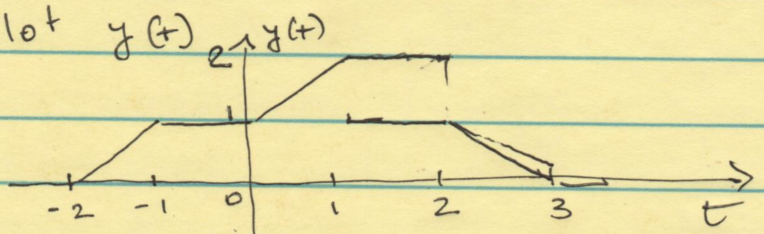
Problem 1

Solution for test practice

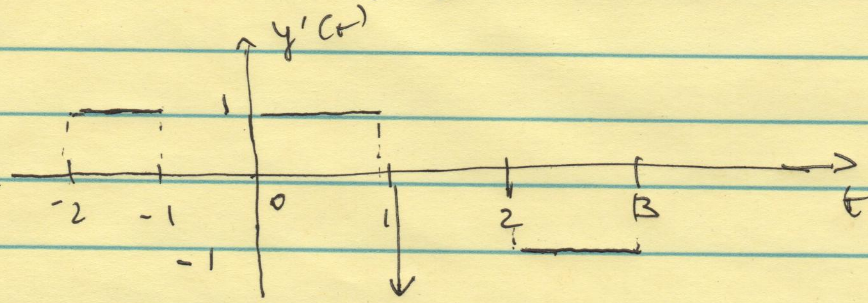
$$y(t) = r(t+2) - r(t+1) + r(t) - r(t-1) - u(t-1) - r(t-2) + r(t-3)$$

a) Plot $y(t)$



b) plot $y'(t)$

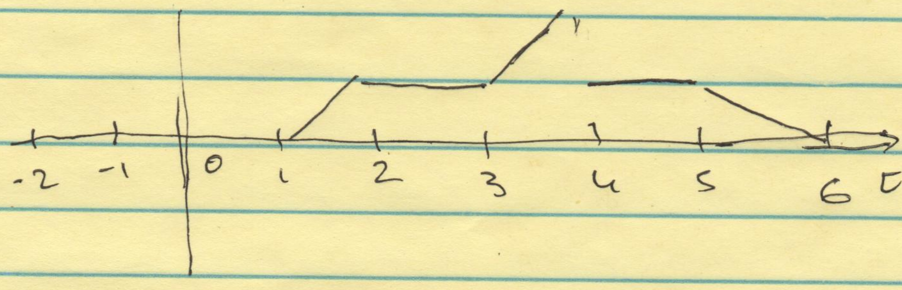
$$y'(t) = u(t+2) - u(t+1) + u(t) - u(t-1) - \delta(t-1) - u(t-2) + u(t-3$$



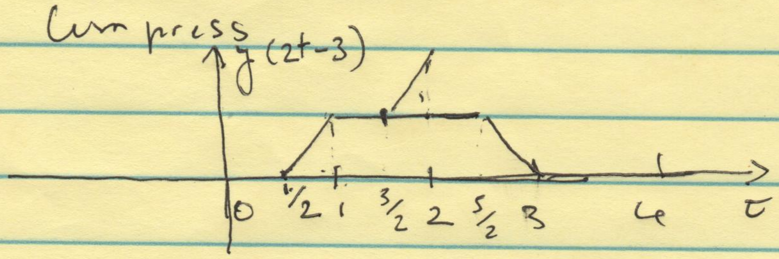
c) Plot $y(2t-3)$

Shift then scale

i) Shift to the right by 3



ii Compress



d) Energy of $y(t)$

$$E = \int_{-\infty}^{+\infty} y^2(t)\,dt.$$

Let's give the expression of $y(t)$ in different intervals $t < -2$ $y(t) = 0$

$-2 \leq t \leq -1$ $y(t) = t+2$

$-1 \leq t \leq 0$ $y(t) = 1$

$0 \leq t \leq 1$ $y(t) = t+1$

$1 \leq t \leq 2$ $y(t) = 1$

$2 \leq t \leq 3$ $y(t) = -t+3$

$t \geq 3$ $y(t) = 0$

$$E = \int_{-\infty}^{+\infty} y^2(t)\,dt = \int_{-2}^{-1} (t+2)^2\,dt + \int_{-1}^{0} 1^2\,dt + \int_{0}^{1} (t+1)^2\,dt$$

$$+ \int_{1}^{2} 1^2\,dt + \int_{2}^{3} (3-t)^2\,dt$$

$t+2 = u$ $\displaystyle\int_{-2}^{-1} (t+2)^2\,dt = \int_{0}^{1} u^2\,du$

$t+1 = u$ $\displaystyle\int_{0}^{1} (t+1)^2\,dt = \int_{1}^{2} u^2\,du$

$$\int_{2}^{3} (3-t)^2\,dt = \int_{2}^{3} (t-3)^2\,dt$$

$u = t-3$ $\displaystyle\int_{2}^{3} (t-3)^2\,dt = \int_{-1}^{0} u^2\,du = \int_{0}^{1} u^2\,du$

$$E = 2\int_0^1 u^2\,du + 2\int_{-1}^{0} 1^2\,dt + \int_1^2 u^2\,du$$

$$= 2 \times \frac{1}{3} + 2 + \frac{1}{3}\left[u^3 \Big|_1^2 \right]$$

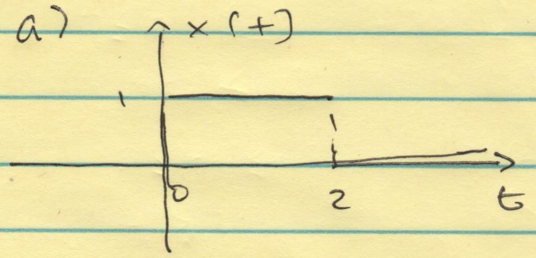
$$= \frac{2}{3} + 2 + \frac{1}{3}\left[8 - 1 \right] = \frac{2}{3} + \frac{7}{3} + 2$$

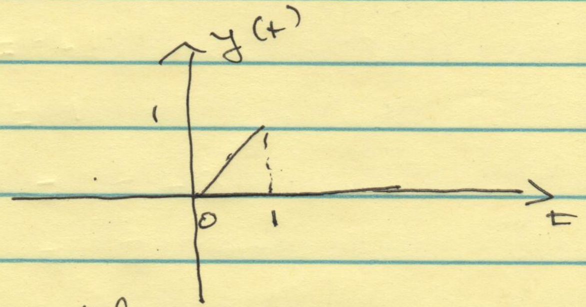
$$= \frac{2 + 7 + 6}{3}$$

$$= \frac{15}{3} = \underline{\underline{5}}$$

Problem 2

a)

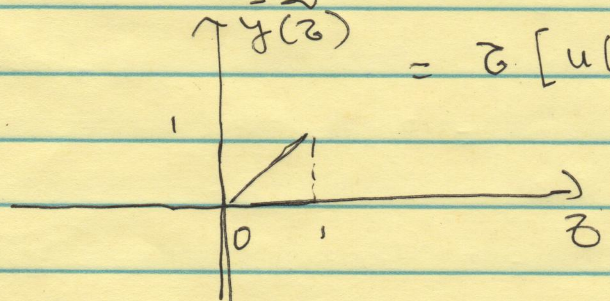




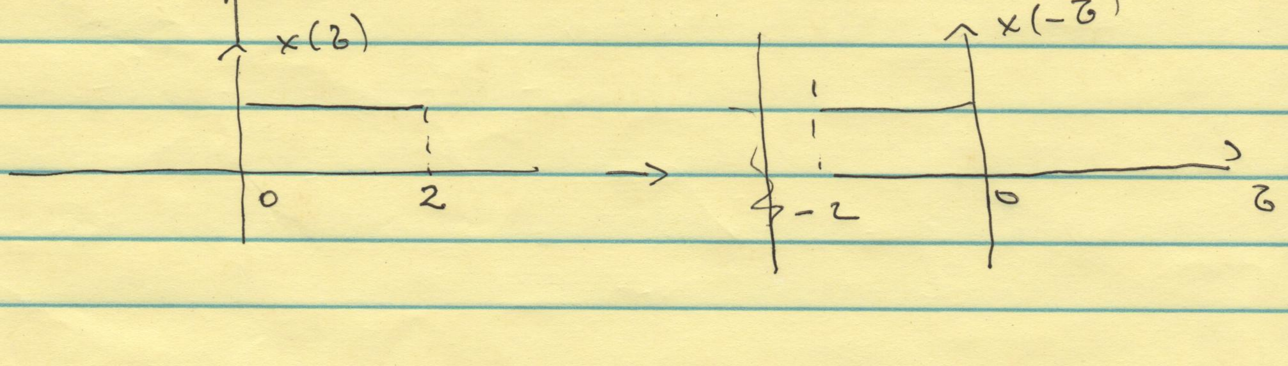
b)

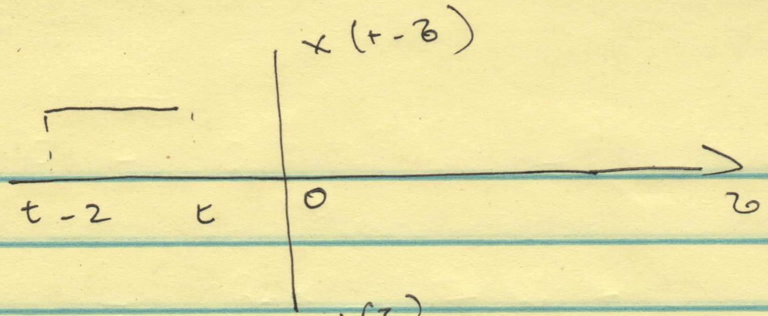
$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau$$

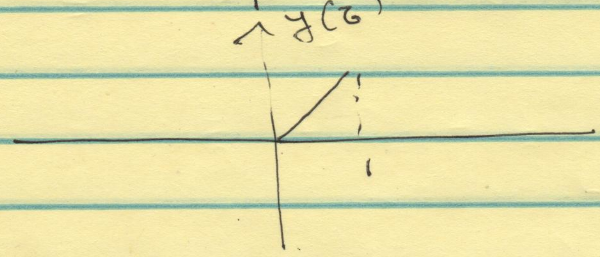
Use $\int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau$

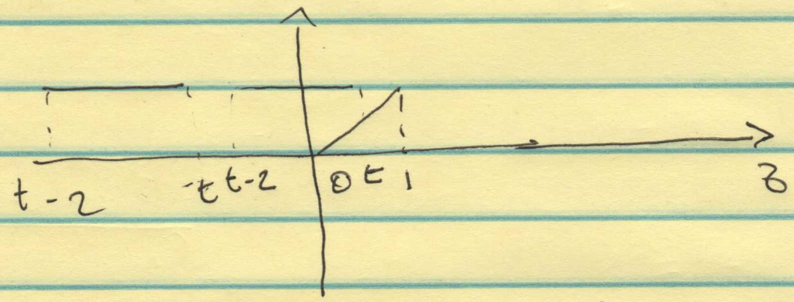


$= \tau\left[u(\tau) - u(\tau - 1) \right]$







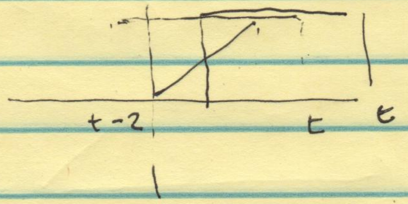


$t < 0 \qquad y(\tau)\, x(t-\tau) = 0 \qquad \text{so} \quad z(t) = 0$

$0 < t < 1 \qquad t-2 < -1$

$$y(\tau)\, x(t-\tau) = \tau \cdot 1 = \tau$$

$$z(t) = \int_0^t \tau \, d\tau = \underline{\underline{\frac{t^2}{2}}}$$



$t > 1$

$t - 2 < 0$

$\underline{1 < t < 2}$

$$z(t) = \int_0^1 \tau \, d\tau = \underline{\underline{\frac{1}{2}}}$$

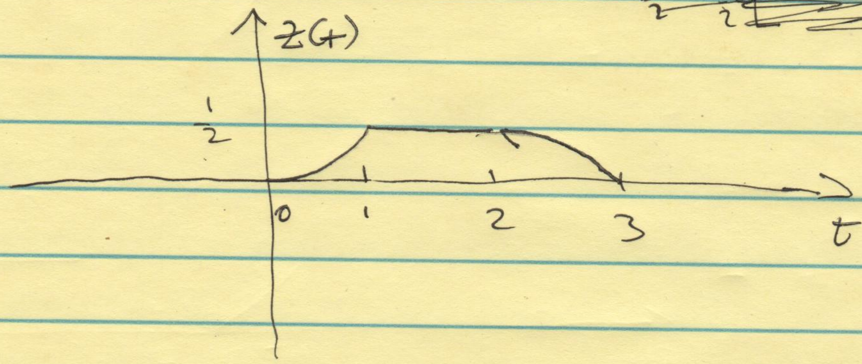
$0 < t-2 < 1$

$2 < t < 3 \qquad z(t) = \int_{t-2}^1 \tau \, d\tau$

$$z(t) = \frac{1}{2}\left[1 - (t-2)^2\right]$$

$$= \frac{1}{2}\left[1 - (t^2 - 4t + 4\right]$$

$t > 3 \qquad z(t) = 0$ $\quad \cancel{= \frac{1}{2}\left[-t^2 + 4t - 3\right]}$



Problem 3

$$h(t) = \underbrace{5e^{-t}u(t)}_{h_1(t)} \underbrace{-16e^{-2t}u(t)}_{h_2(t)} + \underbrace{13e^{-3t}u(t)}_{h_3(t)}$$

$$x(t) = 7\cos(2t)$$

$$y(t) = h(t) * x(t) = x(t) * h_1(t) + x(t) * h_2(t) + h_3(t) * x(t)$$

We need to use the the frequency response

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau) e^{-j\omega\tau} d\tau$$

generic $\quad z(t) = e^{-at} u(t)$

$$Z(j\omega) = \int_{-\infty}^{+\infty} e^{-a\tau} e^{-j\omega\tau} u(\tau) d\tau \qquad a>0$$

$$= \int_0^{\infty} e^{-(a+j\omega)\tau} d\tau$$

$$= \int_0^{+\infty} \frac{1}{-(a+j\omega)} \left[e^{-(a+j\omega)\infty} - 1 \right]$$

$$= \frac{1}{a+j\omega} = \frac{1}{\sqrt{a^2+\omega^2}} e^{-j\theta(\omega)}$$

$\sqrt{8} = \sqrt{\cdot}$

$$\theta(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$$

$$H_1(j\omega) = \frac{5}{\sqrt{1+\omega^2}} e^{-j\tan^{-1}(\omega)}$$

$$H_2(j\omega) = \frac{-16}{\sqrt{4+\omega^2}} e^{-j\tan^{-1}\left(\frac{\omega}{2}\right)}$$

$$\frac{16\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = 4\sqrt{2}$$

$$H_3(j\omega) = \frac{13}{\sqrt{9+\omega^2}} e^{-j\tan^{-1}\left(\frac{\omega}{3}\right)}$$

$\omega = 2$

$$y(t) = \frac{5}{\sqrt{1+4}} \cos\left(2t - \tan^{-1}(2)\right) \cancel{+} \frac{16}{\sqrt{4+4}}$$

$$\cos\left(2t - \tan^{-1}(1)\right) + \frac{13}{\sqrt{13}} \cos\left(2t - \tan^{-1}\left(\frac{2}{3}\right)\right)$$

$$= \frac{5}{\sqrt{5}}$$

$$= \sqrt{5}\cos(2t - 63.43^\circ) \cancel{+} \frac{16}{2\sqrt{2}} \cos(2t - 45^\circ)$$

$$+ \sqrt{13}\cos(2t - 33.7^\circ) - 4\sqrt{2}\cos(2t - 45^\circ)$$

$$y(t) = \sqrt{5}\cos(2t - 63.43^\circ) - 4\sqrt{2}\cos(2t - 45^\circ)$$

$$+ \sqrt{13}\cos(2t - 33.7^\circ)$$

Forgot $x(t) = 7\cos(2t)$

multiply everything by 7

so

$$y(t) = 7\sqrt{5}\cos(2t - 63.43^\circ) - 28\sqrt{2}\cos(2t - 45^\circ)$$

$$+ 7\sqrt{13}\cos(2t - 33.7^\circ)$$

Problem 4

$$z = (-4-3j)^{\frac{2}{5}}$$

$$z^5 = (-4-3j)^2 = (4+3j)^2$$

$4+3j$ in polar form.

$$|4+3j| = \sqrt{16+9} = 5$$

$$\angle 4+3j = \tan^{-1}\left(\frac{3}{4}\right) = 36.869.$$

$$(4+3j) = 5e^{j36.869^\circ}$$

$$(4+3j)^2 = 25e^{j73.74^\circ}$$

therefore $z^5 = \cancel{2}\,25e^{j73.74^\circ}$

Let

$$z = re^{j\theta} \Rightarrow z^5 = r^5 e^{j5\theta} = 25e^{j73.74}$$

$$r^5 = 25 \Rightarrow r = 1.903$$

$$5\theta = 73.74 + 360k$$

$$\theta = \frac{73.74}{5} + 72k \quad k = 0, 1, 2, 3, 4$$

$$= 14.75^\circ + 72k$$

$k=0 \quad \theta_1 = 14.75^\circ$

$z_1 = 1.903(\cos 14.75 + j\sin(14.75))$

$1.903(0.967 + j\,0.2545) = 1.840\; j\,0.484$

$k = 1 \quad \theta_2 = 86.75^0$

$z_2 = 1.903(0.0567 + j\,0.998) = 0.108 + j1.899$

$k = 2 \quad \theta_3 = 158.75$

$z_3 = 1.903(\cos 158.75 + j \sin 158.75)$

$= 1.903(-0.932 + j\,0.3624)$

$= -1.773 + j\,0.689$

$k = 3 \quad \theta_4 = -129.25$

$z_4 = 1.903(-0.6327 - j0.774)$

$-1.204 - j\,1.473$

$k = 4 \quad \theta_5 = -57.25$

$z_5 = 1.903(0.541 - j.841)$

$= 1.03 - j1.6$