Problem 1

A man carries a key to his house, of course, but also has a spare key hidden under a bush in the yard. There is a probability of 0.05 that he will leave his key at work. There is a probability of 0.6 that his wife will not be home to let him in when he gets home from work. There is a 0.4 probability that one of his teenagers will borrow the key under the bush, since they are always losing their own keys. Find the probability that the man will get locked out of his house and have to sit on the front steps until one of his family returns. State assumptions.

Problem 2

A digital chip contains 100 transistors and 400 connections. The probability of a faulty transistor is 10^{-3}, and the probability of a faulty connection is 10^{-4}. What is the probability that a given chip taken at random is defective, assuming all defects are independent?

Problem 3

P1.3.24.

When it snows in Austin, there are three things that can happen at the University of Texas: (1) the university remains open and classes continue; (2) the university closes and classes are canceled for the day; and (3) the university opens at noon, and afternoon classes are held. Under the three conditions and for an afternoon class, the average engineering student will attend an afternoon class with probabilities of 0.9, 0.1 (didn't hear about it), and 0.7, respectively. On a snowy day and at an afternoon class, we know two probabilities: (1) there is a 60% chance that a student is in class; and (2) if the student is in class, there is a 50% probability that classes began at noon. For the sake of communication, let us agree on the following notation: C = classes cancelled, O = university open, H = half day, and S = student in class.

a. Find the probability that if a student is not in class, classes began at noon.
b. Find the probability that the university closes.

Pb 4

An electric circuit consists of a 100-Ω resistor in series with two 50-Ω resistors in parallel. If there is a 0.1 probability that each resistor is an open circuit and a 0.9 probability that each has its correct value, find the probability that the resistance of the combination is less than 200 Ω. Assume that all resistors fail independently.

Pb 5

The Coolsville Police Department has 30 police cars, 10 police motorcycles, and 5 police horses. Further, they have 200 policemen and 50 policewomen. You are at a party in Coolsville and a neighbor calls the police (which means the party is a success, right?). Assume a police officer and a means of transportation are selected independently and at random.

a. How many equally likely outcomes are there for the experiment?
b. What is the probability that a policeman arrives on a motorcycle?

Pb 6

There are 50 people in a class. What is the probability that all have separate birthdays? Ignore leap days.

Solution Pb 2

A1.3.23. 0.1307

1.3.23 solution. Define the events T_i = transistor i fails and C_j = connection j fails and D = chip fails. The relationship between D and the 100 T_i and 400 C_i events is complex because there are so many ways to have a defective chip. There are in fact $2^{100} \times 2^{400} - 1$ ways to have a defective chip. The "1" is the one way the chip can be non defective, that is, if all transistors and connections do not fail. Thus we can write $P[\overline{D}] = P[\overline{T_1} \cap ... \cap \overline{T_{100}} \cap \overline{C_1} \cap ... \cap \overline{C_{400}}]$. But

$P[\overline{T_i}] = 1 - 10^{-3}$ and $P[\overline{C_i}] = 1 - 10^{-4}$, and since all the failures are independent, we can transform $\cap \rightarrow \times$. The result is $P[D] = (0.999)^{100} \times (0.9999)^{400} = 0.869$. Therefore $P[D] = 1 - 0.869 = 0.1307$

Solution Pb 3

A1.3.24. **(a)** 0.3214; **(b)** 0.2679

1.3.24 solution. This problem mainly involves translating the information into probability equations and then manipulating these to get the required solutions. We read the second sentence to say $P[S \mid O] = 0.9$, $P[S \mid C] = 0.1$, and $P[S \mid H] = 0.7$. We read the third sentence to say $P[S] = 0.6$ and $P[H \mid S] = 0.5$. (a) Using the definition we have $P[H \mid \overline{S}] = \frac{P[H \cap \overline{S}]}{P[\overline{S}]} = \frac{P[\overline{S} \mid H]\, P[H]}{1 - P[S]}$. The conditional term in the numerator is $1 - P[S \mid H] = 0.3$, and the denominator is 0.4, so the equation reduces to $P[H \mid \overline{S}] = 0.75\, P[H]$. We can find the $P[H]$ from the last conditional probability given in the problem statement, $P[H \mid S] = \frac{P[S \mid H]\, P[H]}{P[S]} \Rightarrow P[H] = \frac{0.5 \times 0.6}{0.7}$. Put together, these give $P[H \mid \overline{S}] = 0.75 \times 0.429 = 0.3214$. (b) We can get $P[C]$ from the simultaneous solution of two equations: (1) $P[C] + P[O] + P[H] = 1$, since these are a partition, and (2) $P[S] = P[S \mid C]\, P[C] + P[S \mid O]\, P[O] + P[S \mid H]\, P[H]$, which expresses the law of total probability. We know all the terms in both equations except $P[C]$ and $P[O]$, and so we may solve these simultaneously, with the result $P[C] = 0.2679$.

Sol Pb. 4

A1.5.2. 0.891

1.5.2 solution. To have a resistance greater than 200 Ω, either the series resistor fails or both parallel resistors fail, or both. We will designate these S for the failure of the series resistor and B for the failure of both parallel resistors. From the problem statement we have $P[S] = 0.1$ and $P[B] = 0.1 \times 0.1 = 0.01$. Now the failure of the entire system would be $P[S \cup B] = 0.1 + 0.01 - 0.1 \times 0.01 = 0.109$. Thus the probability that the resistance of the combination would be less that 200 Ω would be 0.891

Sol p 5

1.4.22 solution. (a) The possibilities are $(30 + 10 + 5) \times (200 + 50) = 11,250$. (b) Of these possibilities 10×200 are men arriving on motorcycles; thus $\frac{10 \times 200}{11,250} = 0.1778$.

Sol p6

A1.4.29. 0.029626

1.4.29 solution. One approach is to treat the people as distinguishable. This gives 365^{40} outcomes in the experiment, which are the permutations with replacement. Of these $365 \times 364 \times \ldots \times (365-40+1) = \frac{365!}{(365-40)!}$, which are the permutations without replacement. The ratio is 0.108768190.