The generalized Fourier transform employs the Fourier transform of the impulse function. The presence of the impulse function serves to distinguish a generalized Fourier transform from the Fourier transform defined in the last section. Very often it is not important to discriminate between signals that have Fourier transforms and signals that have generalized Fourier transforms. On some occasions, however, this distinction is important.

7.6 CHAPTER SUMMARY

7.6.1 Fourier Series

In this chapter we have introduced the Fourier series as a representation of periodic signals.

Definition 7.1.1: The signal $x(t)$ is <u>periodic</u> if there exists a constant $T_0 > 0$ such that $x(t+T_0) = x(t)$ for all t. The smallest T_0 for which this is true is called the (<u>fundamental</u>) <u>period</u>. All other signals are <u>aperiodic</u>.

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The three different representations of the Fourier series are summarized in Table 7.2.1. The relationships between these representations are summarized in Table 7.6.1. The fundamental frequency of the these Fourier series is computed from the period of the signal.

There are two sets of computational formulas for the coefficients of the trigonometric and exponential Fourier series.

Table 7.6.1 Relationship Between the Three Fourier Series Representations

$x(t) = \sum_{m=0}^{\infty} A_m \cos(m\omega_0 t + \theta_m)$ $A_m > 0$	$a_0 = A_0 \cos\theta_0$ $a_m = A_m \cos\theta_m$ $b_m = -A_m \sin\theta_m$	$X_m = \frac{A_m}{2} e^{j\theta_m}, \ m > 0$ $X_0 = A_0, \quad m = 0$
$A_m = +\sqrt{a_m^2 + b_m^2}$ $\theta_m = -\tan^{-1}\left(\frac{b_m}{a_m}\right)$	$x(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega_0 t)$ $+ \sum_{m=1}^{\infty} b_m \sin(m\omega_0 t)$	$2\,\mathrm{Re}\,X_m = a_m$ $-2\,\mathrm{Im}\,X_m = b_m$ $m > 0$
$A_m = 2\lvert X_m \rvert, \quad m > 0$ $\theta_m = \angle X_m, \quad m > 0$ $A_0 = X_0, \quad m = 0$	$2\,\mathrm{Re}\,X_m = a_m$ $-2\,\mathrm{Im}\,X_m = b_m \quad m > 0$	$x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}$