EEL 3135.
Signals and systems
Test #2 Summer 2013

Solve Problem 1 or Problem 2

Problems 3, 4, and 5 are mandatory

Problem 6 is a bonus problem

All needed documents for this test are attached.

Closed book, closed notes, no laptop, no cell phones, no tablets. Calculators are allowed

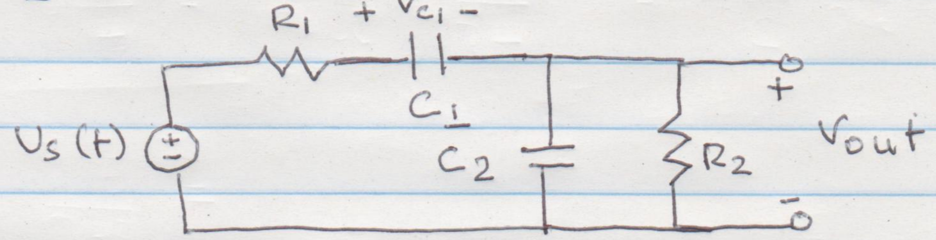
EEL 3135

Signals and Systems

Test # 2 Summer 2013

Problem 1

Using Laplace Transform method, determine $v_{out}(t)$ in the circuit shown below, given that $v_s(t) = 35u(t)$ V, $V_{C_1}(0^-) = 20$ V, $R_1 = 1\,\Omega$, $C_1 = 1$ F, $R_2 = 0.5\,\Omega$, and $C_2 = 2$ F



Problem 2

An LTI system is described by the following differential Equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = \frac{dx}{dt} + 2x$$

Using Laplace Transform method, find $y(t)$ for $x(t) = e^{-2t}u(t)$ $\quad y(0^-) = 1$, $y'(0^-) = 0$

Problem 3

Let $x[n] = u[n] - u[n-3]$ and

$y[n] = n\left[u[n] - u[n-3]\right]$

a) Plot $x[n]$ and $y[n]$

b) Compute graphically $z[n] = x[n] * y[n]$

c) Calculate the energy of $z[n]$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}, \quad u[n-n_0] = \begin{cases} 1 & n \geq n_0 \\ 0 & n < n_0 \end{cases}$$

Problem 4

Let $x(t)$ be a periodic signal with period T_0

such that $x(t) = \sin \omega_0 t$ for $0 \le t \le \frac{T_0}{2}$
and $x(t) = 0$ for $\frac{T_0}{2} \le t \le T_0$

$\omega_0 = \frac{2\pi}{T_0}$

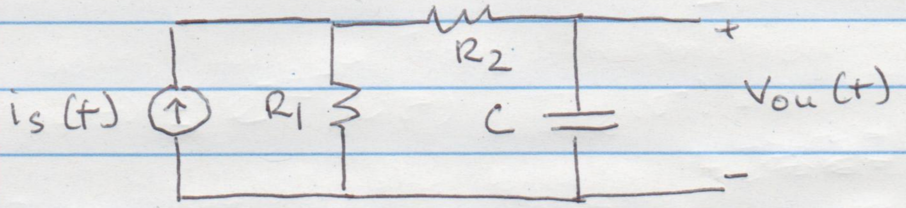
a) Plot $x(t)$

b) Find the Trigonometric Fouriers series coefficients

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

Problem 5

For the circuit shown below



where $i_s(t) = u(t)$, $R_1 = 500\,\Omega$, $R_2 = 2\,K\Omega$, and $C = 0.33 \times 10^{-6}$ F, find $v_{out}(t)$ using Fourier Transform.

Problem 6 (Bonus)

Let $x[n] = u[n]$ and $y[n] = \left(\frac{1}{2}\right)^n u[n]$

Find $z[n] = x[n] * y[n]$ using Z-transform

note: $Z[x[n] * y[n]] = Z[x[n]]\, Z[y[n]]$

Table 3-1: Properties of the Laplace transform for causal functions; i.e., $x(t) = 0$ for $t < 0^-$.

Property	$x(t)$		$\mathbf{X}(\mathbf{s}) = \mathcal{L}[x(t)]$
1. Multiplication by constant	$K\,x(t)$	↔	$K\,\mathbf{X}(\mathbf{s})$
2. Linearity	$K_1\,x_1(t) + K_2\,x_2(t)$	↔	$K_1\,\mathbf{X}_1(\mathbf{s}) + K_2\,\mathbf{X}_2(\mathbf{s})$
3. Time scaling	$x(at), \quad a > 0$	↔	$\frac{1}{a}\,\mathbf{X}\left(\frac{\mathbf{s}}{a}\right)$
4. Time shift	$x(t-T)\,u(t-T),\ T \geq 0$	↔	$e^{-T\mathbf{s}}\,\mathbf{X}(\mathbf{s})$
5. Frequency shift	$e^{-at}\,x(t)$	↔	$\mathbf{X}(\mathbf{s}+a)$
6. Time 1st derivative	$x' = \frac{dx}{dt}$	↔	$\mathbf{s}\,\mathbf{X}(\mathbf{s}) - x(0^-)$
7. Time 2nd derivative	$x'' = \frac{d^2x}{dt^2}$	↔	$\mathbf{s}^2\mathbf{X}(\mathbf{s}) - \mathbf{s}\,x(0^-) - x'(0^-)$
8. Time integral	$\int_{0^-}^{t} x(t')\,dt'$	↔	$\frac{1}{\mathbf{s}}\,\mathbf{X}(\mathbf{s})$
9. Frequency derivative	$t\,x(t)$	↔	$-\frac{d}{d\mathbf{s}}\,\mathbf{X}(\mathbf{s}) = -\mathbf{X}'(\mathbf{s})$
10. Frequency integral	$\frac{x(t)}{t}$	↔	$\int_{\mathbf{s}}^{\infty} \mathbf{X}(\mathbf{s}')\,d\mathbf{s}'$
11. Initial value	$x(0^+)$	=	$\lim_{\mathbf{s}\to\infty} \mathbf{s}\,\mathbf{X}(\mathbf{s})$
12. Final value	$\lim_{t\to\infty} x(t) = x(\infty)$	=	$\lim_{\mathbf{s}\to 0} \mathbf{s}\,\mathbf{X}(\mathbf{s})$
13. Convolution	$x_1(t) * x_2(t)$	↔	$\mathbf{X}_1(\mathbf{s})\,\mathbf{X}_2(\mathbf{s})$

Example 3-6: Laplace Transform

Obtain the Laplace transform of

$$x(t) = t^2 e^{-3t} \cos(4t)\,u(t).$$

Solution:

The given function is a product of three functions. We start with the cosine function which we will call $x_1(t)$:

$$x_1(t) = \cos(4t)\,u(t). \tag{3.39}$$

According to entry #11 in Table 3-2, the corresponding Laplace transform is

$$\mathbf{X}_1(\mathbf{s}) = \frac{\mathbf{s}}{\mathbf{s}^2 + 16}. \tag{3.40}$$

Next we define

$$x_2(t) = e^{-3t} \cos(4t)\,u(t) = e^{-3t}\,x_1(t), \quad (3.4$$

and we apply the frequency shift property (entry #5 in Table 3-1) to obtain

$$\mathbf{X}_2(\mathbf{s}) = \mathbf{X}_1(\mathbf{s}+3) = \frac{\mathbf{s}+3}{(\mathbf{s}+3)^2 + 16}, \quad (3.4$$

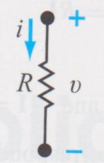
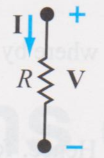
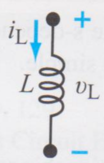
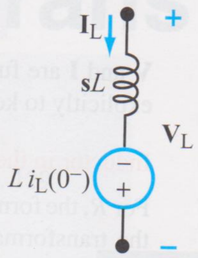
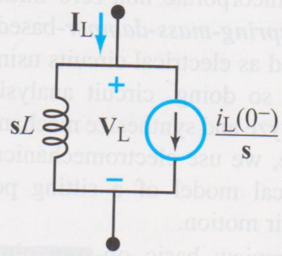
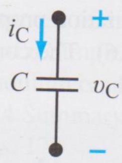
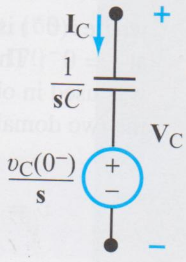
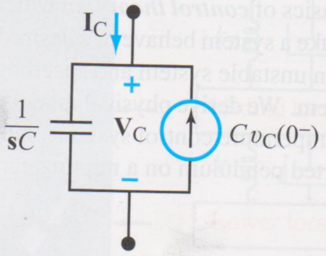
where we replaced s with (s + 3) everywhere in the expression of Eq. (3.40). Finally, we define

$$x(t) = t^2\,x_2(t) = t^2 e^{-3t} \cos(4t)\,u(t), \quad (3.4$$

Table 3-2: Examples of Laplace transform pairs. Note that $x(t) = 0$ for $t < 0^-$ and $T \geq 0$.

	Laplace Transform Pairs		
	$x(t)$		$\mathbf{X}(s) = \mathcal{L}[x(t)]$
1	$\delta(t)$	↔	1
1a	$\delta(t-T)$	↔	e^{-Ts}
2	$u(t)$	↔	$\frac{1}{s}$
2a	$u(t-T)$	↔	$\frac{e^{-Ts}}{s}$
3	$e^{-at}\,u(t)$	↔	$\frac{1}{s+a}$
3a	$e^{-a(t-T)}\,u(t-T)$	↔	$\frac{e^{-Ts}}{s+a}$
4	$t\,u(t)$	↔	$\frac{1}{s^2}$
4a	$(t-T)\,u(t-T)$	↔	$\frac{e^{-Ts}}{s^2}$
5	$t^2\,u(t)$	↔	$\frac{2}{s^3}$
6	$te^{-at}\,u(t)$	↔	$\frac{1}{(s+a)^2}$
7	$t^2e^{-at}\,u(t)$	↔	$\frac{2}{(s+a)^3}$
8	$t^{n-1}e^{-at}\,u(t)$	↔	$\frac{(n-1)!}{(s+a)^n}$
9	$\sin(\omega_0 t)\,u(t)$	↔	$\frac{\omega_0}{s^2+\omega_0^2}$
10	$\sin(\omega_0 t+\theta)\,u(t)$	↔	$\frac{s\sin\theta+\omega_0\cos\theta}{s^2+\omega_0^2}$
11	$\cos(\omega_0 t)\,u(t)$	↔	$\frac{s}{s^2+\omega_0^2}$
12	$\cos(\omega_0 t+\theta)\,u(t)$	↔	$\frac{s\cos\theta-\omega_0\sin\theta}{s^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)\,u(t)$	↔	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$
14	$e^{-at}\cos(\omega_0 t)\,u(t)$	↔	$\frac{s+a}{(s+a)^2+\omega_0^2}$
15	$2e^{-at}\cos(bt-\theta)\,u(t)$	↔	$\frac{e^{j\theta}}{s+a+jb}+\frac{e^{-j\theta}}{s+a-jb}$
15a	$e^{-at}\cos(bt-\theta)\,u(t)$	↔	$\frac{(s+a)\cos\theta+b\sin\theta}{(s+a)^2+b^2}$
16	$\frac{2t^{n-1}}{(n-1)!}\,e^{-at}\cos(bt-\theta)\,u(t)$	↔	$\frac{e^{j\theta}}{(s+a+jb)^n}+\frac{e^{-j\theta}}{(s+a-jb)^n}$

Table 4-1: Circuit models for R, L, and C in the s-domain.

Time-Domain	s-Domain	
Resistor  $\upsilon = Ri$	 $\mathbf{V} = R\mathbf{I}$	
Inductor  $\upsilon_L = L\,\dfrac{di_L}{dt}$ $i_L = \dfrac{1}{L}\displaystyle\int_{0^-}^{t} \upsilon_L\,dt' + i_L(0^-)$	 $\mathbf{V}_L = sL\mathbf{I}_L - L\,i_L(0^-)$	**OR**  $\mathbf{I}_L = \dfrac{\mathbf{V}_L}{sL} + \dfrac{i_L(0^-)}{s}$
Capacitor  $i_C = C\,\dfrac{d\upsilon_C}{dt}$ $\upsilon_C = \dfrac{1}{C}\displaystyle\int_{0^-}^{t} i_C\,dt' + \upsilon_C(0^-)$	 $\mathbf{V}_C = \dfrac{\mathbf{I}_C}{sC} + \dfrac{\upsilon_C(0^-)}{s}$	**OR**  $\mathbf{I}_C = sC\mathbf{V}_C - C\,\upsilon_C(0^-)$

where $\upsilon(0^-)$ is the initial voltage across the capacitor. The s-domain circuit models for the capacitor are available in Table 4-1.

Impedances $\mathbf{Z}_R$, $\mathbf{Z}_L$, and $\mathbf{Z}_C$ are defined in the s-domain in terms of voltage to current ratios, under zero initial conditions $[i(0^-) = \upsilon(0^-) = 0]$:

$$\mathbf{Z}_R = R, \qquad \mathbf{Z}_L = sL, \qquad \mathbf{Z}_C = \frac{1}{sC}.$$

Table 5-3: Fourier series representations for a real-valued periodic function $x(t)$.

Cosine/Sine	Amplitude/Phase	Complex Exponential
$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$	$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n)$	$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t}$
$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt$	$c_n e^{j\phi_n} = a_n - jb_n$	$\mathbf{x}_n = \lvert\mathbf{x}_n\rvert e^{j\phi_n};\ \mathbf{x}_{-n} = \mathbf{x}_n^*;\ \phi_{-n} = -\phi_n$
$a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cos n\omega_0 t\, dt$	$c_n = \sqrt{a_n^2 + b_n^2}$	$\lvert\mathbf{x}_n\rvert = c_n/2;\ \mathbf{x}_0 = c_0$
$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \sin n\omega_0 t\, dt$	$\phi_n = \begin{cases} \tan^{-1}(b_n/a_n), & a_n > 0 \\ \pi - \tan^{-1}(b_n/a_n), & a_n < 0 \end{cases}$	$\mathbf{x}_n = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$
$a_0 = c_0 = \mathbf{x}_0;\ a_n = c_n \cos\phi_n;\ b_n = -c_n \sin\phi_n;\ \mathbf{x}_n = \frac{1}{2}(a_n - jb_n)$		

The exponential representation given by Eq. (5.44) is characterized by ***two-sided line spectra*** because in addition to the dc value at $\omega = 0$, the magnitudes $|\mathbf{x}_n|$ and phases ϕ_n are defined at both positive and negative values of ω_0 and its harmonics.

It is easy to understand what a positive value of the angular frequency ω means, but what does a negative angular frequency mean? It does not have a physical meaning; defining ω along the negative axis is purely a mathematical convenience.

The single-sided line spectra of the amplitude/phase representation given by Eq. (5.39) and the two-sided exponential representation given by Eq. (5.44) are interrelated. For a real-valued periodic function $x(t)$ with fundamental angular frequency ω_0:

(a) The ***amplitude line spectrum*** is a plot of amplitudes c_n at $\omega = 0$ (dc value) and at positive, discrete values of ω, namely $n\omega_0$ with n a positive integer. The outcome is a single-sided spectrum similar to that displayed in Fig. 5-5(a).

(b) The ***magnitude line spectra*** is a plot of magnitudes $|\mathbf{x}_n|$ at $\omega = 0$ (dc value) and at both positive and negative values of ω_0 and its harmonics. The outcome is a two-sided spectrum as shown in Fig. 5-5(b).

(c) The two-sided magnitude spectrum has even symmetry about $n = 0$ (i.e., $|\mathbf{x}_n| = |\mathbf{x}_{-n}|$), and the magnitudes of $\mathbf{x}_n$ are half the corresponding amplitudes c_n. That is, $|\mathbf{x}_n| = c_n/2$, except for $n = 0$ in which case $|\mathbf{x}_0| = |c_0|$.

(d) The ***phase line spectrum*** ϕ_n is single-sided, whereas the phase line spectrum of $\mathbf{x}_n$ is two-sided. The right half of the two-sided phase spectrum [Fig. 5-5(d)] is identical with the one-sided phase spectrum of the amplitude/phase representation [Fig. 5-5(c)].

(e) The two-sided phase spectrum has odd symmetry about $n = 0$ (because $\phi_{-n} = -\phi_n$), as illustrated by Fig. 5-5(d). However, if $a_0 < 0$, then $\phi_0 = \pi$, in which case ϕ_n would no longer be an odd function of n.

Exercise 5-3: A periodic signal $x(t)$ has the complex exponential Fourier series

$$x(t) = (-2 + j0) + (3 + j4)e^{j2t} + (1 + j)e^{j4t} + (3 - j4)e^{-j2t} + (1 - j)e^{-j4t}.$$

Compute its cosine/sine and amplitude/phase Fourier series representations.

Answer: Amplitude/phase representation:

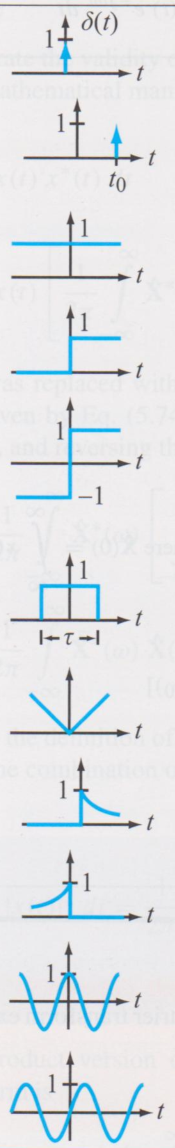
$$x(t) = -2 + 10\cos(2t + 53°) + 2\sqrt{2}\cos(4t + 45°),$$

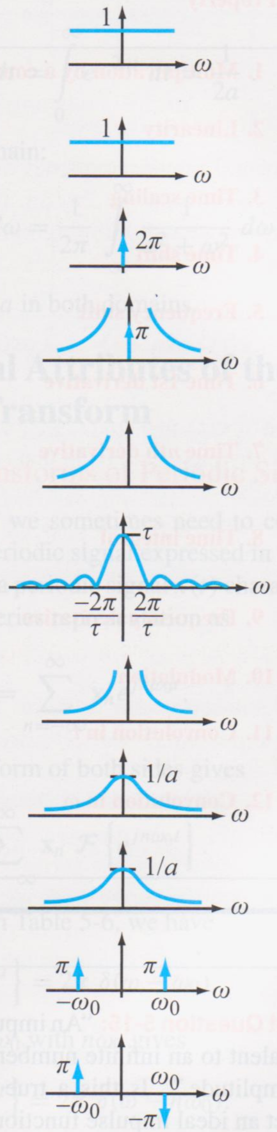
Cosine/sine representation:

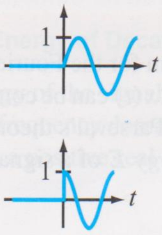
$$x(t) = -2 + 6\cos(2t) - 8\sin(2t) + 2\cos(4t) - 2\sin(4t).$$

(See)

Table 5-6: Examples of Fourier transform pairs. Note that constant $a \geq 0$.







	$x(t)$		$\mathbf{X}(\omega) = \mathcal{F}[x(t)]$
	BASIC FUNCTIONS		
1.	$\delta(t)$	$\leftrightarrow$	1
1a.	$\delta(t - t_0)$	$\leftrightarrow$	$e^{-j\omega t_0}$
2.	1	$\leftrightarrow$	$2\pi\ \delta(\omega)$
3.	$u(t)$	$\leftrightarrow$	$\pi\ \delta(\omega) + 1/j\omega$
4.	$\mathrm{sgn}(t)$	$\leftrightarrow$	$2/j\omega$
5.	$\mathrm{rect}(t/\tau)$	$\leftrightarrow$	$\tau\ \mathrm{sinc}(\omega\tau/2)$
6.	$\lvert t \rvert$	$\leftrightarrow$	$-2/\omega^2$
7a.	$e^{-at}\ u(t)$	$\leftrightarrow$	$1/(a + j\omega)$
7b.	$e^{at}\ u(-t)$	$\leftrightarrow$	$1/(a - j\omega)$
8.	$\cos\omega_0 t$	$\leftrightarrow$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
9.	$\sin\omega_0 t$	$\leftrightarrow$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
	ADDITIONAL FUNCTIONS		
10.	$e^{j\omega_0 t}$	$\leftrightarrow$	$2\pi\ \delta(\omega - \omega_0)$
11.	$te^{-at}\ u(t)$	$\leftrightarrow$	$1/(a + j\omega)^2$
12a.	$[e^{-at}\sin\omega_0 t]\ u(t)$	$\leftrightarrow$	$\omega_0/[(a + j\omega)^2 + \omega_0^2]$
12b.	$[\sin\omega_0 t]\ u(t)$	$\leftrightarrow$	$(\pi/2j)[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + [\omega_0^2/(\omega_0^2 - \omega^2)]$
13a.	$[e^{-at}\cos\omega_0 t]\ u(t)$	$\leftrightarrow$	$(a + j\omega)/[(a + j\omega)^2 + \omega_0^2]$
13b.	$[\cos\omega_0 t]\ u(t)$	$\leftrightarrow$	$(\pi/2)[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + [j\omega/(\omega_0^2 - \omega^2)]$

Table 5-7: Major properties of the Fourier transform.

Property	$x(t)$		$\hat{\mathbf{X}}(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$
1. Multiplication by a constant	$K\, x(t)$	$\leftrightarrow$	$K\, \hat{\mathbf{X}}(\omega)$
2. Linearity	$K_1\, x_1(t) + K_2\, x_2(t)$	$\leftrightarrow$	$K_1\, \hat{\mathbf{X}}_1(\omega) + K_2\, \hat{\mathbf{X}}_2(\omega)$
3. Time scaling	$x(at)$	$\leftrightarrow$	$\frac{1}{\lvert a\rvert}\, \hat{\mathbf{X}}\left(\frac{\omega}{a}\right)$
4. Time shift	$x(t - t_0)$	$\leftrightarrow$	$e^{-j\omega t_0}\, \hat{\mathbf{X}}(\omega)$
5. Frequency shift	$e^{j\omega_0 t}\, x(t)$	$\leftrightarrow$	$\hat{\mathbf{X}}(\omega - \omega_0)$
6. Time 1st derivative	$x' = \frac{dx}{dt}$	$\leftrightarrow$	$j\omega\, \hat{\mathbf{X}}(\omega)$
7. Time *n*th derivative	$\frac{d^n x}{dt^n}$	$\leftrightarrow$	$(j\omega)^n\, \hat{\mathbf{X}}(\omega)$
8. Time integral	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\leftrightarrow$	$\frac{\hat{\mathbf{X}}(\omega)}{j\omega} + \pi\, \hat{\mathbf{X}}(0)\, \delta(\omega)$, where $\hat{\mathbf{X}}(0) = \int_{-\infty}^{\infty} x(t)\, dt$
9. Frequency derivative	$t^n\, x(t)$	$\leftrightarrow$	$(j)^n \frac{d^n \hat{\mathbf{X}}(\omega)}{d\omega^n}$
10. Modulation	$x(t)\, \cos\omega_0 t$	$\leftrightarrow$	$\frac{1}{2}[\hat{\mathbf{X}}(\omega - \omega_0) + \hat{\mathbf{X}}(\omega + \omega_0)]$
11. Convolution in *t*	$x_1(t) * x_2(t)$	$\leftrightarrow$	$\hat{\mathbf{X}}_1(\omega)\, \hat{\mathbf{X}}_2(\omega)$
12. Convolution in ω	$x_1(t)\, x_2(t)$	$\leftrightarrow$	$\frac{1}{2\pi}\, \hat{\mathbf{X}}_1(\omega) * \hat{\mathbf{X}}_2(\omega)$
13. Conjugate symmetry			$\hat{\mathbf{X}}(-\omega) = \hat{\mathbf{X}}^*(\omega)$

Concept Question 5-15: "An impulse in the time domain is equivalent to an infinite number of sinusoids, all with equal amplitude." Is this a true statement? Can one construct an ideal impulse function?

Exercise 5-7: Use the entries in Table 5-6 to determine the Fourier transform of $u(-t)$.

Answer: $\hat{\mathbf{X}}(\omega) = \pi\, \delta(\omega) - 1/j\omega$. (See)

Exercise 5-8: Verify the Fourier transform expression fo entry #10 in Table 5-6.

Answer: (See)

5-9 Parseval's Theorem for Fourier Transforms

Recall that Parseval's theorem for the Fourier series state the *average power* of a signal $x(t)$ can be computed in eit time or frequency domains. Parseval's theorem for the F transform states that the *energy* E of a signal can be com

Table 7-5: Examples of z-transform pairs. ROC stands for region of convergence (validity) in the z-plane.

z-Transform Pairs

	$x[n]$		$\mathbf{X}(\mathbf{z}) = \mathbb{Z}[x[n]]$	ROC
1	$\delta[n]$	⟷	1	All $\mathbf{z}$
1a	$\delta[n-m]$	⟷	$\mathbf{z}^{-m}$, if $m \geq 0$	$\mathbf{z} \neq 0$
2	$u[n]$	⟷	$\dfrac{\mathbf{z}}{\mathbf{z}-1}$	$\lvert\mathbf{z}\rvert > 1$
3	$n\,u[n]$	⟷	$\dfrac{\mathbf{z}}{(\mathbf{z}-1)^2}$	$\lvert\mathbf{z}\rvert > 1$
3a	$n^2\,u[n]$	⟷	$\dfrac{\mathbf{z}(\mathbf{z}+1)}{(\mathbf{z}-1)^3}$	$\lvert\mathbf{z}\rvert > 1$
4	$a^n\,u[n]$	⟷	$\dfrac{\mathbf{z}}{\mathbf{z}-\mathbf{a}}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
4a	$a^{n-1}\,u[n-1]$	⟷	$\dfrac{1}{\mathbf{z}-\mathbf{a}}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
4b	$na^n\,u[n]$	⟷	$\dfrac{\mathbf{a}\mathbf{z}}{(\mathbf{z}-\mathbf{a})^2}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
4c	$(n-1)a^{n-2}\,u[n-2]$	⟷	$\dfrac{1}{(\mathbf{z}-\mathbf{a})^2}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
4d	$n^2a^n\,u[n]$	⟷	$\dfrac{\mathbf{a}\mathbf{z}(\mathbf{z}+\mathbf{a})}{(\mathbf{z}-\mathbf{a})^3}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
5	$\sin(\Omega n)\,u[n]$	⟷	$\dfrac{\mathbf{z}\sin(\Omega)}{\mathbf{z}^2 - 2\mathbf{z}\cos(\Omega) + 1}$	$\lvert\mathbf{z}\rvert > 1$
5a	$a^n\sin(\Omega n)\,u[n]$	⟷	$\dfrac{\mathbf{a}\mathbf{z}\sin(\Omega)}{\mathbf{z}^2 - 2\mathbf{a}\mathbf{z}\cos(\Omega) + \mathbf{a}^2}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
6	$\cos(\Omega n)\,u[n]$	⟷	$\dfrac{\mathbf{z}^2 - \mathbf{z}\cos(\Omega)}{\mathbf{z}^2 - 2\mathbf{z}\cos(\Omega) + 1}$	$\lvert\mathbf{z}\rvert >$
6a	$a^n\cos(\Omega n)\,u[n]$	⟷	$\dfrac{\mathbf{z}^2 - \mathbf{a}\mathbf{z}\cos(\Omega)}{\mathbf{z}^2 - 2\mathbf{a}\mathbf{z}\cos(\Omega) + \mathbf{a}^2}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$
7	$\cos(\Omega n + \theta)\,u[n]$	⟷	$\dfrac{\mathbf{z}^2\cos(\theta) - \mathbf{z}\cos(\Omega - \theta)}{\mathbf{z}^2 - 2\mathbf{z}\cos(\Omega) + 1}$	$\lvert\mathbf{z}\rvert > 1$
7a	$2\lvert\mathbf{a}\rvert^n\cos(\Omega n + \theta)\,u[n]$	⟷	$\dfrac{\mathbf{z}e^{j\theta}}{\mathbf{z}-\mathbf{a}} + \dfrac{\mathbf{z}e^{-j\theta}}{\mathbf{z}-\mathbf{a}^*}$, $\quad \mathbf{a} = \lvert\mathbf{a}\rvert e^{j\Omega}$	$\lvert\mathbf{z}\rvert > \lvert\mathbf{a}\rvert$

where $\mathbf{z}$ is a complex variable analogous to $e^{\mathbf{s}}$ for the Laplace transform.

The z-transform *transforms* $x[n]$ from the discrete-time domain symbolized by the discrete variable n to the z-domain, where, in general, $\mathbf{z}$ is a complex number. The inverse z-transform performs the reverse operation,

$$x[n] = \mathbb{Z}^{-1}[\mathbf{X}(\mathbf{z})]. \tag{7.59}$$

► The uniqueness property of the unilateral z-transform states that a causal signal $x[n]$ has a unique $\mathbf{X}(\mathbf{z})$, and vice versa. ◄

Thus,

$$x[n] \longleftrightarrow \mathbf{X}(\mathbf{z}). \tag{7.60}$$

7-6.2 Examples of z-Transform Pairs

Not all discrete-time signals have z-transforms. Fortunately, however, for the overwhelming majority of signals of practical interest, their z-transforms do exist. Table 7-5 provides a list of z-transform pairs of signals commonly used in science and engineering.