Homework solution

Signals & Systems

1) Find the Laplace Transforms of the following signals

a) $2u(t-10) + 10\,\delta(t-1)$

$$u(t) \longrightarrow \frac{1}{s}$$

$$u(t-10) \longrightarrow \frac{e^{-10s}}{s}$$

$$\delta(t) \longrightarrow 1 \qquad \delta(t-1) \longrightarrow e^{-s}$$

$$F(s) = 2\frac{e^{-10s}}{s} + 10e^{-s}$$

b) $(t-1)e^{-a(t-1)}\,u(t-1)$

$g(t) = t\,f(t)$ where $f(t) = e^{-at}u(t)$

$$G(s) = -\frac{dF(s)}{ds} = \frac{1}{(s+a)^2} \qquad F(s) = \frac{1}{s+a}$$

$$g(t) \longrightarrow G(s) \qquad g(t-1) \longrightarrow e^{-s}G(s)$$

$$(t-1)e^{-a(t-1)}u(t-1) \longrightarrow e^{-s}\frac{1}{(s+a)^2}$$

c) $e^{-3t}u(t) + \frac{t^5}{5!}e^{-10t}u(t) + u(t-5)$

(table)

$$\frac{1}{s+3} + \frac{1}{(s+10)^6} + \frac{e^{-5s}}{s}$$

2) Consider the following system represented by the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = u(t)$$

$$y(0^-) = 2 \quad y'(0^-) = 0$$

find $y(t)$ using Lapla Transform

$$y(t) \longrightarrow Y(s)$$

$$\frac{dy(t)}{dt} \longrightarrow sY(s) - y(0^-)$$

$$\frac{d^2y}{dt^2} \longrightarrow s^2Y(s) - sy(0^-) - y'(0^-)$$

Therefore

$$\left[s^2Y(s) - sY(0^-) - y'(0)\right] + 3\left[sY(s) - y(0^-)\right]$$

$$+ \; 2Y(s) = \frac{1}{s}$$

$$s^2Y(s) - 2s + 3sY(s) - 6 + 2Y(s) = \frac{1}{s}$$

$$(s^2 + 3s + 2)Y(s) = 2s + 6 + 1/s$$

$$Y(s) = \frac{2s + 6 + 1/s}{s^2 + 3s + 2} = \frac{2s^2 + 6s + 1}{s(s^2 + 3s + 2)}$$

$$s^2 + 3s + 2 = (s+1)(s+2)$$

$$Y(s) = \frac{2s^2 + 6s + 1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A = \left.\frac{2s^2 + 6s + 1}{(s+1)(s+2)}\right|_{s=0} = \frac{1}{2}$$

$$B = \left.\frac{2s^2 + 6s + 1}{s(s+2)}\right|_{s=-1} = \frac{2 - 6 + 1}{-1 \times 1} = 3$$

$C = -3/2$ $\quad -3/2 =$ $\quad C = \left.\frac{2s^2 + 6s + 1}{s(s+1)}\right|_{s=-2} = \frac{8 - 12 + 1}{-2 \times -1} = \frac{-3}{+2}$

$$Y(s) = \frac{1}{2}\frac{1}{s} + \frac{3}{s+1} - \frac{3}{2}\frac{1}{s+2}$$

$$y(t) = \frac{1}{2}u(t) + 3e^{-t}u(t) - \frac{3}{2}e^{-2t}u(t)$$

3) Given the following Laplace Transforms for some signals, what are the time domain signals

a) $$\frac{s}{s^2+7s+10} = \frac{s}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$A = \left.\frac{s}{s+5}\right|_{s=-2} = \frac{-2}{3} = -\frac{2}{3}$$

$$B = \left.\frac{s}{s+2}\right|_{s=-5} = \frac{-5}{-3} = \frac{5}{3}$$

$f(t)$ $\cancel{\frac{s}{s^2+7s+10}} = \frac{5}{3}e^{5t}$

$$F(s) = \frac{5}{3}\frac{1}{s+5} - \frac{2}{3}\frac{1}{s+2}$$

$$f(t) = \frac{5}{3}e^{-5t}u(t) - \frac{2}{3}e^{-2t}u(t)$$

c) $$G(s) = \frac{s^2}{s^2+7s+10} = sF(s)$$

$$F(s) = \frac{s}{s^2+7s+10}$$ therefore $g(t) = \mathcal{L}^{-1}[G(s)] = \frac{df(t)}{dt}$

$\cancel{g(t) = \frac{4}{3}e}$

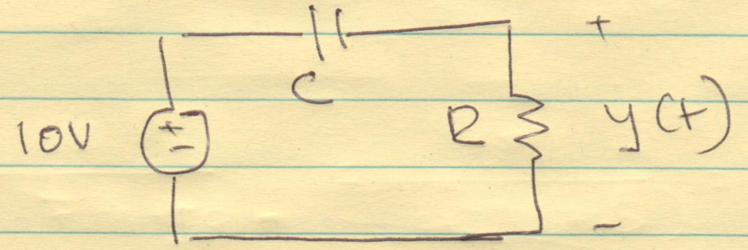
$$g(t) = \frac{df(t)}{dt} = \frac{5}{3}e^{-5t}\delta(t) - \frac{25}{3}e^{-5t}u(t) - \frac{2}{3}e^{-2t}\delta(t) + \frac{4}{3}e^{-2t}u(t)$$

$$= \delta(t) + \frac{4}{3}e^{-2t}u(t) - \frac{25}{3}e^{-5t}u(t)$$

b) $$\frac{1}{s^2+4s+4} = \frac{1}{(s+2)^2} = F(s)$$

using table $f(t) = t\,e^{-2t}\,u(t)$

4) Consider the circuit below



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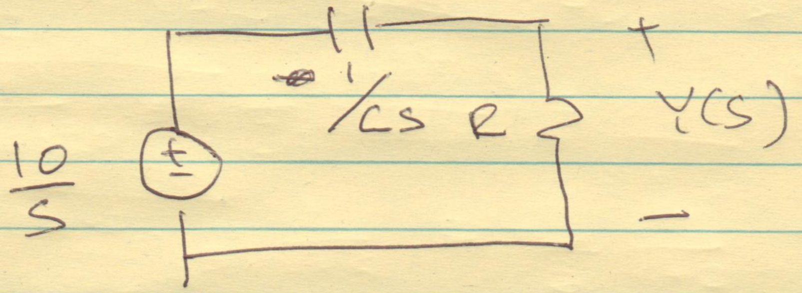
Laplace Domain Circuit



$$Y(S) = \frac{R}{\frac{1}{CS} + R}\,\frac{10}{S}$$

$$= \frac{RCS}{1 + RCS}\,\frac{10}{S} = \frac{10}{S + \frac{1}{RC}}$$

$$y(t) = 10\,e^{-\frac{t}{RC}}\,u(t)$$

The current is $\frac{y(t)}{R} = \frac{10}{R}\,e^{-\frac{t}{RC}}\,u(t)$