Solution HW1

2. $|-0.2i| = |-0.2|\,|i|$

$= 0.2 \times 1 = \underline{\underline{0.2}}$

$|-0.2| = \sqrt{(-0.2)^2 + 0^2} = 0.2$

$|i| = \sqrt{0^2 + 1^2} = 1$

5. $|\cos\theta + i\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta} = \underline{\underline{1}}$

6. $\left|\frac{\bar{z}}{z}\right| = \frac{|\bar{z}|}{|z|}$ if $z = re^{j\theta}$
$\bar{z} = re^{-j\theta}$

and $|z| = r$, $|\bar{z}| = r$ therefore

$\left|\frac{\bar{z}}{z}\right| = \frac{|\bar{z}|}{|z|} = \frac{r}{r} = \underline{\underline{1}}$

7. $\left|\frac{5+7i}{7-5i}\right| = \frac{|5+7i|}{|7-5i|} = \frac{\sqrt{(5)^2+7^2}}{\sqrt{7^2+(-5)^2}} = \underline{\underline{1}}$

9. $\left|\frac{(1+i)^6}{i^3(1+4i)^2}\right| = \frac{|(1+i)^6|}{|i^3(1+4i)^2|} = \frac{|(1+i)^6|}{|i^3|\,|(1+4i)^2|}$

$1+i = \sqrt{2}\,e^{j45^\circ}$ $\quad (1+i)^6 = 2^3 e^{j45^\circ \cdot 6}$

$|(1+i)^6| = 2^3 = 8$

$|i^3| = |i|^3 = 1^3 = 1$

$1+4i = \sqrt{1+16}\,e^{j\theta} \Rightarrow (1+4i)^2 = 17e^{j2\theta}$

so $|(1+4i)^2| = 17$

finally. $\frac{|(1+i)^6|}{|i^3|\,|(1+4i)^2|} = \frac{8}{1\times 17} = \underline{\underline{\frac{8}{17}}}$

22 -

$4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ $\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \underline{\underline{2 + i\,2\sqrt{3}}}$

23. $2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

so $2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \underline{\underline{-2 + 2i}}$

24.

$10(\cos .4 + i\sin .4)$ $\cos .4 = 0.921$

$\sin .4 = 0.389$

$10(\cos .4 + i\sin .4) = \underline{\underline{9.21 + i\,3.89}}$

25.

$\cos(-1.8) + i\sin(-1.8) = \cos(+1.8) - i\sin(1.8)$

(cos is even and sin is odd

$\cos(1.8) = -0.227$

$\sin(1.8) = 0.974$

so $\cos(-1.8) + i\sin(-1.8) = \underline{\underline{-0.227 - i\,0.974}}$

26. $z = \sqrt{i} \iff z^2 = i$

$z = re^{j\theta} \Rightarrow z^2 = r^2 e^{j2\theta} = i = 1e^{j90}$

so $r^2 = 1$ and $2\theta = 90 + 360k$

$r^2 = 1 \Rightarrow r = 1$ and $\theta = 45° + \frac{\cancel{360}k}{2}$

$45° + 180°k$ $k = 0, 1$

$k=0 \quad \theta_1 = 45^\circ$

$z_1 = 1\, e^{j45^\circ} = 1(\cos 45^\circ + j\sin 45^\circ)$

$= \underline{\underline{\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}}$

$k=1$

$\theta_2 = 45^\circ + 180^\circ = 225^\circ = -135^\circ$

$z_2 = 1\, e^{-j135^\circ} = 1(\cos(-135^\circ) + j\sin(-135^\circ))$

$= \underline{\underline{-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}}$

27

$z = \sqrt{-8i} \qquad z^2 = -8i = +8 \times -i$

but $-i = e^{-j90^\circ}$

so $z^2 = -8i = 8\, e^{-j90^\circ}$

~~$z^2 = r^2 e$~~ $\quad z = re^{j\theta}$

$z^2 = r^2 e^{j2\theta} = 8\, e^{-j90^\circ}$

so $r^2 = 8$ then $r = \sqrt{8} = 2\sqrt{2}$

and $2\theta = -90^\circ + 360k$

$\theta = -45^\circ + 180k \quad k = 0, 1$

$k=0 \quad \theta_1 = -45^\circ$

$z_1 = 2\sqrt{2}(\cos(-45^\circ) + j\sin(-45^\circ))$

$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$

$= \underline{\underline{2 - j2}}$

$k=1 \quad \theta_2 = -45^\circ + 180^\circ = 135^\circ$

$z_2 = 2\sqrt{2}(\cos 135^\circ + j\sin 135^\circ$

$= 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$

$= \underline{\underline{-2 + j2}} = -z_1$

28 - $z = \sqrt{-7-24i}$

$z^2 = -7-24i = 25e^{j\varphi}$

$\varphi = \tan^{-1}\left(\frac{-24}{-7}\right) = 73.74^\circ \pm 180 = -106.26$

$z^2 = 25e^{-j106.26^\circ}$

$z = re^{j\theta} \Rightarrow z^2 = r^2 e^{j2\theta} = 25e^{-j106.26^\circ}$

$r^2 = 25 \Rightarrow r = 5$

$2\theta = -106.26 + 360k$

$\theta = -53.13^\circ + 180k \quad k = 0, 1$

$k=0 \quad \theta_1 = -53.13^\circ$

$z_1 = 5(\cos(-53.13^\circ) + j\sin(-53.13^\circ))$

$= 3 - 4j$

$k=1 \quad \theta_2 = -53.13 + 180 = 106.26^\circ$

$z_2 = -z_1 = -3 + 4j$

29.

$z = \sqrt[8]{1} \Rightarrow z^8 = 1 = 1e^{j0^\circ}$

$z = re^{j\theta} \Rightarrow z^8 = r^8 e^{j8\theta} = 1e^{j0}$

so $r^8 = 1 \Rightarrow r = 1$

$8\theta = 0^\circ + 360^\circ k$

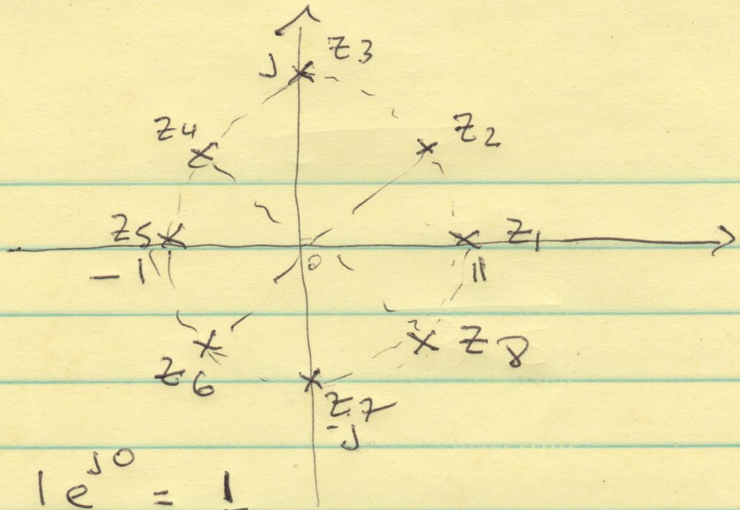
$\theta = 0^\circ + \frac{360}{8} k = 0, 1 \ldots 7$

$\theta = 0 + 45^\circ k^\circ \quad k = 0.$

$\theta_1 = 0^\circ \quad \theta_2 = 45, \ \theta_3 = 90^\circ, \ \theta_4 = 135^\circ \ \theta_5 = 180^\circ$

$\theta_6 = 225^\circ = -135^\circ, \ \theta_7 = 270^\circ = -90^\circ$

$\theta_8 = 315^\circ = -45^\circ$



$$z_1 = 1e^{j\theta_1} = 1e^{j0} = \underline{1}$$

$$z_2 = 1e^{j\theta_2} = 1e^{j45^\circ} = 1(\cos 45^\circ + j\sin 45^\circ) = \underline{\underline{\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}}$$

$$z_3 = 1e^{j\theta_3} = 1e^{j90^\circ} = \underline{\underline{j}}$$

$$z_4 = 1e^{j\theta_4} = 1e^{j135^\circ} = 1(\cos 135^\circ + j\sin 135^\circ)$$

$$= \underline{\underline{-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}}$$

$$z_5 = 1e^{j\theta_5} = 1e^{j180^\circ} = \underline{\underline{-1}} = -z_1$$

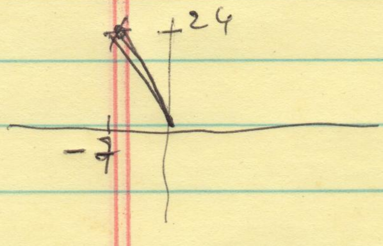
$$z_6 = 1e^{j\theta_6} = 1e^{-j135^\circ} = -z_2 = \underline{\underline{-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}}$$

$$z_7 = 1e^{j\theta_7} = 1e^{-j90^\circ} = -z_3 = \underline{\underline{-j}}$$

$$z_8 = 1e^{j\theta_8} = 1e^{-j45^\circ} = -z_4 = \underline{\underline{\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}}$$

<u>30</u>

$$z = \sqrt[4]{-7+24j} \Rightarrow z^4 = -7+24j$$



$-7+24j$ is in the second quadrant so the phase has to be between 90° and 180°

so $-7+24j = Re^{j\varphi}$

$R = 25$ like in pb 28

$\varphi = -73.74 + 180 = 106.26^\circ$

so $-7 + 24j = 25e^{j106.25}$

$z^4 = r^4 e^{j4\theta} = 25e^{j106.25}$

$r^4 = 25 \Rightarrow r = 2.236$

$4\theta = 106.25 + 360K$

$\theta = \frac{106.25}{4} + \frac{360K}{4}$

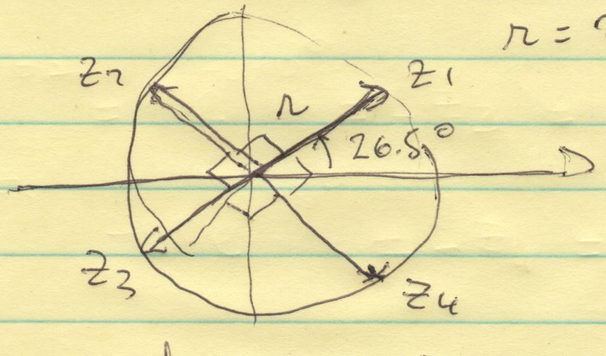
$= 26.56 + 90^\circ K \quad K = 0, 1, 2, 3$

$K = 0 \quad \theta_1 = 26.56$

$z_1 = 2.236(\cos(26.56) + j\sin(26.56))$

$= 2.236(0.894 + j\,0.447) = \cancel{2 + j}\ \underline{\underline{2 + j}}$

$r = 2.236$



$z_3 = -z_1$ and $z_4 = -z_2$

since $\cos(x + 90^\circ) = -\sin x$

an $\sin(x + 90^\circ) = \cos x$ we do not need to ~~re~~ calculate $\cos(26.56 + 90^\circ)$ and $\sin(26.56^\circ + 90^\circ)$

so $z_2 = \underline{-1 + 2j}$

$z_3 = -z_1 = \underline{\underline{-2 - j}}$ and $z_4 = -z_2 = \underline{\underline{1 - 2j}}$

31. $z = \sqrt[4]{-1}$

$z^4 = -1$

$z^4 + 1 = 0$ $\quad j^2 = -1 \Rightarrow 1 = -j^2$

$z^4 + 1 = z^4 - j^2 = 0 \Rightarrow (z^2 - j)(z^2 + j) = 0$

$z^2 = j$ or $z^2 = -j$

from problem 26 $z^2 = j$ has two solutions

$z_1 = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$, $z_2 = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

now $z^2 = -j$ $\quad z^2 = 1e^{-j90^\circ}$

$z = re^{j\theta}$

$z^2 = r^2 e^{j2\theta} = 1e^{-j90^\circ}$ $\quad r^2 = 1 \quad r = 1$

$2\theta = -90^\circ + 360k$

$\theta = -45^\circ + 180k \quad k = 0, 1$

$k = 0 \quad z_3 = 1(\cos(-45) + j\sin(-45^\circ))$

$= \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

and $z_4 = -z_3 = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

The solutions are

$z_1 = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$, $z_2 = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$, $z_3 = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$z_4 = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

32. $z = \sqrt[5]{-1} \Rightarrow z^5 = -1 = e^{j180^\circ}$

$z = re^{j\theta}$

$r^5 e^{j5\theta} = e^{j180^\circ}$ $\quad r^5 = 1 \Rightarrow r = 1$

$5\theta = 180^\circ + k360 \Rightarrow \theta = 36^\circ + k72^\circ$

$k = 0, 1, 2, 3, 4$

$k = 0 \quad \theta_1 = 36^\circ$

$z_1 = 1 e^{j36^\circ} = 1(\cos 36^\circ + j \sin 36^\circ)$

$= 0.809 + j\,0.588$

$k = 1$

$\theta_2 = 36 + 72 = 108$

$z_2 = 1(\cos 108^\circ + j \sin 108^\circ)$

$= -0.309 + j\,0.951$

$k = 2$

$\theta_3 = 108^\circ + 72^\circ = 180^\circ$

$z_3 = \cos 180^\circ + j \sin 180^\circ = -1$

$k = 3$

$\theta_4 = 180^\circ + 72 = -108^\circ$

$\cos(-108) = \cos(108) = -0.309$

$\sin(-108) = -\sin(108) = -0.951$

$z_4 = -0.309 - j\,0.951$

$k = 4$

$\theta_5 = -108 + 72 = -36^\circ$

$\cos(-36) = \cos(36) = 0.809$

$\sin(-36) = -\sin 36 = -0.588$

$z_5 = 0.809 - j\,0.588$

33.

$z = \sqrt[3]{1+j} \implies z^3 = 1 + j$

but $1 + j = \sqrt{2}\, e^{j45^\circ}$

so if $z = re^{j\theta} \quad z^3 = r^3 e^{j3\theta} = \sqrt{2}\, e^{j45^\circ}$

$r^3 = \sqrt{2} \implies r = \sqrt[6]{2} = 1.122$

$3\theta = 45 + 360k$

$\theta = \frac{45}{3} + 120k \quad k = 0, 1, 2$

$\theta = 15 + 120k \quad k = 0, 1, 2$

$k = 0 \quad \theta_1 = 15$

$z_1 = 1.122(\cos 15 + j\sin 15)$

$= 1.122(0.966 + j\,0.259)$

$= 1.08 + j\,0.290$

$k = 1$

$\theta_2 = 15 + 120 = 135^\circ$

$z_2 = 1.122\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$

$= -0.794 + j\,0.794$

$k = 2 \quad \theta_3 = 135 + 120 = 255 = -105^\circ$

$\cos(-105) = \cos(105) = -0.259$

$\sin(-105) = -\sin(105) = -0.966$

$z_3 = 1.122(-0.259 - j\,0.966)$

$= -0.290 - j\,1.08$