Omega$, $R_2 = 2\ \text{k}\Omega$, and $C = 0.33$ mF.