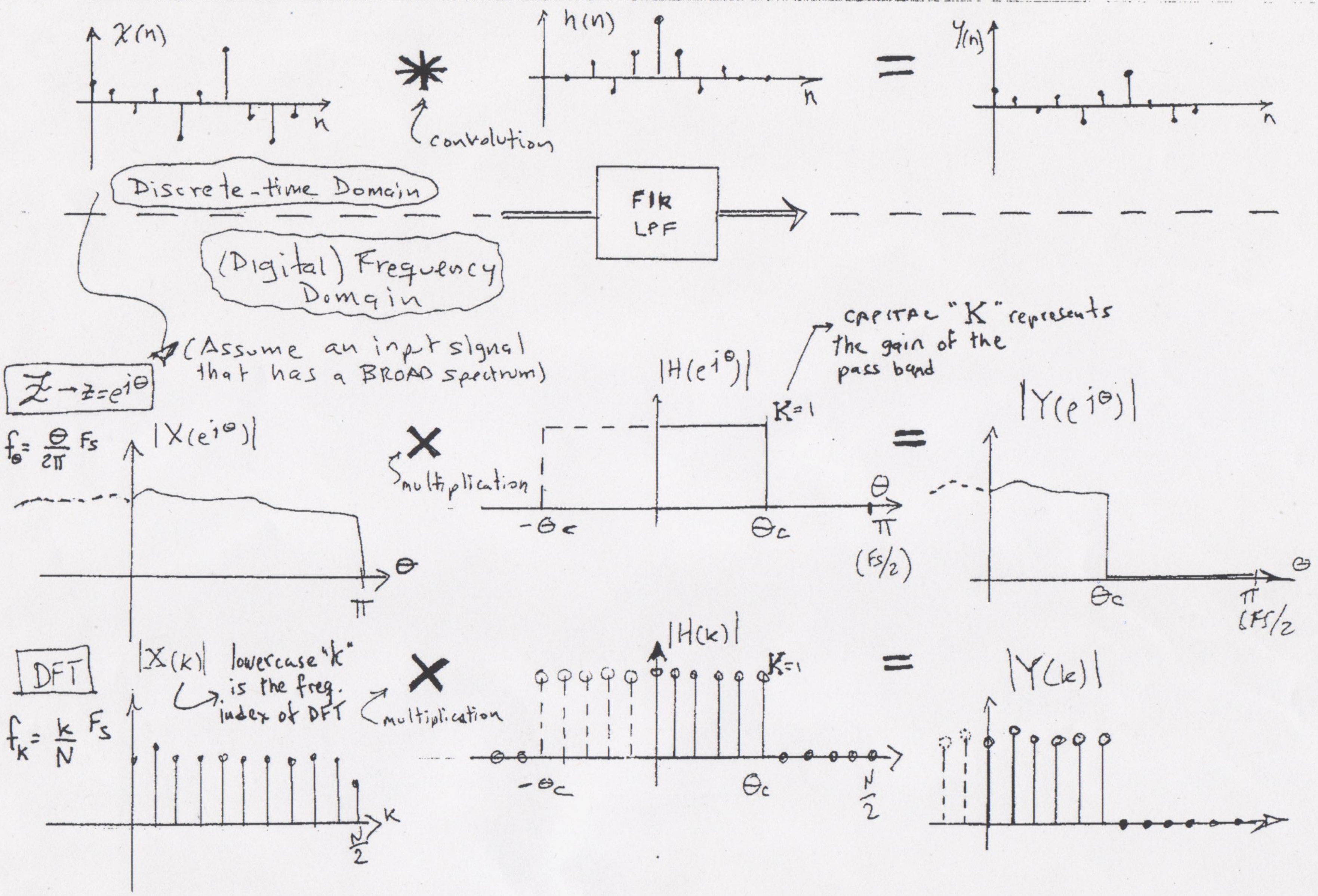
Finite Impulse Response (FIR) Filter Design

by the "Window" or "Fourier Series Coefficients" Method

► What do we need from a "LOW-PASS" FILTER (LPF):



Could we determine $h(n)$, i.e., the FIR filter coefficients $\{b_0, b_1, \ldots b_N\}$, from the $H(k)$ that we want to implement?

→ We know we want $|H(k)| = 1$ from $-\theta_c$ to $+\theta_c$ ✓

→ What about the phase of $H(k)$? – To start let's assume phase = 0.
(all $H(k)$ values are REAL)

⇒ UNDER THAT ASSUMPTION (i.e., $H(k)$ is a real, rectangular pulse, centered at $k=0$), the INVERSE DFT of that function is a SINC $\left(\frac{\sin x}{x}\right)$ function, that is re-scaled in discrete-time according to the desire cutoff frequency θ_c (pulse width)

BUT: a) the sinc function would also be centered at $n=0$ { "positive tail" and "negative tail"

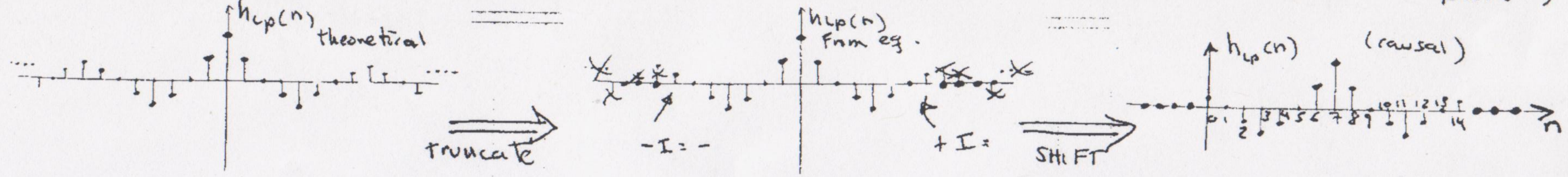
b) the sinc function is, theoretically, "infinite", in both directions, $n \to \infty$, $n \to -\infty$

SPECIFICALLY: $$\boxed{h_{LP}(n) = \frac{K}{\pi n} \sin(n\theta_c) \quad n = 0, \pm 1, \pm 2, \ldots \pm \infty}$$

To implement, practically, a) and b) need to be changed:

b*) → TRUNCATE the sequence to a finite number of terms: $2I+1$:
$$\boxed{h_{LP}(n) = \frac{K}{\pi n} \sin(n\theta_c) \quad n = 0, \pm 1, \pm 2, \ldots \pm I}$$

and a*) → SHIFT ALL THE COEFFICIENTS, "to the right (future" so that none of the coefficients are assigned to negative time indices (which would require a non-causal system)



Windowing

→ to have a frequency response that is as close as possible to the desired (designed) one, a window function, such as a Hanning Window could be applied to the FIR coefficients obtained after truncation and shifting.

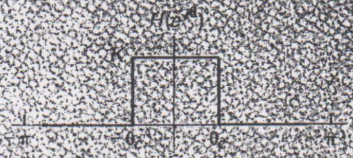
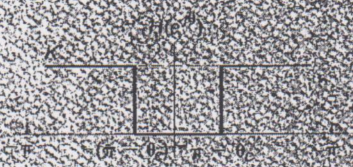
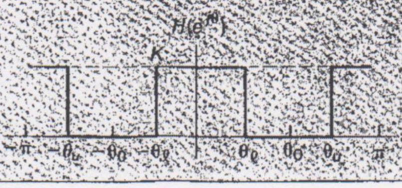
$$h_{LP\,final}(n) = h_{Lp}(n) \times W_d(n)$$

window function

Truncated, shifted FIR coefficients

TO DESIGN HPF, BPF, BSF FIRS: Use $\boxed{\text{FREQUENCY TRASFORMATIONS}}$

TABLE 9.1 FREQUENCY TRANSFORMATIONS

Lowpass		
$h_{LP}(n) = \frac{K}{\pi n}\sin(n\theta_c), \quad n = 0, \pm 1, \pm 2, \ldots, \pm I$		(9.60)
Highpass		
$h_{HP}(n) = (-1)^n h_{LP}(n), \quad n = 0, \pm 1, \pm 2, \ldots, \pm I$		(9.61)
Bandpass		
$h_{BP}(n) = [2\cos(n\theta_0)]h_{LP}(n), \quad n = 0, \pm 1, \pm 2, \ldots, \pm I$ $\theta_u - \theta_\ell = 2\theta_c,\ \theta_0 = \frac{\theta_u + \theta_\ell}{2}$		(9.62)
Bandstop		
$h_{BS}(0) = K - h_{BP}(0)$ $h_{BS}(n) = -h_{BP}(n), \quad n = \pm 1, \pm 2, \ldots, \pm I$ $\theta_u - \theta_\ell = 2\theta_c$ $\theta_0 = \frac{\theta_u + \theta_\ell}{2}$		(9.63)

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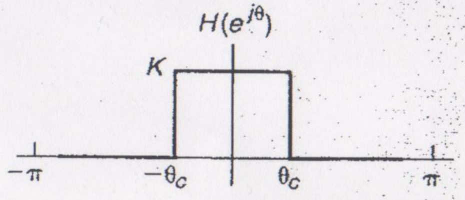
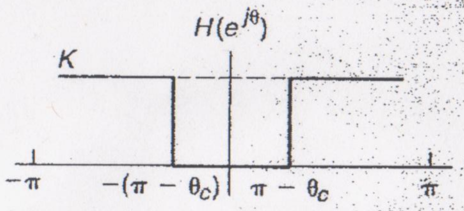
Lowpass

$$h_{LP}(n) = \frac{K}{\pi n}\sin(n\theta_c), \qquad n = 0, \pm 1, \pm 2, \ldots, \pm I \qquad (9.60)$$



Highpass

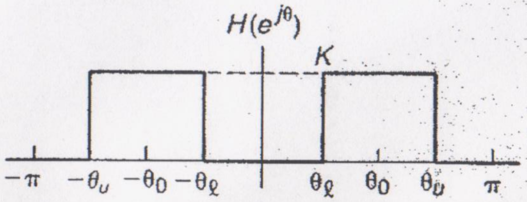
$$h_{HP}(n) = (-1)^n h_{LP}(n), \qquad n = 0, \pm 1, \pm 2, \ldots, \pm I \qquad (9.61)$$



Bandpass

$$h_{BP}(n) = [2\cos(n\theta_0)]h_{LP}(n), \qquad n = 0, \pm 1, \pm 2, \ldots, \pm I$$

$$\theta_u - \theta_\ell = 2\theta_c, \quad \theta_0 = \frac{\theta_u + \theta_\ell}{2} \qquad (9.62)$$



Bandstop

$$h_{BS}(0) = K - h_{BP}(0)$$

$$h_{BS}(n) = -h_{BP}(n), \qquad n = \pm 1, \pm 2, \ldots, \pm I$$

$$\theta_u - \theta_\ell = 2\theta_c$$

$$\theta_0 = \frac{\theta_u + \theta_\ell}{2} \qquad (9.63)$$

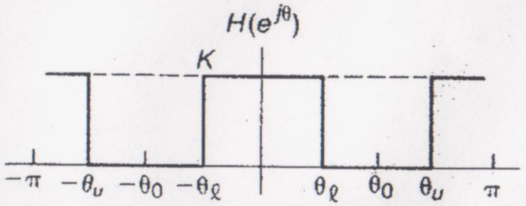


Table 9.1 – FREQUENCY TRANSFORMATIONS, from

First Principles of Discrete Systems and Signal Processing

Robert D. Strum & Donald E. Kirk - Addison-Wesley Publishing Co., 1989. ISBN: 0-201-09518-1

(GENERIC) Infinite Impulse Response (IIR) 2nd ORDER FILTERS :

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2)$$

In all cases : POLES at $re^{j\theta_0}$ and $re^{-j\theta_0}$

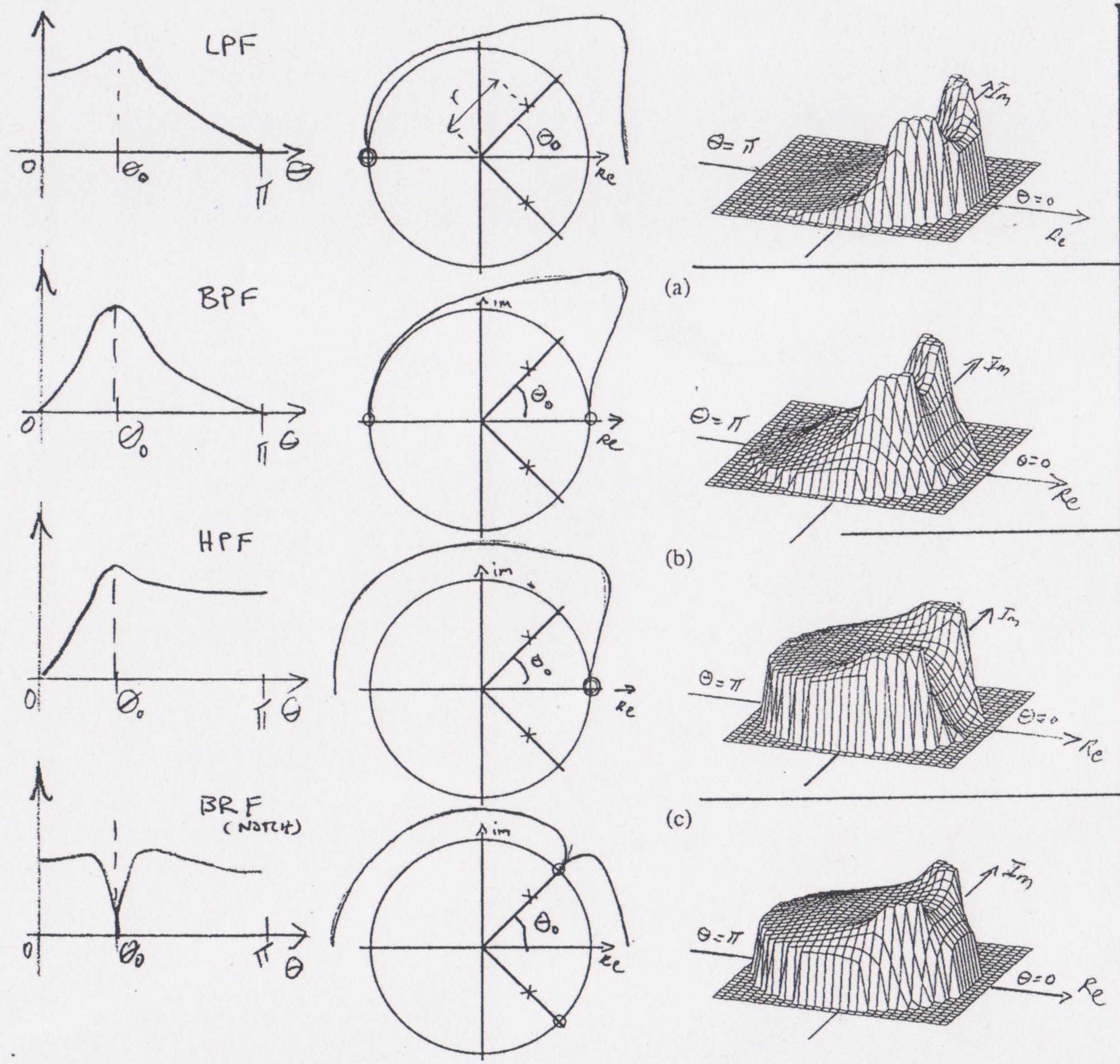
$$(z - re^{j\theta_0})(z - re^{-j\theta_0}) = z^2 - zr\,(e^{j\theta_0} + e^{-j\theta_0}) + r^2 e^{j(\theta_0 - \theta_0)}$$

$$= z^2 - zr\,(2\cos\theta_0) + r^2 \quad \text{SO : } \begin{cases} a_0 = 1 \\ a_1 = -2r\cos\theta_0 \\ a_2 = r^2 \end{cases}$$

HOW TO GET DENOMINATOR

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

HOW TO GET NUMERATOR



LOW-PASS FILTER (LPF)

zeros : $-1, -1$

$(z-(-1))(z-(-1)) = (z+1)(z+1)$

$= z^2 + 2z + 1$

$b_0 = 1$

$b_1 = 2$

$b_2 = 1$

BAND-PASS FILTER (BPF)

ZEROS : $-1, +1$

$(z-(-1))(z-1) = (z+1)(z-1)$

$= z^2 + 0z - 1$

$b_0 = 1$

$b_1 = 0$

$b_2 = -1$

HIGH-PASS FILTER (HPF)

ZEROS : $+1, +1$

$(z-1)(z-1) = z^2 - 2z + 1$

$b_0 = 1$

$b_1 = -2$

$b_2 = 1$

BAND-REJECT FILTER (BRF) - NOTCH

ZEROS : $e^{j\theta_0}, e^{-j\theta_0}$

$(z - e^{j\theta_0})(z - e^{-j\theta_0})$

$= z^2 + z\,(-2\,\text{real}\,e^{j\theta_0}) + 1$

$= z^2 + z\,(-2\cos\theta_0) + 1$

$b_0 = 1$, $b_1 = -2\cos\theta_0$, $b_2 = 1$